Sixth Form Transition Tasks September 2022

Courses at A Level and BTEC are significantly different to GCSE and provide exciting opportunities for independent learning.

To support our application process, we have put together a Transition Programme which aims to prepare you for your time with us. Completion of these tasks will be used to assess your suitability to study with us from September. Our Sixth Form staff have designed tasks and activities to provide you with an insight into life in the Sixth Form. It is vital that you choose courses that interest you, courses that enable you to progress onto your chosen post-18 pathway, and courses where you will want to work independently and challenge yourself to improve your own understanding. The jump from Year 11 to Year 12 is a big one. Whilst September may seem a long way off, it is important that you are fully prepared for your A Level/BTEC studies in order that you can start as you mean to go on.

These tasks are important because they:

- provide an introduction to key ideas within the course.
- help to bridge the gap between GSCE and A Level/BTEC.
- encourage students to think like an A-level/BTEC student
- demonstrate a commitment to study and subject teachers will expect that tasks are completed to a high standard.

Welcome from the Sixth Form Team

All students will be cooperative, confident, ambitious and successful regardless of their background. We expect students to meet academic challenges, discover their passions, develop confidence, contribute positively to society, be resilient and become independent thinkers prepared for successful and fulfilling lives.

Your last few years of secondary school may not have been how you envisaged, but take this opportunity to start the next chapter in your education. You may have just sat your GCSEs, but you will still be sitting your BTEC and A Level exams in the summer of 2024! With this in mind, there is no time like the present to start preparing for these.

Sixth Form is the gateway to life beyond school, and the choices you make are some of the most important you will face. You have made an important decision to study at Gildredge House Sixth Form. You will find aspects of both teaching and learning in the Sixth Form to be different from the way your learning was organised throughout Key Stage 4.

It is imperative that students arrive in September with a positive mind-set and all transition tasks and activities are completed to a high standard. Regardless of GCSE results, the tasks outlined in this booklet will provide you with an opportunity to demonstrate your suitability for your chosen courses. Sixth Form staff will have high expectations of students and the jump from GCSE courses to A Levels/BTEC is significant. The Sixth Form staff will be there to support students fully with this transition. This transition period will inform staff of your ability to cope with the demands of the subjects.

Your time in the Sixth Form will go by very quickly, look how the last five years have flown, so you need to hit the ground running! We aim to do all that we can to support you with making outstanding progress during your post-16 studies. In your two years with us, we hope you develop the confidence and skills to flourish at University and in the workplace. We are here to support, guide and elevate you to fulfil your ambitions and achieve your potential.

Meet the team

Miss Jemma Graffham
Director of Sixth Form

j.graffham@gildredgehouse.org.uk

Mrs Karen Maxwell
Head of Year 12

k.maxwell@gildredgehouse.org.uk

Miss Francesca O'Callaghan
Head of Year 13

f.ocallaghan@gildredgehouse.org.uk

Transition Tasks

Summer Transition Tasks

For each subject that you intend to study, we ask you to complete the tasks set and bring them with you to your first lesson of each subject in September. Each task asks you to research or explore something, and to complete some work which will challenge your existing skills. You may find these activities difficult, but please give them your best attempt. This transition work will help you make a flying start to Sixth Form, as well as providing teachers with information to help understand your level of knowledge, skill and ability. Links to subject tasks can be found on our website.

These tasks are important because they:

- provide an introduction to key ideas within the course.
- help to bridge the gap between GSCE and A Level/BTEC.
- encourage students to think like an A Level/BTEC student
- demonstrate a commitment to study and subject teachers will expect that tasks are completed to a high standard.

Courses at A Level and BTEC are significantly different to GCSE and provide exciting opportunities for independent learning. The first challenge for any student joining Gildredge House Sixth Form or moving from Year 11 into Year 12 is to successfully complete a summer preparation task for each of their chosen subjects.

Option blocks

As you are aware, not all subjects outlined in our Prospectus will be running in September 2022. However, I am delighted to be able to share with you the courses we are running, and the option blocks these fall within.

Block A	Block B	Block C	Block D	Block E
Biology English Literature Further Maths	Chemistry English Language Computer Science	Physics Psychology	Maths History Sociology Business BTEC Geography	BTEC Sport X3 single X6 double X9 triple

Students can select any subject from each block. The majority of our students choose to study the equivalent of three A Levels; however, we do have students who have opted to study four. Subjects within each block run at the same time, therefore students cannot study multiple subjects from the same block.

Staff Contact

Biology	Mr C Applegate	c.applegate@gildredgehouse.org.uk
Business	Mrs S Edwards	s.edwards@gildredgehouse.org.uk
Chemistry	Mr B Young	b.young@gildredgehouse.org.uk
Computer Science	Mr K Kenth	k.kenth@gildredgehouse.org.uk
English Literature English Language	Mrs T Newby	t.newby@gildredgehouse.org.uk
Further Maths	Mrs S Wilshire	s.wilshire@gildredgehouse.org.uk
Geography	Mrs K Nicolle	k.nicolle@gildredgehouse.org.uk
History	Mr G Ince	g.ince@gildredgehouse.org.uk
Mathematics	Mrs J Dowle	j.dowle@gildredgehouse.org.uk
Physics	Mr M Alker	m.alker@gildredgehouse.org.uk
Psychology	Ms H Kitching	h.kitching@gildredgehouse.org.uk
Sociology	Mr C Rogers	c.rogers@gildredgehouse.org.uk
Sport	Mr T Addems	t.addems@gildredgehouse.org.uk

Subject	Qualification	Examination Board
Biology	A Level & Practical Endorsement	Edexcel A (SNAB)
Additional Information:		

Task Overview:

Research and produce four resources to demonstrate your knowledge of (1) circulatory system and cardiovascular disease (2) Movement of substances across a membrane (3) structures and functions of the cell membrane (4) Species and evolution

Success Criteria:

The first resource should cover:

- A diagram of a basic components of the cardiovascular system
- Explain any functions the parts of the cardio vascular system have and how they are adapted for their roles in the system.
- Explain what factors can affect the function of the cardiovascular system. o What is cardiovascular disease?

The second resource should cover:

- Describe the processes of osmosis, diffusion and active transport.
- Compare and contrast each process.
- Explain when each process could be used in a human or plant.

The third resource should cover:

- Diagrams of Eukaryotic, prokaryotic cells.
- Explain the functions of the cell organelles (structures).
- Describe the process of protein synthesis and how we make protein in the body.
- Explain how mutation can cause problems with making these proteins.

The fourth resource should cover:

- Explain what biodiversity is and why it is important to our ecosystem.
- Describe how different species are grouped together and identified.
- Explain why the change in the environment is driving natural selection.

Resources:

https://www.bbc.com/bitesize/examspecs/zcq2j6f
https://www.bbc.co.uk/bitesize/subjects/zm6tyrd -more like A LEVEL standard.
Any existing revision guides or textbooks you own.
Your own (super intelligent) minds.

How will the work produced will fit into subsequent work and the specification as a whole?

These topics lead into the first topics studied at A level and provide a foundation for the rest of Biology

How should the work should be presented?

You may choose to present the work as you wish. Suggestions include a poster, information sheets, leaflets or sample textbook pages.

Who should you contact if you should require further assistance with the work before the end of term?	Mr C Applegate | Gildredge House, Head of Biology c.applegate@gildredgehouse.org.uk
Length of time expected to complete tasks:	12 Hours

Submission Requirements:	Bring in paper copy at the start of term.

What equipment will be needed for the subject?
Standard stationery, a calculator, the course textbook: Salters-Nuffield AS/A level (SNAB) Biology; A Level (book 1-AS) Pearson ISBN: 978-1-4479-9100-7
Optional Extension Task/Further Reading
Extension work: a) What is the difference between Systole and Diastole? Link this to the cardiac cycle. b) What is a risk factor for heart disease? c) What are the features of a good study? d) Explain the what causes the symptoms of Cystic fibrosis. e) What are the different types of stem cell and suggest ethical issues with using of medical research

Transition Task: Business BTEC

Subject	Qualification	Examination Board
Business	BTEC	AQA
Additional Information:		

Task Overview:
Task 1: Make a grid and record all purchases for a week. Include: • The item • How often your purchase that item • What influenced your purchase • Score purchase out of ten after purchase • Give Reason Task 2: Go back to task 1 three weeks later and re-score all items and state whether you purchase again and why. Has your score changed and if so why? Task 3: You will be running enterprise events during Year 12. Make a list of possible ideas and then choose your best. Research cost of purchase resources and items, think about how you would price items and why, work out how much profit per item you would make (sales price- cost of each item to purchase), think about what your challenges might be and what could help to make it a success. Task 4: Design a questionnaire that you could use to ask whether people would purchase your item in Task 3 and check whether they would pay what you plan to charge. Task 5: Watch the news every day, each week pick one news article and pick a business that could use the story as a business opportunity, explain what the business could do. (eg. Covid 19 - Tesco could increase its capacity for home delivery for isolated people) Task 6: Write down all the skills that you think someone needs to be successful in business, self-assess yourself to see which you think you have. Task 7: Find three job adverts in businesses that you would one day like to do and explain why, think about what you need to achieve to be able to do that job. Task 8: Find ten adverts (make links to them if video format or copy if image) that you think make good adverts, explain why with each. Task 9: Write a list of your ten favourite and ten least favourite brands and explain why.
Success Criteria:
Ability to share work for all of the tasks and discuss your thoughts and explain the decisions that you made. Ability to run a successful enterprise event in Year 12.
Resources:
Access to the internet and news.

How will the work produced will fit into subsequent work and the specification as a whole?	
All tasks link in with topics learnt on the specification and will give a good contextual understanding of topics explored through the course.	
How should the work should be presented?	
Digitally.	
Who should you contact if you should require further assistance with the work before the end of term?	k.kenth@gildredgehouse.org.uk
Length of time expected to complete tasks:	2 hours per task
Submission Requirements:	Start of the course

What equipment will be needed for the subject?
Pens, Paper, Calculator, Textbook (Details to follow), Laptop (ideally)
Optional Extension Task/Further Reading
Watch the business news every day.

Subject	Qualification	Examination Board
Chemistry	A Level & Practical Endorsement	AQA

Task Overview:
Summer transition work to prepare you for A level Chemistry Complete task 1 and a minimum of any **three** of the other tasks below

Success Criteria:

Task 1: Structure and bonding

- Produce separate resources showing each type of compound:
    - Ionic, metallic, simple covalent and giant covalent
- For each include
    a. How electrons are shared/given/taken
    b. What forces there are between the particles
    c. The structures formed
    d. The general properties
    e. Dot and cross diagrams (not for metallic or giant covalent)

Task 2: The history of atoms

- Produce an annotated timeline with diagrams showing how ideas about the atom have changed over time
    - Include ideas such as the Dalton model, and the plum pudding model, how these ideas were developed and why they changed.

Task 3: Atomic structure

- Produce a poster or other resource showing:
    - The types and properties of sub-atomic particles
    - How the mass number and atomic number can be used to find information about an atom
    - Electron configuration

Task 4: The Periodic table

- Read and research Mendeleev's periodic table and summarise the key concepts used
- Read and research the **modern** periodic table and summarise the key concepts used
- How electron configuration is linked to the position of the element on the modern periodic table
- Make a table comparing the similarities and differences between Mendeleev's table and the modern table

Task 5: Laboratory techniques

- By researching online produce a simple flowchart outline the main steps involved in the following procedures:
    a. Preparation of a standard solution
    b. Carrying out a simple acid-base titration

Task 6: Amount of substance

- Research and produce a resource showing what the following terms mean and how they can each be calculated or worked out:
    - Mass number
    - Relative atomic mass
    - Relative formula mass
    - Moles
        - Include how this could be calculated from:
        i. Mass and relative mass
        ii. Concentration and volume
        iii. Volume of gas at room temperature and pressure
    - Reacting mass/theoretical yield

- Empirical formulae
- Atom economy
- Actual yield
    - You would have to measure this directly, rather than calculating it
- Percentage yield

Task 7: Further reading

- By using the AQA Chemistry textbooks, or any other suitable resources (see below, Seneca is excellent) read around the following topics:

a. Structure and bonding
b. Electron shells, sub-shells and orbitals
c. Moles and mass calculations
d. The ideal gas equation
e. Electronegativity
f. Intermolecular forces
    - Limited to Van der Waals forces, dipole attractions and hydrogen bonding.

Task 8: Seneca, reading A level content

- Use https://www.senecalearning.com/
- Register for an account if you do not already have one and use the Chemistry AQA A Level unit to start reading and preparing for the first units – we will start with physical chemistry.

Resources:

- https://www.bbc.com/bitesize/examspecs/zy984j6
- https://www.senecalearning.com/ (Chemistry: Edexcel *(or AQA)* GCSE Higher)
- AQA A-level chemistry unit on seneca
- Required textbook for the course: ***AQA Chemistry, A Level (Year 1 and AS), 2nd Edition. Ted Lister and Janet Renshaw. Oxford press.***
- Any existing revision guides or textbooks you own.

How the work produced will fit into subsequent work and the specification as a whole?

These topics lead into the first units studied at A level and provide a foundation for the rest of the Chemistry A level.

How the work should be presented?

Your choice of presentation but must be able to show evidence that the work has been completed (screenshots or photos of completed work are fine)

Who to contact if you should require further assistance with the work before the end of term?	Mr B Young, Head of Chemistry b.young@gildredgehouse.org.uk
Length of time expected to complete tasks:	1-2 hours per task
Submission Requirements:	Bring evidence with you to the first lessons in September

What equipment will be needed for the subject?

Standard stationery, a calculator, the course textbook
(AQA Chemistry; A Level – Year 1 and AS. Oxford University Press)

Gildredge House

Subject	Qualification	Examination Board
Computer Science	A Level	OCR
Additional Information:		

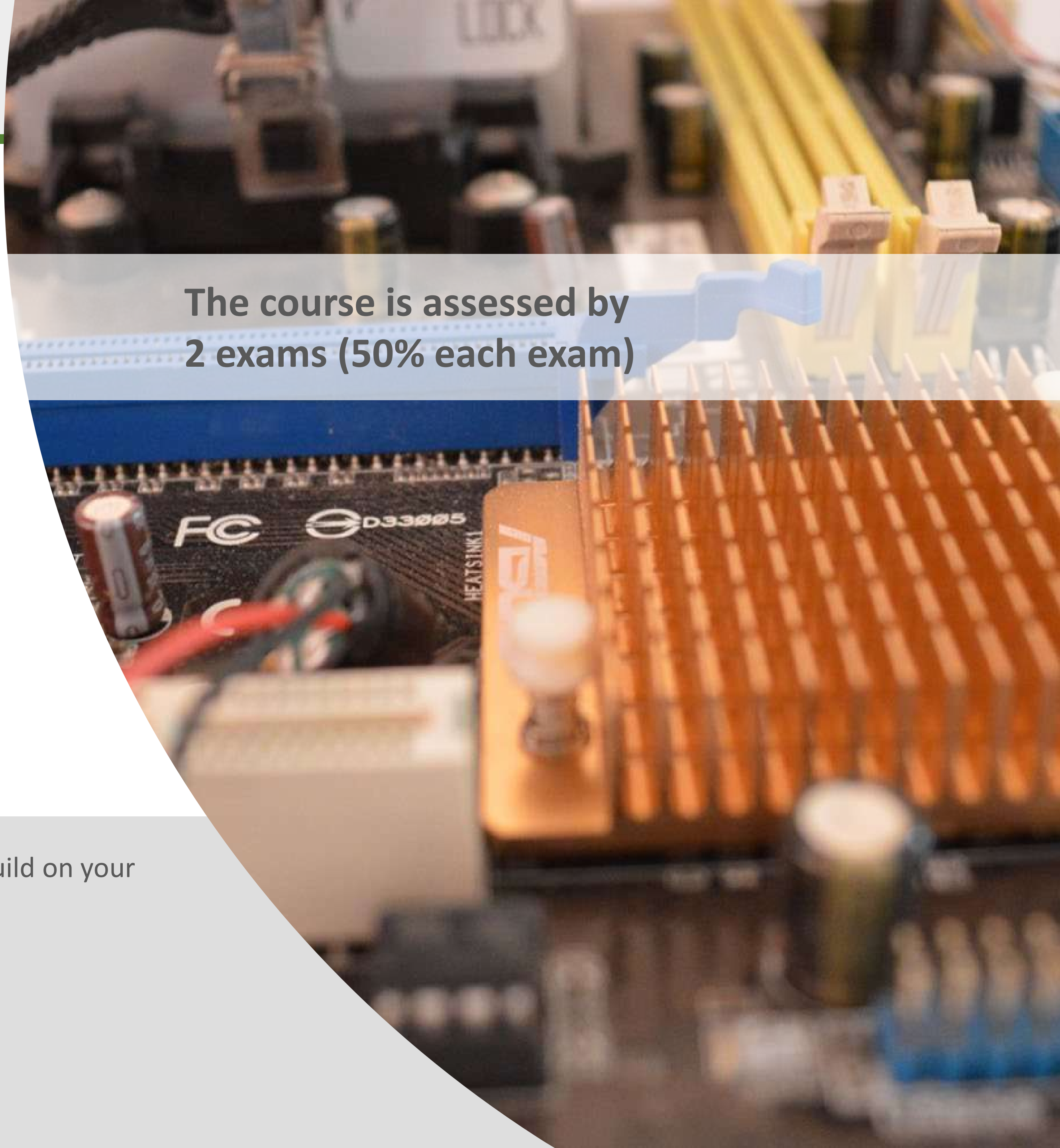
Task Overview:
Work this transition workbook, it covers many of the topics on the course.
Success Criteria:
Completion of all tasks in workbook.
Resources:
Transition workbook
How will the work produced will fit into subsequent work and the specification as a whole?
Ensures that all students have an understanding of what the course is about. The workbook covers many of the topics covered in the course, you will have a chance to review some of the theory and topics that will be delivered and have a chance to do some wider reading.
How should the work should be presented?
Students should work through the workbook and complete sections of it in the workbook.

Who should you contact if you should require further assistance with the work before the end of term?	Mr K Kenth : Head of Computer Science k.kenth@gildredgehouse.org.uk Mr G Newport: Teacher of Computer Science g.newport@gildredgehouse.org.uk
Length of time expected to complete tasks:	50 hours to complete all tasks
Submission Requirements:	Bring completed work on USB or online file store when completed

What equipment will be needed for the subject?
Title: OCR A Level Computer Science Author: George Rouse, Jason Pitt and Sean O'Byrne Publisher: Hodder Education ISBN-10: 1471839761 ISBN-13: 978-1471839764
Optional Extension Task/Further Reading
Read the Introduction to Computing and Study Hints sections of the OCR A Level Computer Science Text Book pages 1 - 10.

Computer Science Transition workbook

- The topic of **Computer Science** is at the heart of the modern world
- Studying it can make you extremely sought after in todays job market
- The transition from GCSE to A level is significant, this includes:
  - An increased emphasis on **technical content**
  - An increased emphasis **independent research**



The course is assessed by 2 exams (50% each exam)

This workbook is designed to allow you to practice some of these skills and build on your existing knowledge.

Please complete by your first lesson back in September.

1

"Tell me about yourself"

Craig'n'Dave

Why did you choose Computer Science?

Expected time to complete: ½ hour

In this simple task you get the opportunity to tell me your choices and reasons behind choosing to study Computer Science. Please answer all questions as best you can.

1. Why did you choose to study A level Computer Science?

2. What other courses have you chosen to study at Key Stage 5, and what made you choose this combination?

3. What are you hoping to achieve from studying Computer Science?

4. How would you describe yourself as a learner at GCSE? What skills where you good at, what areas would you like to improve on?

5. What are your other hobbies and interests outside of school? Anything related to Computing?

Emerging computer technology

Expected time to complete: 2 hours

In this task you get to investigate any area of emerging computer technology which interests you.

You can pick any area which interests you, but examples could be:

- Artificial intelligence
- Robotics
- Automated self driving cards
- Quantum computing

In no more than ONE side of A4 summarise the area you have chosen under the following four headings:

1. What is it?
2. What are the possible Social, Moral, Cultural and Ethical **benefits** of this technology on society
3. What are the possible Social, Moral, Cultural and Ethical **risks** of this technology on society
4. My conclusion on this technology and what it will mean for our world 10 years from now

Additional help:

For additional help and support in structuring your answer you might like to watch some of the videos from the following Craig 'n' Dave playlists:

OCR:
SLR 17 – Ethical, morale and cultural issues
https://student.craigndave.org/videos/slr-17-ethical-moral-and-cultural-issues

Thinking like a computer

Expected time to complete: 2 hours

At the heart of Computer Science is the ability to look at problems, analyse them, break them down and solve them in a way which involves a variety of "Computational Thinking" skills.

1. Download the "Computational thinking and Computational methods placemats" from Craig n Dave:
   - https://student.craigndave.org/specification-key-terminology-and-cheat-sheets

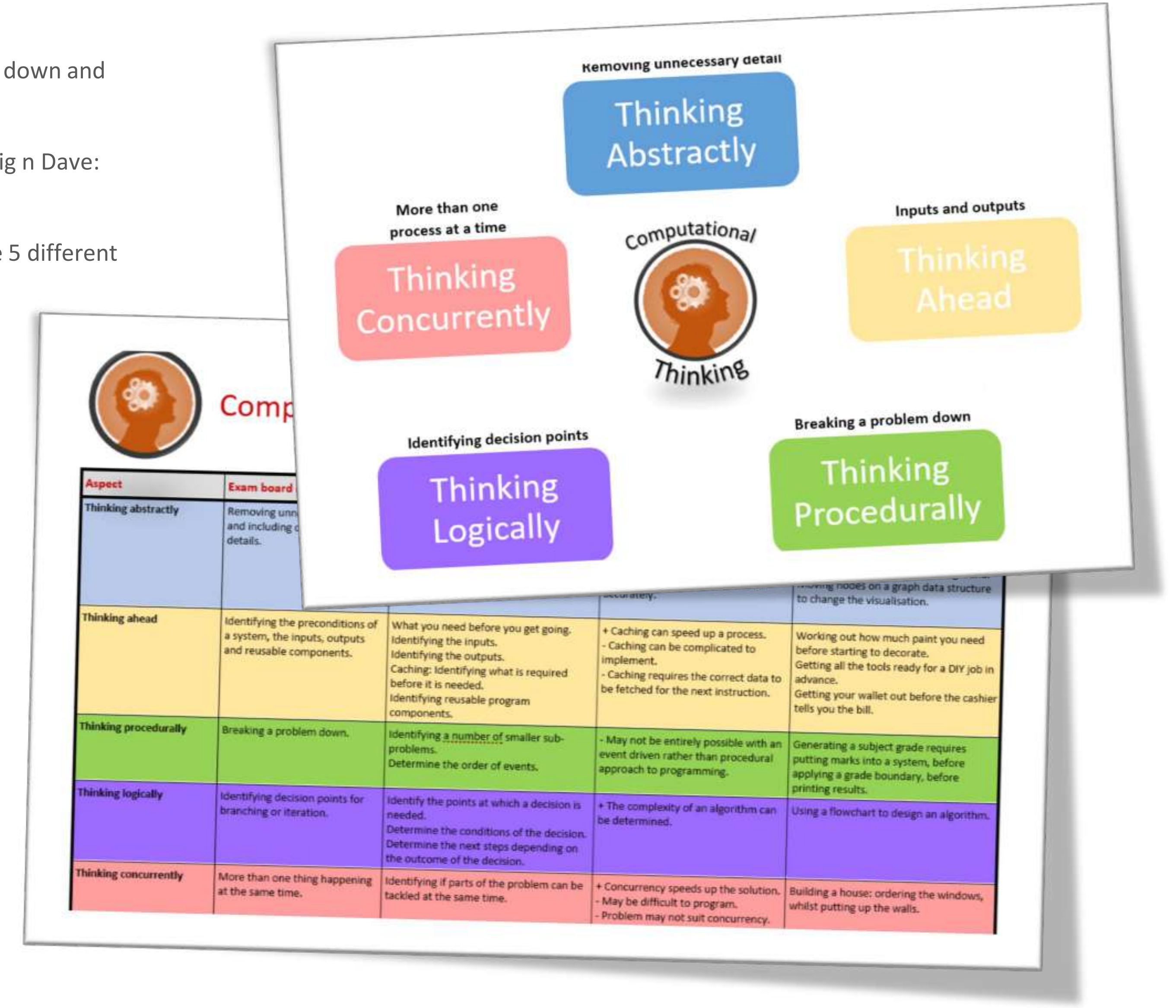
2. Create your own spider diagram / mind map which shows your clear understanding of the 5 different computational thinking strands
   - Keep it to a single side of A4 / A3

3. Your goal is to imagine someone else has to revise from your mind map. Ask yourself:
   - Does it make sense?
   - Is it clear?
   - Does it cover all of the important concepts?

Note:

Although the five strands listed (and the download resources provided for this task) are from the OCR AS / A'Level specification, the concepts of "Computational Thinking" are just as applicable to the AQA course.

Indeed many of the strands listed are explicitly covered in the AQA specification in different locations.



4 Note taking practice task

Craig'n'Dave

The Cornell method of note taking

Expected time to complete: 1½ hours

The expectation to do independent research at A Level will increase dramatically from GCSE.

There is a real skill to taking decent notes outside of lesson which are of value. Research has proven that one of the most effective methods is the "Cornell" note taking method.

1. To start download the "Cornell note taking template" from Craig n Dave:
   - https://craigndave.org/product/cornell-note-taking-template/

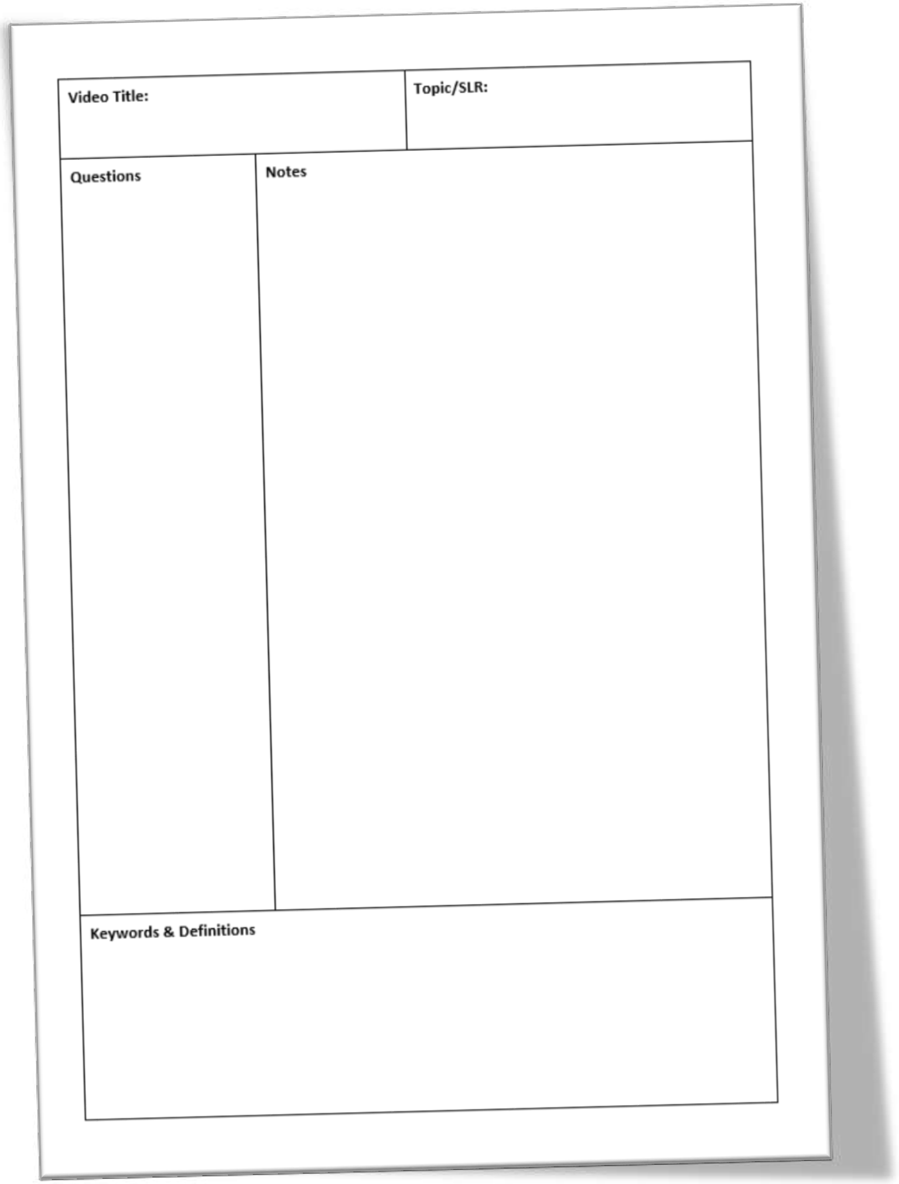
2. Pick any two of the following videos from Craig 'n' Dave:
   - OCR: https://student.craigndave.org/videos/ocr-alevel-slr01-alu-cu-registers-and-buses
   - OCR: https://student.craigndave.org/videos/ocr-alevel-slr04-paging-segmentation-and-virtual-memory
   - OCR: https://student.craigndave.org/videos/ocr-alevel-slr05-stages-of-compilation
   - OCR: https://student.craigndave.org/videos/ocr-alevel-slr14-data-structures-part-2-graphs

3. Write the title of the video and its topic in the top boxes (use a different sheet for each video).

4. In the main "Notes" section, write notes from the video. You can do this in any way you like, a suggestion might be to rewind slightly when the canvas changes, thinking carefully about what was important in the previous few minutes.

5. Having recorded the notes, review them:
   - Turn each part into a question in the section on the left.
   - For example, the notes may say, "The value of the program counter is passed to the memory address register".
   - The question then becomes, "which register is the value of the program counter passed to?"
   - Sometimes these questions are easy, and at times they are more difficult to write.
   - There may also be more than one valid question.
   - You will need to decide for yourself which are the most appropriate questions for revision.

6. Finally pull out all the key words and their definitions words the notes and list them in the bottom section.



5 Key terms task

Craig'n'Dave

Getting to grips with terminology

Expected time to complete: 2 hours

An important aspect of being successful with your study of Computer Science is getting to grips with subject related terminology. There are over 240 specific terms you will need to learn!

Below are a handful of the key terms you will need to become familiar with.

Control Unit	**Register**	**Busses**
Von Neuman Architecture	**Optical Storage**	**Operating System**
Intermediate Code	**Device Driver**	**Compiler**
Assembly Language	**Machine Code**	**Lossy Compression**
Hashing	**Normalisation**	**TCP/IP Stack**
Packet Switching	**ASCII**	**Problem Decomposition**

1. Research each of the key terms and write a definition.
2. Resist the urge to simply cut and paste a definition from the first website you find. Many definitions found on The Internet are overly complicated and wordy.
3. Ask yourself:
   - Does my definition make sense?
   - Is it succinct, to the point?
   - Does the definition have appropriate depth and detail for A'Level?
   - Could I give this definition to another student so they could revise from it?

6 An introduction to the basics of programming tasks

Craig'n'Dave

Programming basics

Expected time to complete: 6 hours

Learning to "code" is a fun and essential part of A Level Computer Science. This task is ideal if you haven't done the GCSE in Computer Science or you simply want a nice refresher ahead of starting your A Level course.

1. Head over to the web site: https://www.learnpython.org/
2. Complete the following python tutorials under the heading:
   - Hello, World!
   - Variables and Types
   - Lists
   - Basic Operators
   - String Formatting
   - Basic String Operations
   - Conditions
   - Loops
   - Functions
3. Each section presents you with theory, code to run and exercises to try out.
4. If you want to practice writing your own python programs you can download and install a simple python development tool here: https://www.python.org/downloads/

Additional note:

This task is most suited to students who intend to do the A Level and have not previously gained much / or any programming experience from the GCSE Computer Science course.

Although the language chosen here is Python, and that may not be what you will be using at A Level, it is the underlying programming concepts which are important.

The list of topics above cover the standard set of programming concepts you would be expected to know having completed a GCSE and Computer Science and so will prepare you well for the A level.

7 Critical thinking task

Craig'n'Dave

Why is Computer Science important?

Expected time to complete: 2 hours

It is easy to say, "Computer Science is essential in todays world", but are you able to think critically about this statement and back it up? "Thinking Critically" is an essential skill at A Level.
It involves you:

- Looking at a topic / concept in depth
- Taking account of different views / perspectives
- Considering positives and negatives
- Evaluating links and effects on other concepts
- Drawing your own conclusions backed up with evidence

1. On the following slide answer the questions:
    - What is Computer Science?
    - What are the benefits and risks of Computer Science at a local level (think about your local community / town / city / county)
    - What are the benefits and risks of Computer Science at a national level
    - What are the benefits and risks of Computer Science at a global level

Additional help:

For additional help and support in structuring your answer you might like to watch some of the videos from the following Craig 'n' Dave playlists:

OCR:
SLR 17 – Ethical, morale and cultural issues
https://student.craigndave.org/videos/slr-17-ethical-moral-and-cultural-issues

Critical thinking task

Why is Computer Science important?

Expected time to complete: 2 hours

•What is Computer Science?	• Enter your answer here
What are the benefits and risks of Computer Science at a local level	• Enter your answer here • Try to make at least 4 valid points • At least 2 of your points should be about the potential risks of Computer Science • At least 2 of your points should be about the potential benefits of Computer Science
•What are the benefits and risks of Computer Science at a national level	• Enter your answer here • Try to make at least 4 valid points • At least 2 of your points should be about the potential risks of Computer Science • At least 2 of your points should be about the potential benefits of Computer Science
•What are the benefits and risks of Computer Science at a global level	• Enter your answer here • Try to make at least 4 valid points • At least 2 of your points should be about the potential risks of Computer Science • At least 2 of your points should be about the potential benefits of Computer Science

Craig'n'Dave

Augmented reality

Expected time to complete: 1½ hours

A key skill at A Level is being able to take a topic and then discuss it in the context of different scenarios.

Most theory-based exam questions will be asked in the form of a scenario, simply regurgitating what you know on the topic without contextualising your answer to the scenario will often result in low marks!

The topic for this exercise is “Augmented Reality”. It is a truly fascinating area of technology which has the potential to change almost every aspect of our daily lives.

Watch this brief video to learn more:
https://www.youtube.com/watch?v=vQtwWzfzKXI

After watching the video complete the next slide which asks you to discuss the benefits, limitations and risks of augmented reality in the context of:

- Medicine & health care
- Gaming & entertainment
- Schools & learning
- Travel & tourism
- Social media
- Transport & navigation

8 Applying technical knowledge in context task

Craig'n'Dave

Augmented reality

Expected time to complete: 1½ hours

•Medicine & health care

Discuss the **benefits**, **limitations** and **risks** of augmented reality in this context:

- Enter your answer here

•Gaming & entertainment

Discuss the **benefits**, **limitations** and **risks** of augmented reality in this context:

- Enter your answer here

•Schools & learning

Discuss the **benefits**, **limitations** and **risks** of augmented reality in this context:

- Enter your answer here

•Travel & tourism

•Discuss the **benefits**, **limitations** and **risks** of augmented reality in this context:

- Enter your answer here

•Social media

•Discuss the **benefits**, **limitations** and **risks** of augmented reality in this context:

- Enter your answer here

•Transport & navigation

•Discuss the **benefits**, **limitations** and **risks** of augmented reality in this context:

- Enter your answer here

Looking under the hood of the processor

Expected time to complete: 2 hours

The CPU "Central Processing Unit" is the central core of any computer system. You will study what it contains and how it works it in depth at A Level.

1. Start by watching the following 3 videos from Craig 'n' Dave (choose from OCR or AQA exam board)
    1. **OCR:** https://student.craigndave.org/videos/ocr-alevel-slr01-alu-cu-registers-and-buses
    2. **OCR:** https://student.craigndave.org/videos/ocr-alevel-slr01-fetch-decode-execute-cycle
    3. **OCR:** https://student.craigndave.org/videos/ocr-alevel-slr01-performance-of-the-cpu

2. Produce a fully annotated diagram on a single sheet of A4 / A3 paper which shows how the CPU works.

3. Make sure the diagram includes and covers:
    - Major CPU components and what they are for:
        - Arithmetic Logic Unit (ALU)
        - Control Unit (CU)
        - Cache
    - The main registers
        - Program Counter (PC)
        - Memory Address Register (MAR)
        - Current Instruction Register (CIR)
        - Memory Data/Buffer Register (MDR / MBR)
    - Fetch-decode-execute cycle
    - Include annotations which explain how the performance of a CPU can be improved by:
        - Increasing the clock speed
        - Increasing the cache size
        - Increasing the number of cores

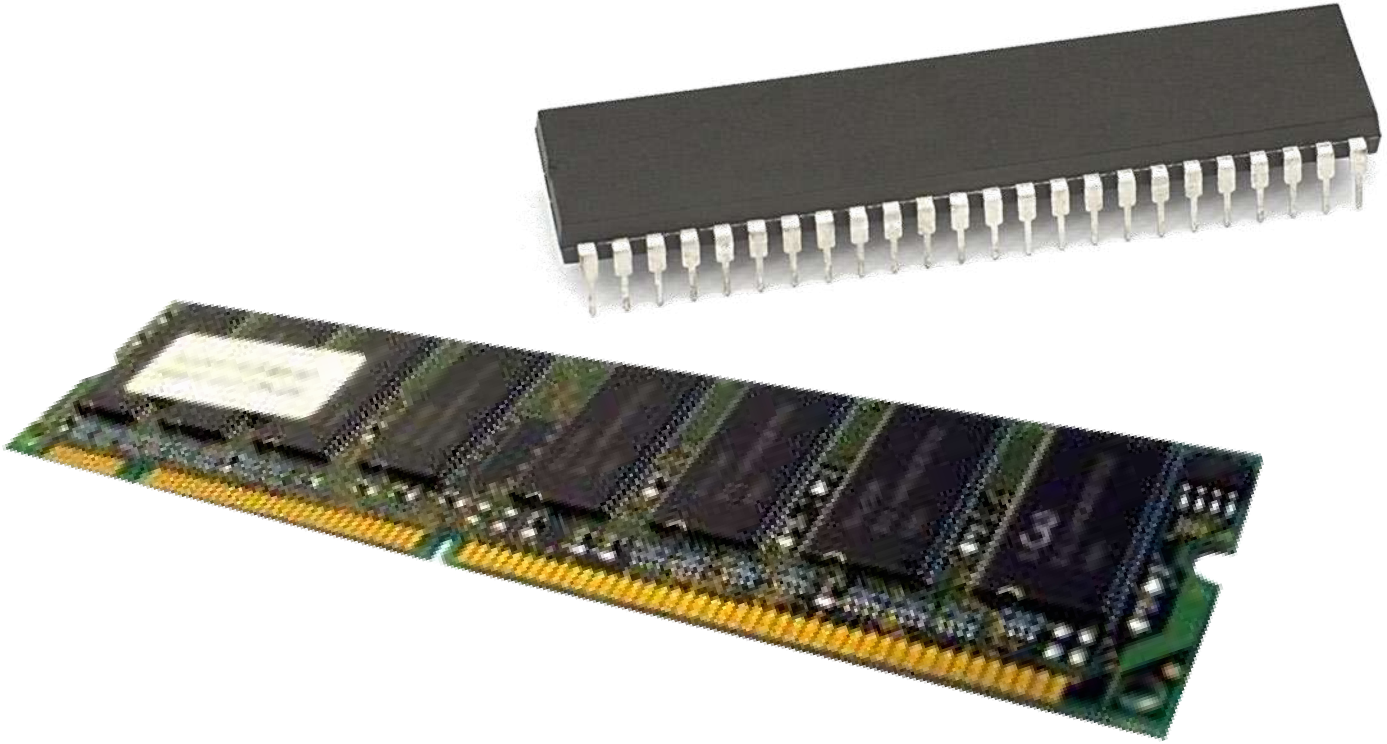
Different types of memory

Expected time to complete: 1 hour

Computer memory comes in many different forms, some of the main ones are:

- Random Access Memory (RAM)
- Read Only Memory (ROM)
- Virtual Memory

Carry out some research into these forms of memory and then complete the tasks on the following slide.



Additional help:

For additional help and support in completing this task you might like to watch some of the following videos from Craig 'n' Dave:

RAM and ROM:
https://student.craigndave.org/videos/ocr-gcse-slr1-2-ram-and-rom

The need for Virtual Memory:
https://student.craigndave.org/videos/ocr-gcse-slr1-2-the-need-for-virtual-memory

10 Memory task

Craig'n'Dave

Different types of memory

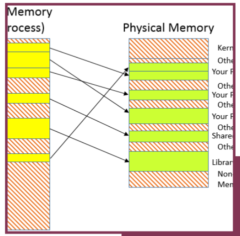
Expected time to complete: 1 hour

Random Access Memory (RAM)

- <Enter a definition for RAM>
- <State if RAM is volatile or non volatile>
- <State if RAM is read only or read & writeable>
- <Describe what sort of information RAM typically holds>

Read Only Memory (ROM)

- <Enter a definition for ROM>
- <State if ROM is volatile or non volatile>
- <State if ROM is read only or read & writeable>
- <Describe what sort of information ROM typically holds>



Virtual Memory

- <Enter a definition for Virtual Memory>
- <Explain why virtual memory is needed>

Types of secondary storage

Expected time to complete: 1 hour

Virtually all secondary storage devices in use today fit into one of three broad categories:

- Magnetic
- Optical
- Solid state

Carry out some research into these categories and then complete the tasks on the following slide.

Additional help:

For additional help and support in completing this task you might like to watch some of the following videos from Craig 'n' Dave:

Magnetic, Flash and Optical storage:
https://student.craigndave.org/videos/ocr-alevel-slr03-magnetic-flash-and-optical-storage

Comparing capacity and speed of storage media:
https://student.craigndave.org/videos/aqa-alevel-slr18-comparing-capacity-and-speed-of-storage-media

11 Storage task

Craig'n'Dave

Types of secondary storage

Expected time to complete: 1 hour

How does it work?

Enter your answer here...

Advantages / Positives

Enter your answer here...

Disadvantages / Negatives

Enter your answer here...

Examples of typical usage

Enter your answer here...

Typical storage capacity range

Enter your answer here...

Optical Storage

How does it work?

Enter your answer here...

Advantages / Positives

Enter your answer here...

Disadvantages / Negatives

Enter your answer here...

Examples of typical usage

Enter your answer here...

Typical storage capacity range

Enter your answer here...

Magnetic Storage

How does it work?

Enter your answer here...

Advantages / Positives

Enter your answer here...

Disadvantages / Negatives

Enter your answer here...

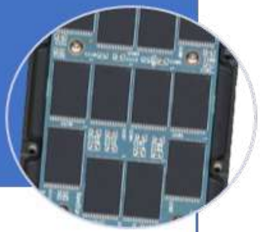
Examples of typical usage

Enter your answer here...

Typical storage capacity range

Enter your answer here...

Solid State Storage



Types of networks

Expected time to complete: 2 hours

Carry out some research on computer networks, in particular LANs, WiFi, Network topologies and connectivity devices.
Use the symbols on the right (feel free to revise them) to create an appropriate network over the floorplan on the next slide.
Make sure your network meets all the following requirements:

1. Each member of the main office needs a desktop PC
2. Angela, Pam, Dwight and Oscar also use an office issued smart phone
3. The following rooms new access to WiFi:
    1. Meeting room (top right)
    2. Reception
    3. Conference Room
    4. Main office
4. Use a circle with a transparent fill (so you can see the network underneath) with a width and height of 12.5cm to provide the WiFi coverage needed to cover the rooms above:
    1. The circles need to have a WAP at the centre
    2. The 12.5cm diameter circles represent the maximum range of each WAP
    3. They WAP icons must be attached to a wall
    4. You must use the minimum number of WAP possible to provide the coverage needed
5. All desktop PCs use wired connections in a star network configuration
    1. The top left server room, conference room and main office need to be on one subnet with its own switch
    2. All other rooms are on a separate subnet and will require its own hardware for this
    3. The two subnets need to be appropriately connected together
6. The top left room needs to have a server placed in it and connected appropriately to the local subnet
7. The server room needs hardware to appropriately connected the LAN to “The Internet”
8. Reception needs a photocopier and it needs connecting to the local subnet
9. A firewall should be placed somewhere appropriate

Use the following symbols:

Additional help:

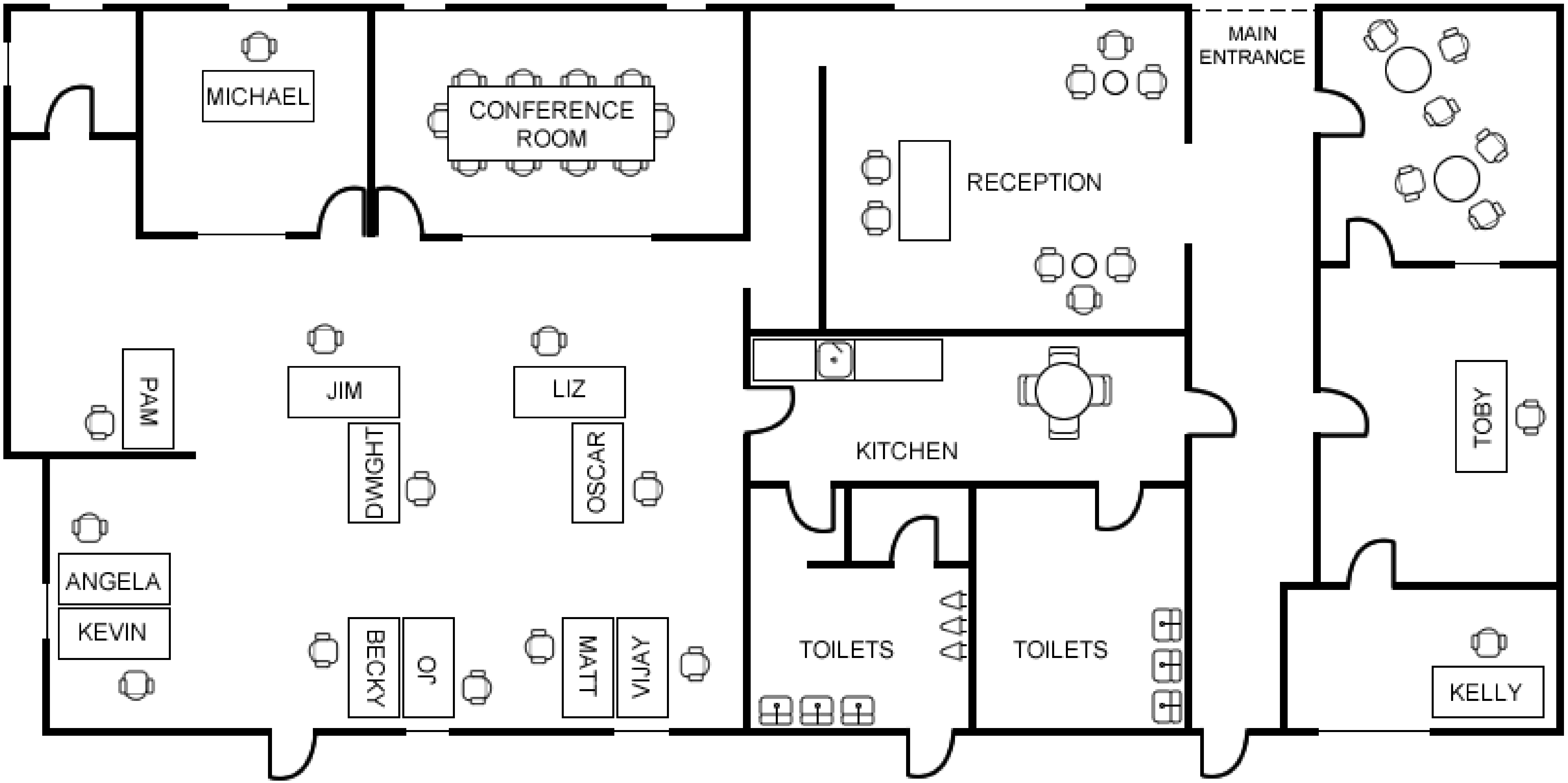
For additional help and support in structuring your answer you might like to watch some of the videos from the following Craig ‘n’ Dave playlists:

OCR: SLR 11 – Networks
https://student.craigndave.org/videos/slr-11-networks

Craig'n'Dave

Types of networks

Expected time to complete: 2 hours



13 Systems software task

Craig'n'Dave

Operating systems

Expected time to complete: 2 hours

Operating systems are arguably the most important piece of software installed on a computer.

Carry out some research into the following areas:

- The purpose of operating systems
- The roles of operating systems
- The purpose of interrupts
- How interrupts work as part of the fetch-decode-execute cycle

Complete the tasks on the following slides.

Additional help:

For additional help and support in structuring your answer you might like to watch some of the videos from the following Craig 'n' Dave playlists:

OCR: SLR 4 – Operating systems – Systems software
https://student.craigndave.org/videos/slr-4-operating-systems-systems-software

Operating systems

Expected time to complete: 2 hours

1. List at least 8 different roles an Operating System perform.

The purpose and roles of an Operating System

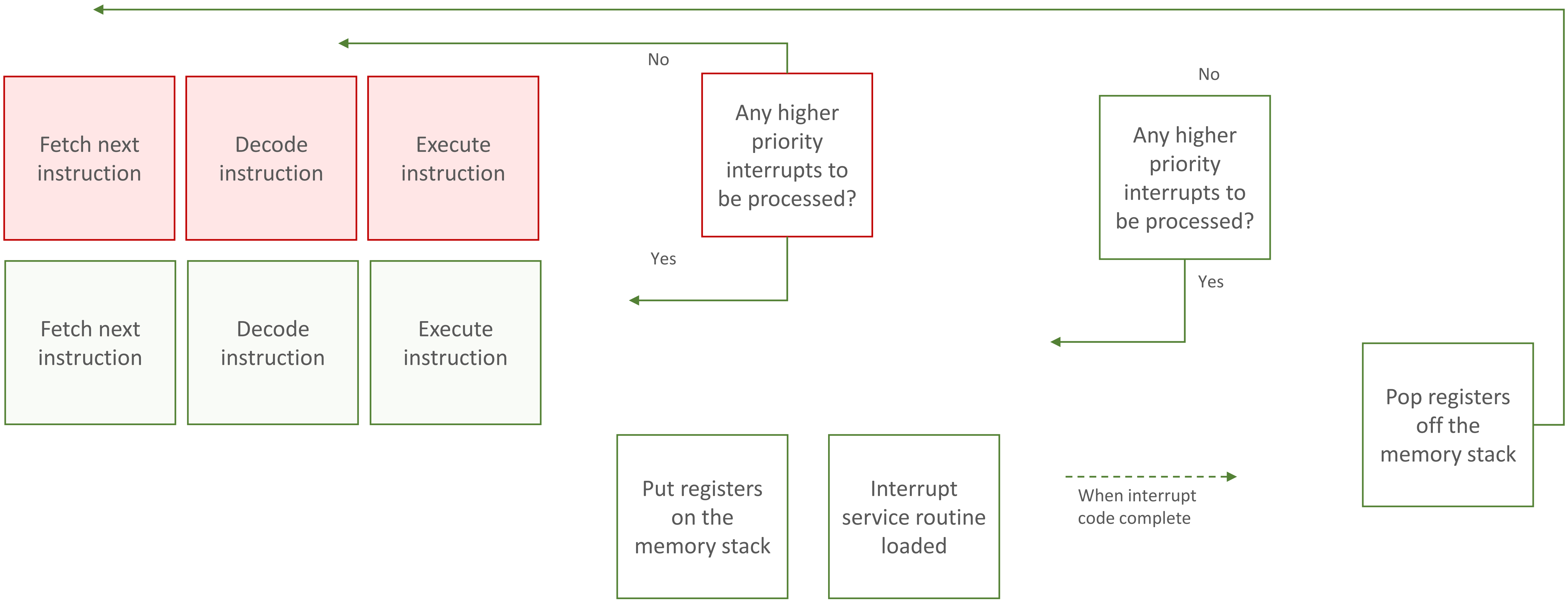
<Enter answer here>	<Enter answer here>	<Enter answer here>	<Enter answer here>	<Enter answer here>	<Enter answer here>	<Enter answer here>	<Enter answer here>

Craig'n'Dave

Operating systems

Expected time to complete: 2 hours

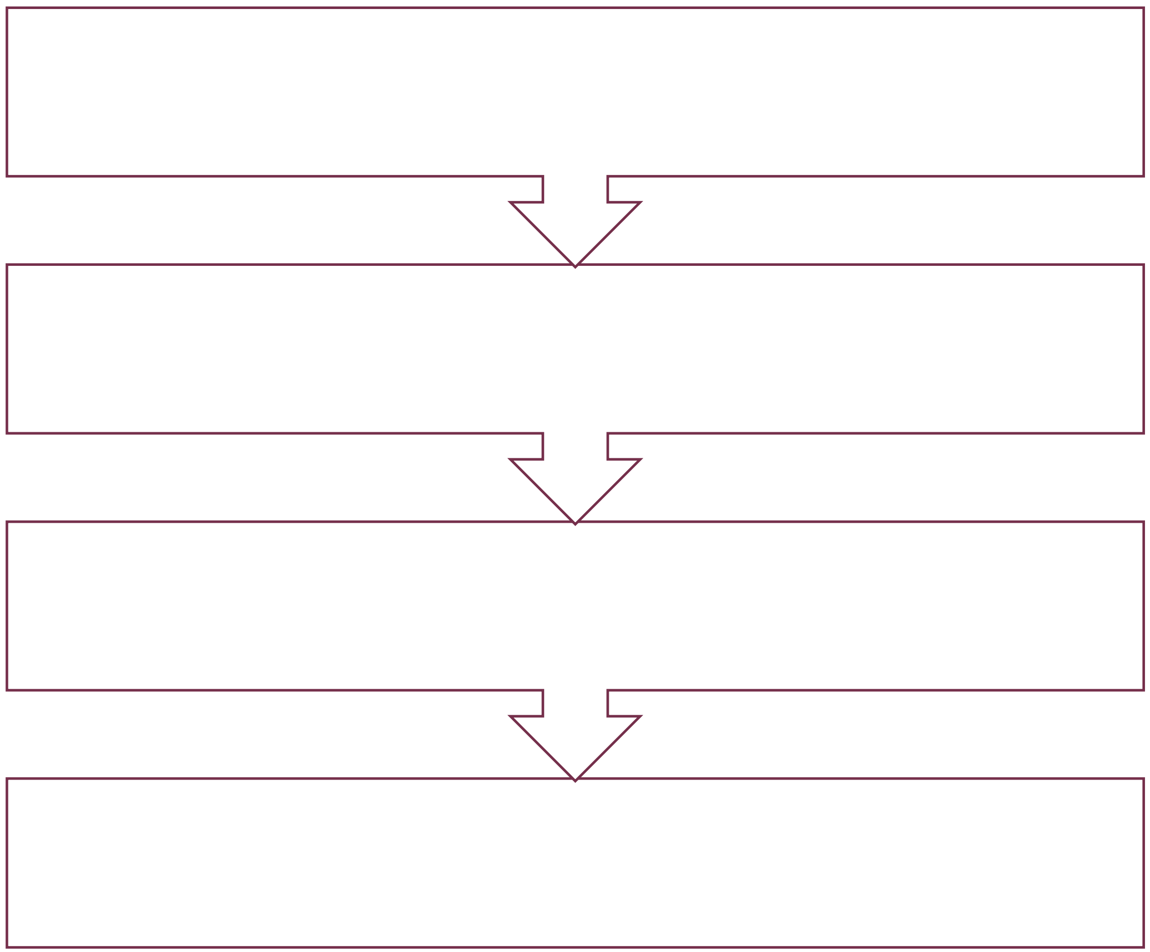
2. Rearrange the elements in the diagram below to show hoe the fetch-decode-execute cycle handles interrupts.



Operating systems

Expected time to complete: 2 hours

3. Drag the labels into their correct place on the following diagram:



4. Explain what this diagram is showing:

Hardware

User

Applications

Operating System

Computational logic task

Truth tables to circuit diagrams

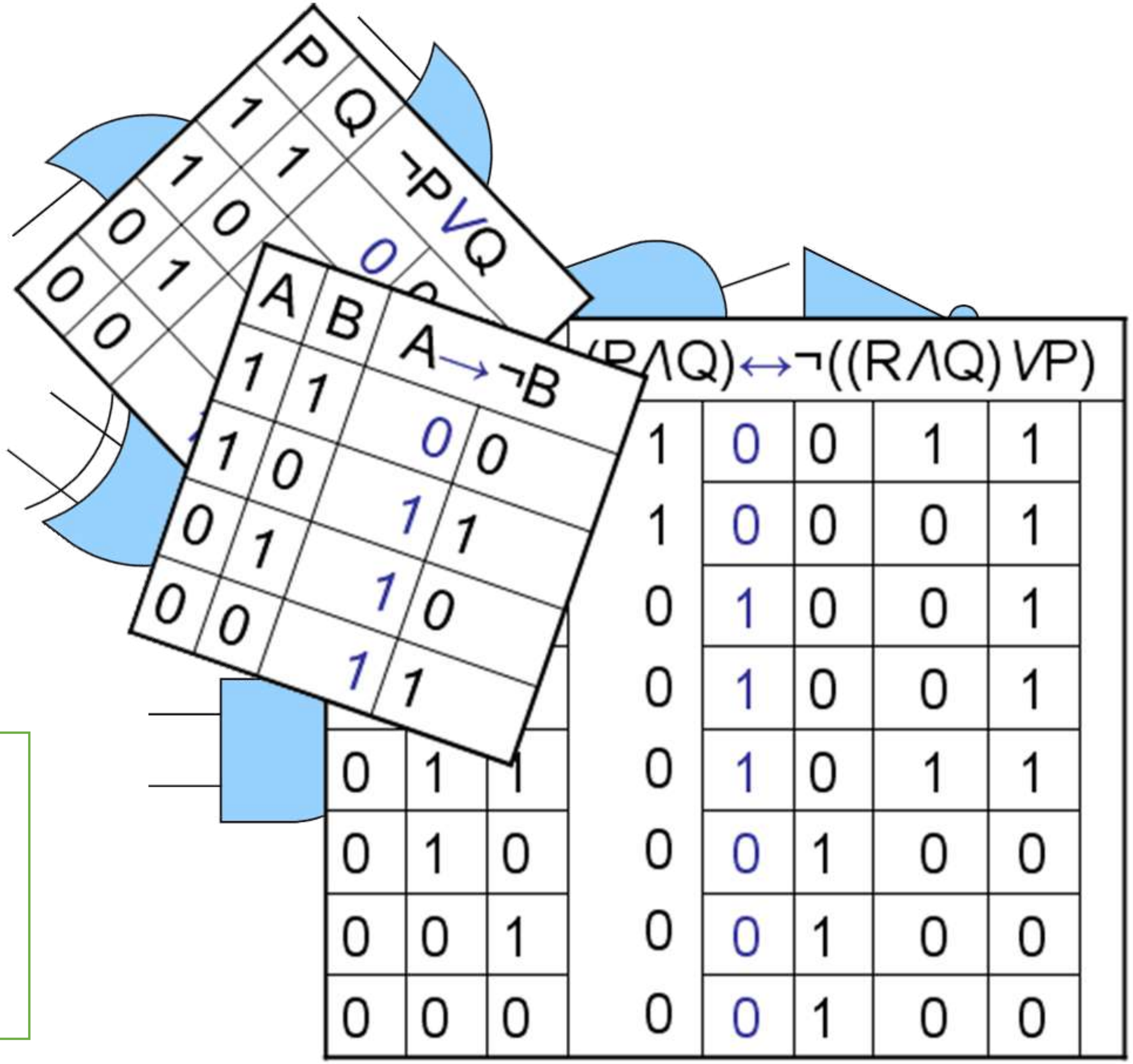
Expected time to complete: 2 hours

An important area of computer science is understanding the logic gates and diagrams which are used to represent the physical circuitry of computer systems.

Carry out some research into the following areas:

- Logic gates:
    - AND
    - NAND
    - NOR
    - NOT
    - OR
    - XOR
- Truth tables
- Boolean expressions
- Circuit diagrams

Complete the tasks on the following slides.



Additional help:

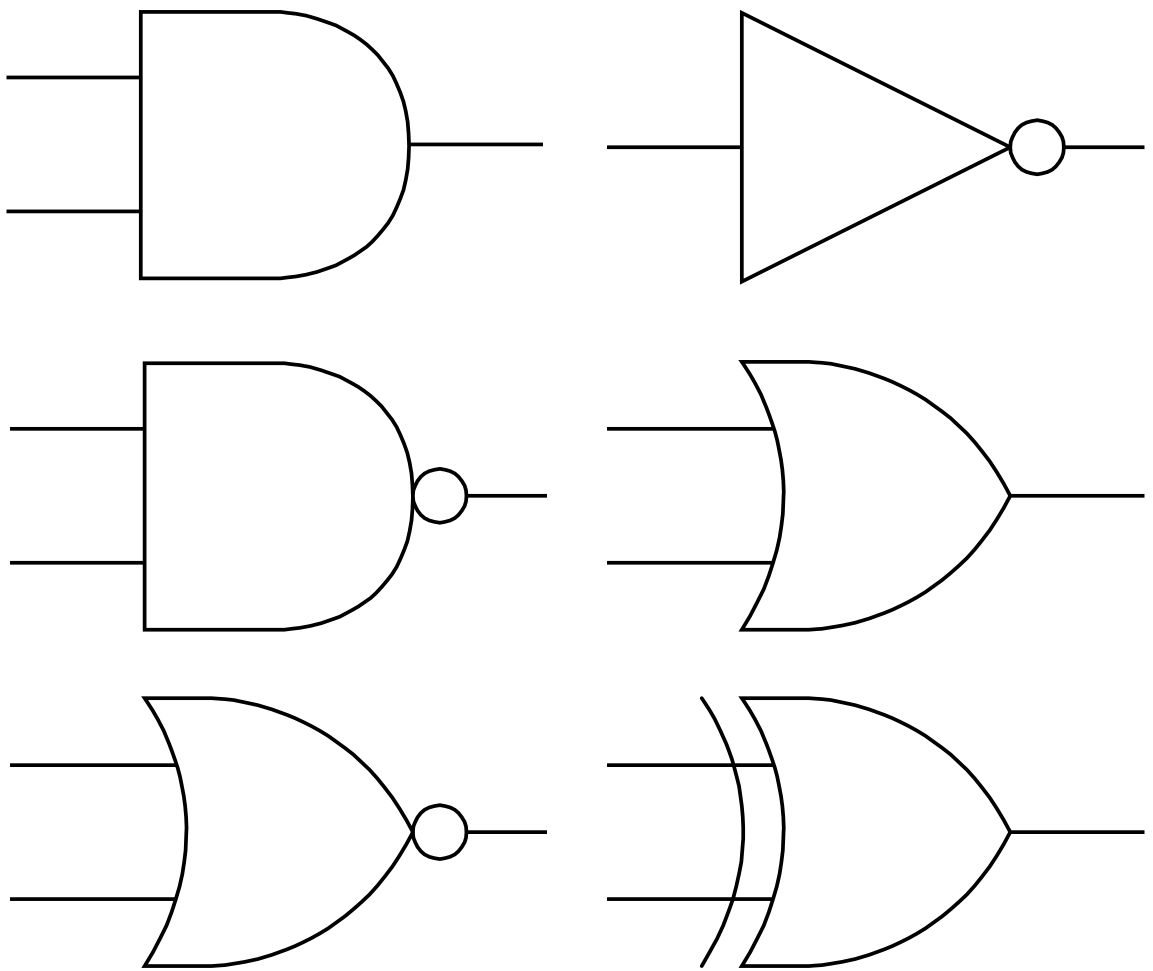
For additional help and support in structuring your answer you might like to watch some of the videos from the following Craig 'n' Dave playlists:

OCR: SLR 15 – Boolean algebra
https://student.craigndave.org/videos/slr-15-boolean-algebra

Truth tables to circuit diagrams

Expected time to complete: 2 hours

1. Drag the labels into their correct place on the following diagram:



Truth tables to circuit diagrams

Expected time to complete: 2 hours

2. Draw the circuit diagram which would represent the following Boolean expression:

OCR Boolean Expression: $\mathbf{F = \neg(\neg C \wedge (A\ \underline{\vee}\ B)}$

A

B

C

F

3. Complete the truth table for the circuit diagram you have drawn

A	B	C	D	E	F
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

Representing negative numbers in binary

Expected time to complete: 1½ hours

In GCSE computer science you will have learnt how to represent positive whole numbers in binary e.g. 47

At A Level you will need to know how to represent negative as well e.g. -47

Start to recapping (or learning if you didn't do the GCSE) how to represent positive whole numbers between 0-255 in binary

Now research how to represent negative numbers in binary using the method known as:

- Two's complement

Complete the tasks on the following slides.

Additional help:

For additional help and support in structuring your answer you might like to watch some of the following videos from Craig 'n' Dave:

GCSE recap: How to represent positive binary values 0-255
https://student.craigndave.org/videos/aqa-gcse-slr13-number-bases

A Level: Representing negative binary values using Two's Complement
https://student.craigndave.org/videos/aqa-alevel-slr11-twos-complement

Craig'n'Dave

Representing negative numbers in binary

Expected time to complete: 1½ hours

1. Write out the positive binary number 107, the answer should be displayed in 8 bits.

128	64	32	16	8	4	2	1	Binary weighting
								Binary value

2. Write out the negative binary number -107 using Two's Complement, the answer should be displayed in 8 bits.

								Binary weighting
								Binary value

3. How would you represent the lowest negative number possible using Two's Complement, given 8 bits.

								Binary weighting
								Binary value

4. How would you represent the largest positive number possible using Two's Complement, given 8 bits.

								Binary weighting
								Binary value

16 Binary task

Craig'n'Dave

Converting between base-2, base-10 and base-16

Expected time to complete: 1½ hours

As humans we have use the decimal or denary number system (base-10), made up of the unique digits 0-9.

Computer systems at the most basic level use only binary 1's and 0's (base-2).

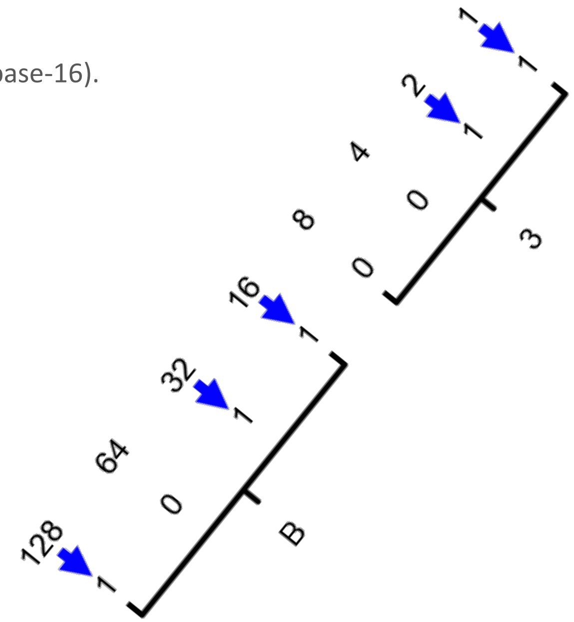
As a computer scientist you will also need to become familiar with the hexadecimal number system (base-16).

You will also need to be comfortable with converting numbers between these three base systems.

Research the following areas:

- Base-2 binary number system
- Base-10 decimal/denary number system
- Base-16 hexadecimal number system
- How to convert between base-2, base-10 and base-16

Complete the tasks on the following slides.



Denary	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Additional help:

For additional help and support in structuring your answer you might like to watch some of the following videos from Craig 'n' Dave:

Base 2, 10 and 16 number systems:
https://student.craigndave.org/videos/aqa-alevel-slr10-base-2-10-and-16-number-systems

Converting between binary, hex and decimal:
https://student.craigndave.org/videos/aqa-alevel-slr11-aqa-converting-between-binary-hex-and-decimal

Converting between base-2, base-10 and base-16

Expected time to complete: 1½ hours

1. Convert the base-2 binary number 11000101 into base-10 and base-16.
(Show all your working)

2. Convert the base-16 hexadecimal number 9F into base-2 and base-10.
(Show all your working)

You may wish to cut and paste this standard binary weighting line to help lay out parts of your answer

128	64	32	16	8	4	2	1	Binary weighting
								Binary value

Linear search programming task

Algorithms: from theory to practice

Expected time to complete: 4 hours

A core concept of computer science is that of data structures and algorithms.

It is also an area which many students struggle with during examinations.

Probably the most basic algorithm is that of the "linear search".

If you have done the GCSE course you will have learnt about this searching algorithm already.

Start by learning or refreshing your knowledge of the linear search algorithm by using the videos on this page:

- https://www.craigndave.org/algorithms-linear-search

Once you are happy with the theory complete the exercises on the following slides.

Algorithms: from theory to practice

Expected time to complete: 2 hours

1. Describe what the linear search algorithm does.

2. What are the applications of the linear search algorithm?

3. Write out the steps of the linear search algorithm in simple-structured English.

4. Draw a simple diagram which illustrates the linear search algorithm.

Algorithms: from theory to practice

Expected time to complete: 2 hours

5. Write out pseudocode for the linear search algorithm.

- The algorithm should use an array called items which is pre-populated with the following values: "Florida","Georgia","Delaware","Alabama","California“
- The algorithm should ask the user to “Enter the state to find:”
- If the algorithm locates the state entered by the user in the array it should report back to the screen “Item found at position n”
- If the algorithms can not locate the state entered by the user in the array it should report back to the screen “Item not found”

17 Linear search programming task

Craig'n'Dave

Algorithms: from theory to practice

Expected time to complete: 2 hours

6. Have a go at coding the linear searching algorithm in a programming language of your choice.

- The program should work use an array called items which is pre-populated with the following values: "Florida","Georgia","Delaware","Alabama","California“
- The program should ask the user to “Enter the state to find:”
- If the program locates the state entered by the user in the array it should report back to the screen “Item found at position n”
- If the program can not locate the state entered by the user in the array it should report back to the screen “Item not found”

Cut and paste the code you have written into the box below:

Gildredge House

ASPIRE

A Level English Language Induction Task

Welcome to your induction work for English Language A Level!

We are really looking forward to meeting you and getting to know you all better next year. This work will give us, and you, a flavour of what you can achieve and what English Language A Level is all about.

What work is being set?

- There are three main tasks outlined below. Two involve analysing pieces of Spoken Language and the third is to analyse a series of written texts.

How long should I spend on this work?

- It is expected that you spend five hours on this work

Who should I contact with any questions?

- Mrs Newby – t.newby@gildredgehouse.org.uk
- Miss McCart – c.mccart@gildredgehouse.org.uk

How and when to submit

- Your work should be brought to your Language lesson in September by the end of your first full week.

The English world is broad and diverse, and the more you are aware of its various guides the better! You need to tune yourself into as many different kinds of language as you can, so you will spend this summer collecting them.

In addition, the most important skill to have as a successful A Level candidate is that of INDEPENDENT LEARNING: i.e. that you work alone and come to class with your own ideas about what you are studying. So the best place to start is here….

Tasks:

1. **Choose a prepared speech:**
   (Perhaps a political speech or one made to commemorate a particular occasion. You can find *transcriptions* of speeches all over the internet – UK MPs speeches are all held on either government or political party websites, and newspapers transcribe a good number of internationally significant ones. Pick something contemporary – but please don't *all* choose Donald Trump or Boris Johnson!)
   a) Find out and make a note of the following:
      - The intended audience
      - The purpose of the speech
      - How does the speaker show power through their words?
   b) Annotate on the speech the persuasive/rhetorical techniques used.
   c) Write about 500 words on the above and comment on how effective you think the speech is and why

2. **Records and 'transcribe' a short conversation between two people.**

This must be spontaneous speech (i.e. not rehearsed or 'scripted' but what two people have actually said). You could record this on a mobile phone if it's a 'live' situation; or choose a piece from TV or radio spontaneous discussion programme e.g. Question Time, Interviews on the News, a chat show etc. Don't choose a sit-com or drama – they are scripted and not good examples of spontaneous speech.

a) Choose the part of the conversation which interests you most, this should be 10-15 lines
b) Write down a very brief description of the context, the subject and the two people involved
c) Write down (transcribe) the speech. Attempt to capture in writing as accurately as possible what you hear; this should include the following non-verbal features
   - Pauses
   - The stress on particular words
   - Fillers (ums, ers, etc.)
   - Stuttering
   - Drawn out words
   - Coughs, giggles, clearing of throat etc.
   - Volume of voice (raised, whispering etc.)
   - Speed of speech (quickly, slowly, hesitantly etc.)
   - Manner of speech (angrily, lovingly, snappily etc.)

3. **Make yourself a 'Language Scrapbook'. Choose yourself a broad subject e.g. Food, University Life, Music…. (This can then be used as a start for your coursework later in Y12)**
   a) Find 5 different texts which are connected with your topic. E.g. for the topic of food:
      - Recipe
      - Food advertisement
      - Transcript from a cooking programme
      - An extract from a blog
      - A formal article about different varieties of potato
      - A healthy eating leaflet
      - An email about a birthday meal out
      - A conversation between children about school dinners
      - A menu
      - An extract from a novel describing a meal
      - A restaurant review
      - A page from a children's story about a picnic
      - A verse from Craig David's 'What's your Flava?'
      - A poem
   b) Paste each text into a scrapbook – you could simple staple all the texts together and then annotate them or glue them into a notebook or exercise book to analyse. Feel free to copy ad paste them into a Word document or pdf and make notes using a computer – any of these is acceptable
   c) Write brief (but detailed and useful) notes next to each text about anything which strikes you as important in a language or communication sense. For instance, you might want to consider:

- The level of formality
- Purpose
- Audience
- The context in which it was produced
- The context in which it was intended to be received
- Use of images
- Individual qualities
- Use of accent/dialect
- Use of layout/fonts/logos
- Genre
- Spontaneous or prepared? Evidence?
- Sentences
- Use of jargon
- Lexis (vocabulary)
- Sound patterns (alliteration/onomatopoeia etc.)

These notes will be used during the first few weeks of lessons and we will come back to them for coursework so please bring them in with you when you start the course – there is no need to email the work to your teacher.

Have a wonderful summer; take care of yourselves and Miss McCart and I are really looking forward to meeting you all in September.

Mrs Newby and Miss McCart.

Gildredge House

English Literature

Induction Tasks: Time to start thinking like an A Level Student!

Welcome to the exciting first stage in your new Advanced Level course! We are delighted you have chosen to study English Literature at Gildredge House Sixth Form and this pack will help you to make the best possible start to studying this subject.

This work is VITAL for you to make a good start on your new course. It is directly linked to the syllabus and to the skills that you will be developing across your A-Level studies.

We will be collecting in your work at the beginning of September, so please make sure you keep the work safe until then. We will not be marking this work and awarding an A-Level grade but we will be looking to see whether you have the right attitude and resilience for completing the course. You will be provided with feedback on what you have done well and the process by which you have tackled your work, as well as given suggestions for how to build on these strategies so as to become a more successful Literature student in the future. This will form part of an initial assessment of your progress your teacher will make in the first two weeks of the course.

All of the materials you need to complete the tasks are in this pack or available on the Internet – just follow the links in the document.

If you need help: The tasks are designed to get a bit more difficult as you work through them, as they are preparing you for studying at a higher level and to become an effective independent learner. If you get stuck, simply make a note of it and we can discuss in the first week of lessons.

For an overview of the course, please see the Edexcel website: https://qualifications.pearson.com/content/dam/pdf/A%20Level/English%20Literature/2015/Specification%20and%20sample%20assessments/GCE2015-A-level-Eng-Lit-spec-Issue-6.pdf

An Overview of the Induction Tasks

1. Reading and making preparatory notes on your set texts.
2. Close analysis of key extracts from set texts, looking at the methods by which a writer communicates their views.
3. Research tasks on the contextual background of the set texts.
4. Analysing a poem.
5. Wider reading.
6. Application of critical reading to a chosen text.

Please ensure that your work is neatly labelled and formatted, ready to be submitted to your teacher and so it is easy to read and assess.

1. READING YOUR SET TEXTS

On the course, you will study the following texts:

PROSE:

Frankenstein by Mary Shelley (Wordsworth Classics, ISBN:978-1-85326-023-0)
The Handmaid's Tale by Margaret Atwood (Vintage, ISBN: 9780099740919)

DRAMA:

A Shakespearean play (to be confirmed)
A Streetcar Named Desire by Tennessee Williams (ISBN:)

POETRY:

A selection of poems from *Poems of the Decade: An Anthology of the Forward Books of Poetry 2002–2011* (ISBN: 978-0571325405)
The Wife of Bath's Prologue and Tale by Geoffrey Chaucer (ISBN: 978-1316615607)

Please ensure that you have bought **TWO copies** of the texts in advance of September – one for class annotations and one for the exam which must be a 'clean' copy (i.e. no annotations). Prior to starting in September, you should read ***Frankenstein* by Mary Shelley**, as this will be the first novel you will study. If you want to prepare further, you should also read ***The Handmaid's Tale* by Margaret Atwood** and read/watch a version of ***A Streetcar Named Desire* by Tennessee Williams.**

a) Read the novel in its entirety.
b) Write a detailed plot summary for each chapter. This will help you to navigate the text and will be enormously helpful to you in the Spring of Year 13 before you have to revise for your exams. **Do not simply copy one from the internet - you need to make your own notes!**
c) Write a set of character notes for each of the main characters. How you format these notes is up to you but we recommend that you include the following information: appearance; personality; how they behave; how they speak; their motivation for their behaviour; how they develop over the course of the novel; how they interact with others and the specifics of their relationships with other characters. You may also want to note down anything you notice about the themes of the novel.

2. CLOSE ANALYSIS OF TWO EXTRACTS

The following extract comes from Chapter 5 of Mary Shelley's *Frankenstein*. In it, the narrator, Victor Frankenstein, describes the moment he brings his creation to life. Analyse the extract and then answer the following question: **"Explore how the writer presents ideas about creation. Write 2 A4 pages (max.) in answer to the question."** Analyse and annotate the extract thoroughly before answering the question. Pay particular attention to the choices the writer has made about language devices, sentence structure and other language features, and what the effects of these choices are on the reader.

Extract 1: *Frankenstein* - Chapter 5

It was on a dreary night of November that I beheld the accomplishment of my toils. With an anxiety that almost amounted to agony, I collected the instruments of life around me, that I might infuse a spark of being into the lifeless thing that lay at my feet. It was already one in the morning; the rain pattered dismally against the panes, and my candle was nearly burnt out, when, by the glimmer of the half-extinguished light, I saw the dull yellow eye of the creature open; it breathed hard, and a convulsive motion agitated its limbs.

How can I describe my emotions at this catastrophe, or how delineate the wretch whom with such infinite pains and care I had endeavoured to form? His limbs were in proportion, and I had selected his features as beautiful. Beautiful! Great God! His yellow skin scarcely covered the work of muscles and arteries beneath; his hair was of a lustrous black, and flowing; his teeth of a pearly whiteness; but these luxuriances only formed a more horrid contrast with his watery eyes, that seemed almost of the same colour as the dun-white sockets in which they were set, his shrivelled complexion and straight black lips.

The different accidents of life are not so changeable as the feelings of human nature. I had worked hard for nearly two years, for the sole purpose of infusing life into an inanimate body. For this I had deprived myself of rest and health. I had desired it with an ardour that far exceeded moderation; but now that I had finished, the beauty of the dream vanished, and breathless horror and disgust filled my heart. Unable to endure the aspect of the being I had created, I rushed out of the room and continued a long time traversing my bed-chamber, unable to compose my mind to sleep. At length lassitude succeeded to the tumult I had before endured, and I threw myself on the bed in my clothes, endeavouring to seek a few moments of forgetfulness. But it was in vain; I slept, indeed, but I was disturbed by the wildest dreams. I thought I saw Elizabeth, in the bloom of health, walking in the streets of Ingolstadt. Delighted and surprised, I embraced her, but as I imprinted the first kiss on her lips, they became livid with the hue of death; her features appeared to change, and I thought that I held the corpse of my dead mother in my arms; a shroud enveloped her form, and I saw the grave-worms crawling in the folds of the flannel. I started from my sleep with horror; a cold dew covered my forehead, my teeth chattered, and every limb became convulsed; when, by the dim and yellow light of the moon, as it forced its way through the window shutters, I beheld the wretch—the miserable monster whom I had created. He held up the curtain of the bed; and his eyes, if eyes they may be called, were fixed on me. His jaws opened, and he muttered some inarticulate sounds, while a grin wrinkled his cheeks. He might have spoken, but I did not hear; one hand was stretched out, seemingly to detain me, but I escaped and rushed downstairs. I took refuge in the courtyard belonging to the house which I inhabited, where I remained during the rest of the night, walking up and down in the greatest agitation, listening attentively, catching and fearing each sound as if it were to announce the approach of the demoniacal corpse to which I had so miserably given life.

Oh! No mortal could support the horror of that countenance. A mummy again endued with animation could not be so hideous as that wretch. I had gazed on him while unfinished; he was ugly then, but when those muscles and joints were rendered capable of motion, it became a thing such as even Dante could not have conceived.

1. The following extract is the first chapter of Margaret Atwood's *The Handmaid's Tale*. Read the extract and answer the following questions:

    a. What impressions have you gained from this chapter of the narrator and her circumstances and by what means?
    b. What do you not know that you would have expected to find out from the first chapter of a novel? What might be the author's purpose in keeping such knowledge from her readers?

> We slept in what had once been the gymnasium. The floor was of varnished wood, with stripes and circles painted on it, for the games that were formerly played there; the hoops for the basketball nets were still in place, though the nets were gone. A balcony ran around the room, for the spectators, and I thought I could smell, faintly like an afterimage, the pungent scent of sweat, shot through with the sweet taint of chewing gum and perfume from the watching girls, felt-skirted as I knew from pictures, later in mini-skirts, then pants, then in one earring, spiky green-streaked hair. Dances would have been held there; the music lingered, a palimpsest of unheard sound, style upon style, an undercurrent of drums, a forlorn wail, garlands made of tissue-paper flowers, cardboard devils, a revolving ball of mirrors, powdering the dancers with a snow of light.
>
> There was old sex in the room and loneliness, and expectation, of something without a shape or name. I remember that yearning, for something that was always about to happen and was never the same as the hands that were on us there and then, in the small of the back, or out back, in the parking lot, or in the television room with the sound turned down and only the pictures flickering over lifting flesh.
>
> We yearned for the future. How did we learn it, that talent for insatiability? It was in the air; and it was still in the air, an afterthought, as we tried to sleep, in the army cots that had been set up in rows, with spaces between so we could not talk. We had flannelette sheets, like children's, and army-issue blankets, old ones that still said U.S. We folded our clothes neatly and laid them on the stools at the ends of the beds. The

lights were turned down but not out. Aunt Sara and Aunt Elizabeth patrolled; they had electric cattle prods slung on thongs from their leather belts.

No guns though, even they could not be trusted with guns. Guns were for the guards, specially picked from the Angels. The guards weren't allowed inside the building except when called, and we weren't allowed out, except for our walks, twice daily, two by two around the football field which was enclosed now by a chain-link fence topped with barbed wire. The Angels stood outside it with their backs to us. They were objects of fear to us, but of something else as well. If only they would look. If only we could talk to them. Something could be exchanged, we thought, some deal made, some trade-off, we still had our bodies. That was our fantasy.

We learned to whisper almost without sound. In the semi-darkness we could stretch out our arms, when the Aunts weren't looking, and touch each other's hands across space. We learned to lip-read, our heads flat on the beds, turned sideways, watching each other's mouths. In this way we exchanged names, from bed to bed:

Alma. Janine. Dolores. Moira. June.

RESEARCH TASK ON THE SOCIAL, HISTORICAL AND LITERARY CONTEXTS OF YOUR SET TEXTS

The Year 12 course focuses on texts written between the early 1800s and the present day. Therefore, a key part of being able to understand and analyse these texts is an awareness of the contexts in which they were written and received. Research the context of two of your set texts; the format in which you record your research is up to you. If you wish to research the other texts, feel free to do so.

The Wife of Bath

1. Explore why The Canterbury Tales were written and the context of them
2. Research the key historical and political events that influenced Chaucer.
3. Research Geoffrey Chaucer - create a biography, ensuring you explore his biography, his social, political and religious beliefs and his most noted works.

Frankenstein

1. Research what Romanticism is and what the core ideas and beliefs behind this movement are.
2. Research the key historical and political events that influenced the Romantic Movement.
3. Research what the Gothic Literary tradition and explore how it relates to Romanticism.
4. Research the life of Mary Shelley.
5. Research scientific advancements in the early 19th century and how they influenced Shelley's *Frankenstein*.

A Streetcar Named Desire

1. Research Tennessee Williams' life and background. Include: family relationships, sexuality, mental health, themes and issues in his plays.
2. Social, historical and cultural context: American Civil War, the 'old' south versus the north, social class in the southern American states, post-second world war America.
3. Literary context: American theatre (1900-1950), realism, expressionism, Southern Gothic and the genre of tragedy.

The Handmaid's Tale

1. Research Margaret Atwood's life and background.
2. Political, social and historical contexts: Reaganism, the rise of the conservative religious right (the 'Christian right'), second wave feminism and the 1980s anti-feminist backlash, and the growing environmental concerns following the Second World War.
3. The features of dystopian literature.

4. ANALYSING A POEM

The following poem is one of the poems you will study from the 'Poems of the Decade' anthology. Analyse and annotate the poem before writing one A4 page answering the following question: "How does the poet present ideas about identity?"

'To My Nine-Year-Old Self' by Helen Dunmore

You must forgive me. Don't look so surprised,
perplexed, and eager to be gone,
balancing on your hands or on the tightrope.
You would rather run than walk, rather climb than run
rather leap from a height than anything.

I have spoiled this body we once shared.
Look at the scars, and watch the way I move,
careful of a bad back or a bruised foot.
Do you remember how, three minutes after waking
we'd jump straight out of the ground floor window
into the summer morning?

That dream we had, no doubt it's as fresh in your mind
as the white paper to write it on.
We made a start, but something else came up –
a baby vole, or a bag of sherbet lemons –
and besides, that summer of ambition
created an ice-lolly factory, a wasp trap
and a den by the cesspit.

I'd like to say that we could be friends
but the truth is we have nothing in common
beyond a few shared years. I won't keep you then.
Time to pick rosehips for tuppence a pound,
time to hide down scared lanes
from men in cars after girl-children,

or to lunge out over the water
on a rope that swings from that tree
long buried in housing –
but no, I shan't cloud your morning. God knows
I have fears enough for us both -

I leave you in an ecstasy of concentration
slowly peeling a ripe scab from your knee
to taste it on your tongue.

5. WIDER READING

As part of your Unit 2 exam, you will study two texts which will be linked by the concept of 'science and society'. As part of that exam, you will be expected to write an essay comparing the two texts and with a good knowledge of other ideas and texts written at the same time.

Good preparation for this will involve ensuring that you have read widely around your set texts, increasing your understanding of typical themes and concerns, so that you have a more perceptive understanding of literature within this area for the exam.

Over the course of Year 12, you will be asked to select wider reading texts to help you develop as a more independent and critical reader. *This wider reading might also help you to select one of your coursework texts for Year 13.* You will be asked to come to lessons ready to engage in active discussions and debates about what you have read. The following task will help you to prepare for this more mature way of working in Literature.

Task
Choose at least one of the texts listed to read for enjoyment. Using whatever format you choose, create a logbook to record your thoughts and useful quotes and information. Complete at least one entry for the text you have chosen to read. If you read more than one text, please complete an entry for each text.

Please note that the following list is extremely detailed and covers a very broad range of texts, many of which you would find on university reading lists. We don't expect you to read all or even many of them, far from it, instead we want to give you an introduction to the sheer range and depth of English Literature. Use this list as a starting point and enjoy your reading this summer. It's okay to not like a book – in fact, maybe you should expect that – and it's okay to stop reading and start something else.

Text List - The Struggle for Identity in Modern Literature

(* denotes text published after 1990)
(+ denotes text published between 1800-1945)

PROSE FICTION	Any of the ten named prose texts for Unit 2, or any other novel by Morrison.
Chinua Achebe	*Things Fall Apart* (Penguin, 1958)
James Baldwin	*Go Tell it on the Mountain* (Penguin)
Nadine Gordimer	*July's People* (Bloomsbury, 1981)
Radclyffe Hall	*The Well of Loneliness* + (Virago, 1928)
Zora Neale Hurston	*Their Eyes Were Watching God* + (Virago, 1937)
Andrea Levy	*Small Island* * (Headline, 2004)
Patrick McCabe	*Breakfast on Pluto* * (Picador, 1998)
Anne Michaels	*Fugitive Pieces* * (Bloomsbury, 1996)
Arundhati Roy	*The God of Small Things* * (Harper Perennial, 1997)
Robert Tressell	*The Ragged-Trousered Philanthropists* + (Flamingo, 1914)
Irvine Welsh	*Trainspotting* * (Vintage, 1993)
Jeanette Winterson	*Oranges are not the only fruit* (Vintage, 1984)
Richard Wright	*Native Son* + (Vintage, 1940)
Kurt Vonnegut	*Slaughterhouse 5* (Vintage, 1969)
Rose Tremain	*The Road Home* (Chatto and Windus)
Kathryn Stockett	*The Help* (Penguin, 2009)

PROSE NON-FICTION

Autobiographies and Biography, Diaries

Maya Angelou	*Autobiography,* especially *I Know Why The Caged Bird Sings* (Virago, 1969)
Diana Souhami	*The Trials of Radclyffe Hall* * (Virago, 1999)
Nelson Mandela	*Long Walk to Freedom* (Abacus, 1994)

Memoirs and Interviews

Silvia Calamati	*Women's stories from the North of Ireland* * (Beyond the Pale Publications, 2002)
Bobby Sands	*Skylark Sing Your Lonely Song* (Mercier Press, 1982)
Malcolm X	*Malcolm X Talks to Young People* (Pathfinder, 1964-1965)
Alice Walker	*The Same River Twice: Honoring the Difficult* * (Phoenix, 1996)

Travelogues

Salman Rushdie	*The Jaguar Smile: A Nicaraguan Journey (Vintage, 1987)*

History and cultural commentary, essays and speeches

David Beresford	*Ten Men Dead: The Story of the 1981 Irish Hunger Strike* (Harper Collins,1987)
Beverley Bryan, Suzanne Scafe, Stella Dadzie	*The Heart of the Race* (Virago, 1985)
Germaine Greer	*The Female Eunuch* (Harper Perennial, 1970)
Martin Luther King Jr.	*I Have A Dream: Writings And Speeches That Changed The World* (Harper, 1956-68)
Adhaf Soueif	*Mezzaterra-Fragments from the Common Ground* * (Bloomsbury, 2004)
Amrit Wilson	*Dreams, Questions, Struggles South Asian Women in Britain* (Pluto Press, 2006)

Laws

Parliament	'Section 28 of the Education Act' 1988

Literary Criticism

Ralph Ellison	*Shadow and Act* (Vintage, 1967)
Dolly A. McPherson	*Order out of Chaos: The Autobiographical Works of Maya Angelou* (Virago, 1990)
Kate Millet	*Sexual Politics* (Virago, 1977)
Amrit Wilson	*Finding a Voice: Asian Women in Britain* (Virago, 1978)
Richard Wright	*Blueprint for Negro Writing* + (1937)
Jeremy Hawthorn ed.	*The British Working Class Novel in the Twentieth Century* (Hodder Arnold, 1984)

DRAMA

Brendan Behan	*The Hostage* (Methuen, 1958)
Sudhar Bhuchar	*Child of the Divide* * (Methuen Modern Plays)

Jim Cartwright	*Road* (Methuen Modern Plays, 1986)
Caryl Churchill	All plays * (some will be post 1990)
Claire Dowie	*Why is John Lennon Wearing a Skirt?* * (Methuen Modern Plays, 1996)
Brian Friel	*Dancing at Lughnasa* * (Faber, 1990)
Lorraine Hansberry	*A Raisin in the Sun* (Methuen Modern Plays, 1959)
Sarah Kane	*Complete Plays* * (Methuen Drama, 1998-2006)
Tony Kushner	*Angels in America* * (Nick Herne Books, 1992)
Martin McDonagh	*Beauty Queen of Leenane* * (Methuen, 1996)
Sean O'Casey	*Three Dublin Plays: Juno and the Paycock* + (1924), *The Plough and the Stars* + (1926), *Shadow of a Gunman* + (1923) (Faber)
Arthur Miller	
Mark Ravenhill	*Death of a Salesman* (Penguin, 1949)
Ntozake Shange	*Citizenship* * (Methuen Modern Plays, 2006)
Timberlake Wertenbaker	*Shange Plays 1- (Includes For Colored Girls Who Have Considered Suicide When the Rainbow is Enough)*
Tennessee Williams	*Our Country's Good* (Methuen, 1988)
	A Streetcar Named Desire (Methuen, 1947)
International Connections (contributor Jackie Kay)	*New Plays for Young People* * (Faber 2003)

POETRY

Simon Armitage	*Dead Sea Poems* * (Faber, 1995)
W.H Auden	e.g 'The Quarry', 'Funeral Blues', 'Refugee Blues' + (1930s)
Gillian Clarke	*Letter From a Far Country* (1985)
Carol Ann Duffy	*The Other Country* * (Anvil, 1990)
Allan Ginsberg	*Howl* (City Lights Pocket Poet Series, 1956)
Langston Hughes	*Collected Poems* + (Vintage, 1930-1960)
Jackie Kay	*Life Mask* * (Bloodaxe Books, 2005)
Liz Lockhead	*Dreaming Frankenstein and Collected Poems* (Polygon, 1984)
Audre Lorde	Any – (some will be post 1990)
Grace Nichols	*The Fat Black Woman's Poems* (Virago, 1984)
Adrienne Rich	*The School Among the Ruins* * (Norton, 2004)
Lemn Sissay	*Morning Breaks in the Elevator* * (Payback Press, 1999)
Gertrude Stein	*Tender Buttons* + (Dover, 1914)
Alice Walker	*Revolutionary Petunias and other Poems* (Harcourt Brae Jovanovitch, 1970)
Benjamin Zephaniah	*Too Black, Too Strong* * (Bloodaxe Books, 2001)
Edited by Lemn Sissay	*The Fire People: A Collection of Contemporary Black British Poets* * (Payback Press, 1998)
	Woman (Waterways, 2003)
Agnes Meadows	*A Recipe for Water* (Carcaret, 2009)
Gillian Clarke	*The Thing in the Gap Stone Stile* (Faber, 1996)
Alice Oswald	*I Have Crossed an Ocean* (Bloodaxe Books Ltd, 2010)
Grace Nichols	*Love Poems* (Picador, 2010)
Carol Ann Duffy	*Darling* (Bloodaxe Books Ltd, 2007)
Jackie Kay	*The Colour of Black and White* (Polyfon, 2003)
Liz Lochhead	*Rebel Without Applause* (Bloodaxe Books Ltd, 1992
Lenin Sissay	

TEXTS IN TRANSLATION

Novels

Isabel Allende	*The House of the Spirits* (Chile/Spanish) (Black Swan, 1985)
Alexandra Kollontai	*Love of Worker Bees* + (USSR/Russian) (Virago, 1930)
Manuel Puig	[illegible] (Vintage, 1976)
Alexander Solzenichen	[illegible] (USSR/Russian) (Penguin, 1962)

Poetry

Pablo Neruda	*Residence on Earth + (Chile/Spanish) (Souvenir Press, 1933)*

Drama

Bertolt Brecht	*Mother Courage and her Children* + (German) (Methuen, 1940)
Federico Garcia Lorca	*The House of Bernarda Alba* + (1936), *Yerma* + (1934), *Blood Wedding* + (1933) (Spanish) (Penguin)

Non fiction autobiography/diary/travelogue

Anne Frank	*The Diary of a Young Girl* (Dutch) (Penguin, 1947)
Che Guevara	*The Motorcycle Diaries* (Argentina/Spanish) (Harper Perennial, 1952)
Nawal al-Saadawi	*Memoirs from the Women's Prison* (Egypt/Arabic) (1984)

6. APPLYING CRITICAL VIEWS TO YOUR WIDER READING

One of the key skills you are going to have to develop in your study of English Literature is the ability to deal with other peoples' critical views of the text. You will be asked to research and find your own critical views to include in your essays and you will be asked to judge a text based on the extent to which your interpretation agrees with a given critical view.

The following activities will help you to begin practising these skills.

Task

It has been said that: "Modern literature shows isolated characters as being profoundly damaged". Choose one of the texts you have read for the previous task and find evidence from the text that both supports and/or refutes this critical view. Write a paragraph which tries to either prove or disprove the critical view using evidence from your chosen text.

Well done for completing these tasks. Every task has helped you practise a skill you will develop over the course of A-Level Literature and has helped you to build knowledge you will be relying on.

We look forward to seeing you in September!

Further Mathematics

Subject	Qualification	Examination Board
Further Mathematics	A Level (or an AS level studied over two years).	Edexcel
Additional Information:	You must also be taking Mathematics A level.	

Task Overview:
To begin to explore the topic of matrices in mathematics. To become familiar with some of the technology you will use as part of your Further Mathematics course. To complete exercises on this topic and make use of relevant technology. To be able to experiment and discover some new mathematics.
Success Criteria:
Exercises successfully completed (evidence on paper ready to hand in, marked and corrected as far as possible). Notes made on any areas of difficulty that need further work or extra support. Ability to use the new Classwiz calculator. Have gained an understanding of matrices and matrix calculations.
Resources:
We will provide the exercises, instructions for the task and an outline of the investigation extension. Note that most of this can be completed without the technology. You will need to buy the Casio fx-991EX Classwiz calculator (this is a requirement for both the Mathematics and Further Mathematics courses). However, don't worry if you don't have this until later.
How will the work produced will fit into subsequent work and the specification as a whole?
The knowledge you gain of matrices will give you a head start in your studies in both the Core Pure Maths and Decision Maths parts of your Further Mathematics course. Familiarity with the new technology will enhance your studies throughout the course.
How should the work should be presented?
Evidence on A4 paper clearly labelled and communicated / explained as appropriate. Remember to mark all exercises in red as you go, stopping to correct in red where you have made mistakes. You should make a note of any problem areas and problems or successes with using the technology.

Who should you contact if you should require further assistance with the work before the end of term?	Mrs Wilshire. Teacher of Further Mathematics. s.wilshire@gildredgehouse.org.uk
Length of time expected to complete tasks:	3-5 hours.
Submission Requirements:	Work should be ready to hand in at your first lesson in Further Mathematics.

What equipment will be needed for the subject?
See above for calculator requirement. You will also need to obtain the following textbooks ready for the start of the course: Pearson Core Pure Mathematics Book 1/AS ISBN 978 – 1 – 292 – 18333-6. Pearson Decision Mathematics 1 D1 ISBN 978-1-292-18329-9. You will need the usual mathematics equipment you used for GCSE e.g. blue or black pen(s), pencil, ruler, rubber, protractor, compass, red pen for marking, and whiteboard pen(s) for use on small whiteboards. In addition for A Level you need a supply of A4 lined paper and a filing system e.g. A4 folder and dividers. There are two further textbooks required later for year 13.
Optional Extension Task/Further Reading
Extend your work into matrix transformations in 2 and possibly 3 dimensions and / or research how matrices are used in mathematics and related areas. There are applications in Computer Science, Business and Engineering.

Transition Task: Further Mathematics

You will be learning about the topic of Matrices.

The skills you learn here will be used in both the Core Pure and Decision Mathematics areas of study in your A Level Further Mathematics course.

There are three sections to this booklet

1. Matrices
2. Multiplying Matrices
3. Transformations.

You are required to complete the first two sections. Section 3 is an optional extension to this work.

For sections 1 and 2:

Read through the information given, stopping at each discussion point, technology box and activity. Here you should answer the questions or find out how to use the technology (your calculator).

There are answers at the back. Please mark and correct your work in red to make sure you have understood before moving on.

Then complete the exercise. You should answer all questions in Exercise 1.1 and questions 1 to 9 of Exercise 1.2. (Show any working clearly). Where a question has multiple very similar parts you can just do the first and last part, mark your work, and if it's all correct and you fully understand move on to the next question. If it's not all correct, or you need more practice, do all parts of the question.

If you are feeling confident you could try Exercise 1.2 questions 10 and 11 which are more challenging.

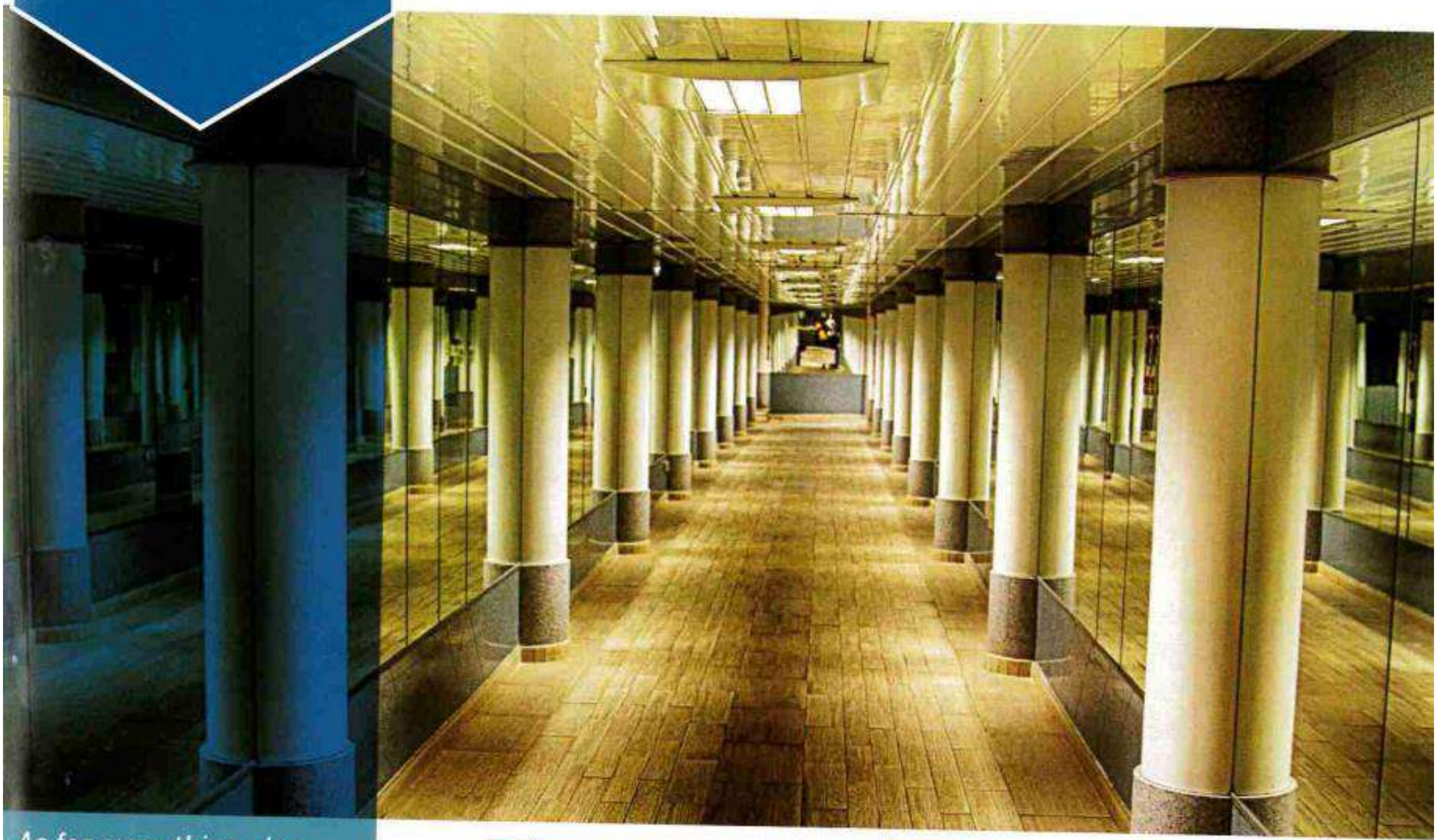
Use the answers at the back. Be sure to mark and correct your work in red to make sure you have understood before moving on.

Optional extension / investigation: Section 3:

If you would like to you can investigate how matrices can be used to transform shapes. Or you might want to explore how matrices are used in business, engineering or computer science.

Please hand in all work on A4 paper clearly named and labelled with topic and section titles. Work should be handed in at your first Further Mathematics lesson.

1 Matrices and transformations



As for everything else, so for a mathematical theory – beauty can be perceived but not explained.

Arthur Cayley 1883

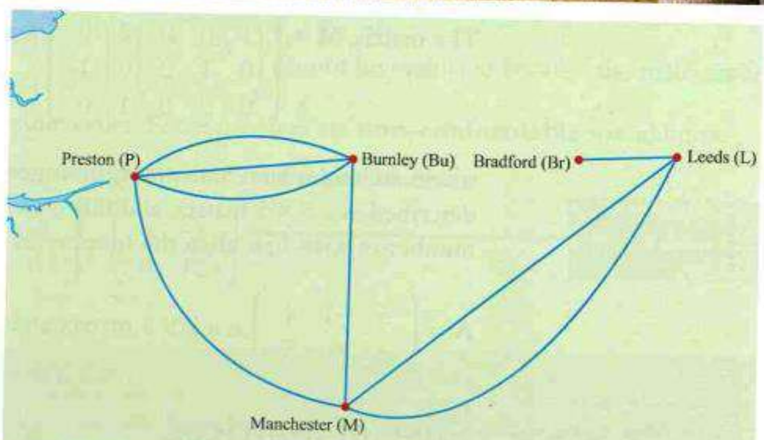


Figure 1.1 Illustration of some major roads and motorways joining some towns and cities in the north of England.

Discussion point

→ How many direct routes (without going through any other town) are there from Preston to Burnley? What about Manchester to Leeds? Preston to Manchester? Burnley to Leeds?

1 Matrices

You can represent the number of direct routes between each pair of towns (shown in Figure 1.1) in an array of numbers like this:

	Br	Bu	L	M	P
Br	0	0	1	0	0
Bu	0	0	0	1	3
L	1	0	0	2	0
M	0	1	2	0	1
P	0	3	0	1	0

This array is called a matrix (the plural is matrices) and is usually written inside curved brackets.

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 3 & 0 & 1 & 0 \end{pmatrix}$$

It is usual to represent matrices by capital letters, often in bold print.

A matrix consists of rows and columns, and the entries in the various cells are known as **elements**.

The matrix $\mathbf{M} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 3 & 0 & 1 & 0 \end{pmatrix}$ representing the routes between the towns and cities has 25 elements, arranged in five rows and five columns. **M** is described as a 5×5 matrix, and this is the **order** of the matrix. You state the number of rows first, then the number of columns. So, for example, the matrix

$\mathbf{A} = \begin{pmatrix} 3 & -1 & 4 \\ 2 & 0 & 5 \end{pmatrix}$ is a 2×3 matrix and $\mathbf{B} = \begin{pmatrix} 4 & -4 \\ 3 & 4 \\ 0 & -2 \end{pmatrix}$ is a 3×2 matrix.

Special matrices

Some matrices are described by special names which relate to the number of rows and columns or the nature of the elements.

Matrices such as $\begin{pmatrix} 4 & 2 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 3 & 5 & 1 \\ 2 & 0 & -4 \\ 1 & 7 & 3 \end{pmatrix}$ which have the same number of rows as columns are called **square matrices**.

The matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is called the 2×2 **identity matrix** or **unit matrix**, and similarly $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is called the 3×3 identity matrix. Identity matrices must be square, and are usually denoted by **I**. An identity matrix consists of 1's on the leading diagonal (the diagonal from top left to bottom right) and 0's everywhere else.

The matrix $\mathbf{O} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is called the 2×2 **zero matrix**. Zero matrices can be of any order.

Two matrices are said to be **equal** if and only if they have the same order and each element in one matrix is equal to the corresponding element in the other matrix. So, for example, the matrices **A** and **D** below are equal, but **B** and **C** are not equal to any of the other matrices.

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 4 & 0 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

Working with matrices

Matrices can be added or subtracted if they are of the same order.

$$\begin{pmatrix} 2 & 4 & 0 \\ -1 & 3 & 5 \end{pmatrix} + \begin{pmatrix} 1 & -1 & 4 \\ 2 & 0 & -5 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 4 \\ 1 & 3 & 0 \end{pmatrix}$$

Add the elements in corresponding positions.

$$\begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix} - \begin{pmatrix} 7 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 5 & -1 \end{pmatrix}$$

Subtract the elements in corresponding positions.

But $\begin{pmatrix} 2 & 4 & 0 \\ -1 & 3 & 5 \end{pmatrix} + \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}$ cannot be evaluated because the matrices are not of the same order. These matrices are **non-conformable** for addition.

You can also multiply a matrix by a **scalar** number:

$$2\begin{pmatrix} 3 & -4 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 6 & -8 \\ 0 & 12 \end{pmatrix}$$

Multiply each of the elements by 2.

TECHNOLOGY

You can use a calculator to add and subtract matrices of the same order and to multiply a matrix by a number. For your calculator, find out:

- the method for inputting matrices
- how to add and subtract matrices
- how to multiply a matrix by a number for matrices of varying sizes.

Associativity and commutativity

When working with numbers the properties of **associativity** and **commutativity** are often used.

Associativity

Addition of numbers is **associative**.

$$(3+5)+8=3+(5+8)$$

When you add numbers, it does not matter how the numbers are grouped, the answer will be the same.

Commutativity

Addition of numbers is **commutative**.

$$4+5=5+4$$

When you add numbers, the order of the numbers can be reversed and the answer will still be the same.

Discussion points

- Give examples to show that subtraction of numbers is not commutative or associative.
- Are matrix addition and matrix subtraction associative and/or commutative?

Exercise 1.1

① Write down the order of these matrices.

(i) $\begin{pmatrix} 2 & 4 \\ 6 & 0 \\ -3 & 7 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & 8 & 4 \\ -2 & -3 & 1 \\ 5 & 3 & -2 \end{pmatrix}$ (iii) $\begin{pmatrix} 7 & -3 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$

(v) $\begin{pmatrix} 2 & -6 & 4 & 9 \\ 5 & 10 & 11 & -4 \end{pmatrix}$ (vi) $\begin{pmatrix} 8 & 5 \\ -2 & 0 \\ 3 & -9 \end{pmatrix}$

② For the matrices

$\mathbf{A}=\begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix}$ $\mathbf{B}=\begin{pmatrix} 7 & -3 \\ 1 & 4 \end{pmatrix}$ $\mathbf{C}=\begin{pmatrix} 3 & 5 & -9 \\ 2 & 1 & 4 \end{pmatrix}$ $\mathbf{D}=\begin{pmatrix} 0 & -4 & 5 \\ 2 & 1 & 8 \end{pmatrix}$

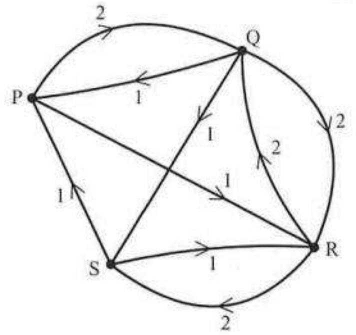
$\mathbf{E}=\begin{pmatrix} -3 & 5 \\ -2 & 7 \end{pmatrix}$ $\mathbf{F}=\begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$

find, where possible

(i) $\mathbf{A}-\mathbf{E}$ (ii) $\mathbf{C}+\mathbf{D}$ (iii) $\mathbf{E}+\mathbf{A}-\mathbf{B}$ (iv) $\mathbf{F}+\mathbf{D}$ (v) $\mathbf{D}-\mathbf{C}$
(vi) $4\mathbf{F}$ (vii) $3\mathbf{C}+2\mathbf{D}$ (viii) $\mathbf{B}+2\mathbf{F}$ (ix) $\mathbf{E}-(2\mathbf{B}-\mathbf{A})$

③ The diagram in Figure 1.2 shows the number of direct flights on one day offered by an airline between cities P, Q, R and S.

The same information is also given in the partly completed matrix **X**.



$$\mathbf{X} = \begin{array}{c} \\ \text{From} \\ \\ \\ \\ \end{array} \begin{array}{c} \\ \text{P} \\ \text{Q} \\ \text{R} \\ \text{S} \end{array} \begin{array}{c} \text{To} \\ \begin{array}{cccc} \text{P} & \text{Q} & \text{R} & \text{S} \end{array} \\ \left(\begin{array}{cccc} 0 & 2 & 1 & 0 \\ 1 & & & \\ & & & \\ & & & \end{array}\right) \end{array}$$

Figure 1.2

(i) Copy and complete the matrix **X**.

A second airline also offers flights between these four cities. The following matrix represents the total number of direct flights offered by the two airlines.

$$\begin{pmatrix} 0 & 2 & 3 & 2 \\ 2 & 0 & 2 & 1 \\ 2 & 2 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{pmatrix}$$

(ii) Find the matrix **Y** representing the flights offered by the second airline.

(iii) Draw a diagram similar to the one in Figure 1.2, showing the flights offered by the second airline.

④ Find the values of w, x, y and z such that

$$\begin{pmatrix} 3 & w \\ -1 & 4 \end{pmatrix} + x\begin{pmatrix} 2 & -1 \\ y & z \end{pmatrix} = \begin{pmatrix} -9 & 8 \\ 11 & -8 \end{pmatrix}.$$

⑤ Find the possible values of p and q such that

$$\begin{pmatrix} p^2 & -3 \\ 2 & 9 \end{pmatrix} - \begin{pmatrix} 5p & -2 \\ -7 & q^2 \end{pmatrix} = \begin{pmatrix} 6 & -1 \\ 9 & 4 \end{pmatrix}.$$

⑥ Four local football teams took part in a competition in which they each played each other twice, once at home and once away. Figure 1.3 shows the results matrix after half of the games had been played.

	Win	Draw	Lose	Goals for	Goals against
City	2	1	0	6	3
Rangers	0	0	3	2	8
Town	2	0	1	4	3
United	1	1	1	5	3

Figure 1.3

(i) The results of the next three matches are as follows:

City 2	Rangers 0
Town 3	United 3
City 2	Town 4

Find the results matrix for these three matches and hence find the complete results matrix for all the matches so far.

(ii) Here is the complete results matrix for the whole competition.

$$\begin{pmatrix} 4 & 1 & 1 & 12 & 8 \\ 1 & 1 & 4 & 5 & 12 \\ 3 & 1 & 2 & 12 & 10 \\ 1 & 3 & 2 & 10 & 9 \end{pmatrix}$$

Find the results matrix for the last three matches (City vs United, Rangers vs Town and Rangers vs United) and deduce the result of each of these three matches.

⑦ A mail-order clothing company stocks a jacket in three different sizes and four different colours.

The matrix $\mathbf{P} = \begin{pmatrix} 17 & 8 & 10 & 15 \\ 6 & 12 & 19 & 3 \\ 24 & 10 & 11 & 6 \end{pmatrix}$ represents the number of jackets in stock at the start of one week.

The matrix $\mathbf{Q} = \begin{pmatrix} 2 & 5 & 3 & 0 \\ 1 & 3 & 4 & 6 \\ 5 & 0 & 2 & 3 \end{pmatrix}$ represents the number of orders for jackets received during the week.

(i) Find the matrix $\mathbf{P} - \mathbf{Q}$.

What does this matrix represent? What does the negative element in the matrix mean?

A delivery of jackets is received from the manufacturers during the week.

The matrix $\mathbf{R} = \begin{pmatrix} 5 & 10 & 10 & 5 \\ 10 & 10 & 5 & 15 \\ 0 & 0 & 5 & 5 \end{pmatrix}$ shows the number of jackets received.

(ii) Find the matrix which represents the number of jackets in stock at the end of the week after all the orders have been dispatched.

(iii) Assuming that this week is typical, find the matrix which represents sales of jackets over a six-week period. How realistic is this assumption?

2 Multiplication of matrices

When you multiply two matrices you do not just multiply corresponding terms. Instead you follow a slightly more complicated procedure. The following example will help you to understand the rationale for the way it is done.

There are four ways of scoring points in rugby: a try (five points), a conversion (two points), a penalty (three points) and a drop goal (three points). In a match Tonga scored three tries, one conversion, two penalties and one drop goal.

So their score was

$3 \times 5 + 1 \times 2 + 2 \times 3 + 1 \times 3 = 26.$

You can write this information using matrices. The tries, conversions, penalties and drop goals that Tonga scored are written as the 1×4 row matrix $(3\ \ 1\ \ 2\ \ 1)$ and the points for the different methods of scoring as the 4×1 column matrix

$$\begin{pmatrix} 5 \\ 2 \\ 3 \\ 3 \end{pmatrix}.$$

These are combined to give the 1×1 matrix $(3 \times 5 + 1 \times 2 + 2 \times 3 + 1 \times 3) = (26)$.

Combining matrices in this way is called **matrix multiplication** and this example is written as $(3\ \ 1\ \ 2\ \ 1)\begin{pmatrix} 5 \\ 2 \\ 3 \\ 3 \end{pmatrix} = (26)$.

The use of matrices can be extended to include the points scored by the other team, Japan. They scored two tries, two conversions, four penalties and one drop goal. This information can be written together with Tonga's scores as a 2×4 matrix, with one row for Tonga and the other for Japan. The multiplication is then written as

$$\begin{pmatrix} 3 & 1 & 2 & 1 \\ 2 & 2 & 4 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 26 \\ 29 \end{pmatrix}.$$

So Japan scored 29 points and won the match.

This example shows you two important points about matrix multiplication. Look at the orders of the matrices involved.

The two 'middle' numbers, in this case 4, must be the same for it to be possible to multiply two matrices. If two matrices can be multiplied, they are conformable for multiplication.

$2 \times 4 \quad \times \quad 4 \times 1$

The two 'outside' numbers give you the order of the product matrix, in this case 2×1.

You can see from the previous example that multiplying matrices involves multiplying each element in a row of the left-hand matrix by each element in a column of the right-hand matrix and then adding these products.

Multiplication of matrices

Example 1.1

Find $\begin{pmatrix} 10 & 3 \\ -2 & 7 \end{pmatrix}\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

Solution

The product will have order 2×1.

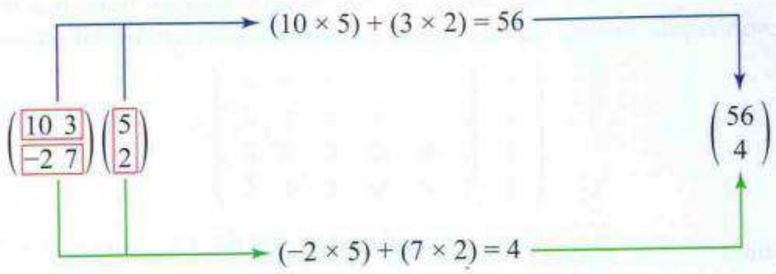


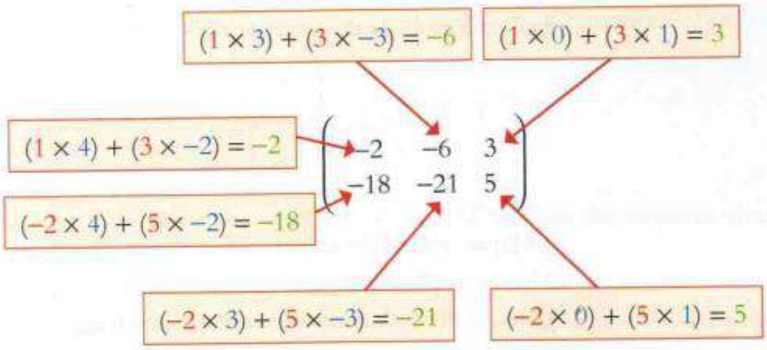
Figure 1.4

Example 1.2

Find $\begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 4 & 3 & 0 \\ -2 & -3 & 1 \end{pmatrix}$.

Solution

The order of this product is 2×3.



So $\begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}\begin{pmatrix} 4 & 3 & 0 \\ -2 & -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & -6 & 3 \\ -18 & -21 & 5 \end{pmatrix}$

Discussion point

→ If $\mathbf{A} = \begin{pmatrix} 1 & 3 & 5 \\ -2 & 4 & 1 \\ 0 & 3 & 7 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 8 & -1 \\ -2 & 3 \\ 4 & 0 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 5 & 0 \\ 3 & -4 \end{pmatrix}$

which of the products **AB**, **BA**, **AC**, **CA**, **BC** and **CB** exist?

Example 1.3

Find $\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

What do you notice?

Solution

The order of this product is 2×2.

$$\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$$

$(3 \times 1) + (2 \times 0) = 3$

$(3 \times 0) + (2 \times 1) = 2$

$(-1 \times 0) + (4 \times 1) = 4$

$(-1 \times 1) + (4 \times 0) = -1$

Multiplying a matrix by the identity matrix has no effect.

Properties of matrix multiplication

In this section you will look at whether matrix multiplication is:

- commutative
- associative.

On page 4 you saw that for numbers, addition is both associative and commutative. Multiplication is also both associative and commutative. For example:

$$(3 \times 4) \times 5 = 3 \times (4 \times 5)$$

and

$$3 \times 4 = 4 \times 3$$

ACTIVITY 1.1

Using $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -4 & 0 \\ -2 & 1 \end{pmatrix}$ find the products **AB** and **BA** and hence comment on whether or not matrix multiplication is commutative.

Find a different pair of matrices, **C** and **D**, such that **CD** = **DC**.

TECHNOLOGY

You could use the matrix function on your calculator.

ACTIVITY 1.2

Using $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 & 0 \\ -2 & 1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, find the matrix products:

(i) $\mathbf{AB}$
(ii) $\mathbf{BC}$
(iii) $(\mathbf{AB})\mathbf{C}$
(iv) $\mathbf{A}(\mathbf{BC})$

Does your answer suggest that matrix multiplication is associative?

Is this true for all 2×2 matrices? How can you prove your answer?

Exercise 1.2

In this exercise, do not use a calculator unless asked to. A calculator can be used for checking answers.

① Write down the orders of these matrices.

(i) (a) $\mathbf{A} = \begin{pmatrix} 3 & 4 & -1 \\ 0 & 2 & 3 \\ 1 & 5 & 0 \end{pmatrix}$ (b) $\mathbf{B} = \begin{pmatrix} 2 & 3 & 6 \end{pmatrix}$

(c) $\mathbf{C} = \begin{pmatrix} 4 & 9 & 2 \\ 1 & -3 & 0 \end{pmatrix}$ (d) $\mathbf{D} = \begin{pmatrix} 0 & 2 & 4 & 2 \\ 0 & -3 & -8 & 1 \end{pmatrix}$

(e) $\mathbf{E} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$. (f) $\mathbf{F} = \begin{pmatrix} 2 & 5 & 0 & -4 & 1 \\ -3 & 9 & -3 & 2 & 2 \\ 1 & 0 & 0 & 10 & 4 \end{pmatrix}$

(ii) Which of the following matrix products can be found? For those that can state the order of the matrix product.

(a) $\mathbf{AE}$ (b) $\mathbf{AF}$ (c) $\mathbf{FA}$ (d) $\mathbf{CA}$ (e) $\mathbf{DC}$

② Calculate these products.

(i) $\begin{pmatrix} 3 & 0 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} 7 & 2 \\ 4 & -3 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 & -3 & 5 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 5 & 8 \\ -3 & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 2 & 5 & -1 & 0 \\ 3 & 6 & 4 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ -9 \\ 11 \\ -2 \end{pmatrix}$

T Check your answers using the matrix function on a calculator if possible.

③ Using the matrices $\mathbf{A} = \begin{pmatrix} 5 & 9 \\ -2 & 7 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -3 & 5 \\ 2 & -9 \end{pmatrix}$, confirm that matrix multiplication is not commutative.

④ For the matrices

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -3 & 7 \\ 2 & 5 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 7 & 1 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 3 & 4 \\ 7 & 0 \\ 1 & -2 \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 4 & 7 \\ 3 & -2 \\ 1 & 5 \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} 3 & 7 & -5 \\ 2 & 6 & 0 \\ -1 & 4 & 8 \end{pmatrix}$$

calculate, where possible, the following:

(i) **AB** (ii) **BA** (iii) **CD** (iv) **DC** (v) **EF** (vi) **FE**

T

⑤ Using the matrix function on a calculator, find $\mathbf{M}^4$ for the matrix

$$\mathbf{M} = \begin{pmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 4 & 3 \end{pmatrix}.$$

> **Note**
>
> $\mathbf{M}^4$ means $\mathbf{M} \times \mathbf{M} \times \mathbf{M} \times \mathbf{M}$

⑥ $\mathbf{A} = \begin{pmatrix} x & 3 \\ 0 & -1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 2x & 0 \\ 4 & -3 \end{pmatrix}$:

(i) Find the matrix product **AB** in terms of x.

(ii) If $\mathbf{AB} = \begin{pmatrix} 10x & -9 \\ -4 & 3 \end{pmatrix}$, find the possible values of x.

(iii) Find the possible matrix products **BA**.

⑦ (i) For the matrix $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$, find

(a) $\mathbf{A}^2$

(b) $\mathbf{A}^3$

(c) $\mathbf{A}^4$

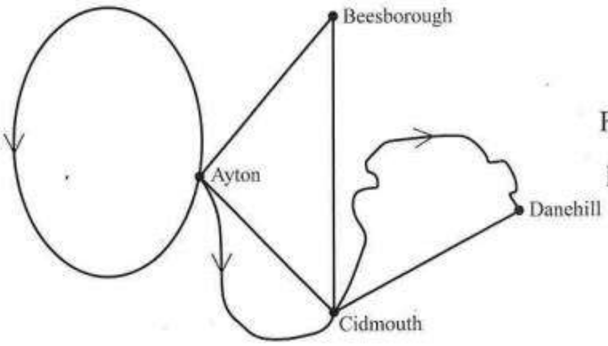
(ii) Suggest a general form for the matrix $\mathbf{A}^n$ in terms of n.

T

(iii) Verify your answer by finding $\mathbf{A}^{10}$ on your calculator and confirming it gives the same answer as using (iv).

⑧ The map in Figure 1.5 below shows the bus routes in a holiday area. Lines represent routes that run each way between the resorts. Arrows indicated one-way scenic routes.

M is the partly completed 4 × 4 matrix which shows the number of direct routes between the various resorts.



$$\begin{array}{cc} & \text{To} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \text{From} \quad \mathbf{M} = \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 1 & 1 & 2 & 0 \\ & & & \\ & & & \\ & & & \end{pmatrix} \end{array}$$

Figure 1.5

(i) Copy and complete the matrix **M**.

(ii) Calculate $\mathbf{M}^2$ and explain what information it contains.

(iii) What information would $\mathbf{M}^3$ contain?

⑨ $\mathbf{A} = \begin{pmatrix} 4 & x & 0 \\ 2 & -3 & 1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 2 & -5 \\ 4 & x \\ x & 7 \end{pmatrix}$:

(i) Find the product **AB** in terms of x.

A symmetric matrix is one in which the entries are symmetrical about the leading diagonal, for example $\begin{pmatrix} 2 & 5 \\ 5 & 0 \end{pmatrix}$ and $\begin{pmatrix} 3 & 4 & -6 \\ 4 & 2 & 5 \\ -6 & 5 & 1 \end{pmatrix}$.

(ii) Given that the matrix **AB** is symmetric, find the possible values of x.

(iii) Write down the possible matrices **AB**.

⑩ The matrix **A**, in Figure 1.6, shows the number of sales of five flavours of ice cream: Vanilla(V), Strawberry(S), Chocolate(C), Toffee(T) and Banana(B), from an ice cream shop on each of Wednesday(W), Thursday(Th), Friday(F) and Saturday(Sa) during one week.

$$\mathbf{A} = \begin{matrix} & \begin{matrix} V & S & C & T & B \end{matrix} \\ \begin{matrix} W \\ Th \\ F \\ Sa \end{matrix} & \begin{pmatrix} 63 & 49 & 55 & 44 & 18 \\ 58 & 52 & 66 & 29 & 26 \\ 77 & 41 & 81 & 39 & 25 \\ 101 & 57 & 68 & 63 & 45 \end{pmatrix} \end{matrix}$$

Figure 1.6

(i) Find a matrix **D** such that the product **DA** shows the total number of sales of each flavour of ice cream during the four-day period and find the product **DA**.

(ii) Find a matrix **F** such that the product **AF** gives the total number of ice cream sales each day during the four-day period and find the product **AF**.

The Vanilla and Banana ice creams are served with strawberry sauce; the other three ice creams are served with chocolate sprinkles.

(iii) Find two matrices, **S** and **C**, such that the product **DAS** gives the total number of servings of strawberry sauce needed and the product **DAC** gives the total number of servings of sprinkles needed during the four-day period. Find the matrices **DAS** and **DAC**.

The price of Vanilla and Strawberry ice creams is 95p, Chocolate ice creams cost £1.05 and Toffee and Banana ice creams cost £1.15 each.

(iv) Using only matrix multiplication, find a way of calculating the total cost of all of the ice creams sold during the four-day period.

⑪ Figure 1.7 shows the start of the plaiting process for producing a leather bracelet from three leather strands a, b and c.

The process has only two steps, repeated alternately:

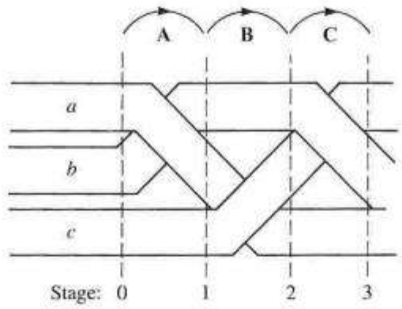


Figure 1.7

Step 1: cross the top strand over the middle strand

Step 2: cross the middle strand under the bottom strand.

At the start of the plaiting process, Stage 0, the order of the strands is given by $\mathbf{S}_0 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

(i) Show that pre-multiplying $\mathbf{S}_0$ by the matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ gives $\mathbf{S}_1$, the matrix which represents the order of the strands at Stage 1.

(ii) Find the 3×3 matrix $\mathbf{B}$ which represents the transition from Stage 1 to Stage 2.

(iii) Find matrix $\mathbf{M} = \mathbf{BA}$ and show that $\mathbf{MS}_0$ gives $\mathbf{S}_2$, the matrix which represents the order of the strands at Stage 2.

(iv) Find $\mathbf{M}^2$ and hence find the order of the strands at Stage 4.

(v) Calculate $\mathbf{M}^3$. What does this tell you?

3 Transformations

You are already familiar with several different types of transformation, including reflections, rotations and enlargements.

- The original point, or shape, is called the **object**.
- The new point, or shape, after the transformation, is called the **image**.
- A transformation is a **mapping** of an object onto its image.

Some examples of transformations are illustrated in Figures 1.8 to 1.10 (note that the vertices of the image are denoted by the same letters with a dash, e.g. A′, B′).

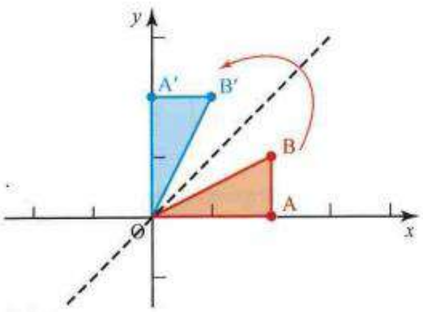


Figure 1.8 Reflection in the line $y = x$

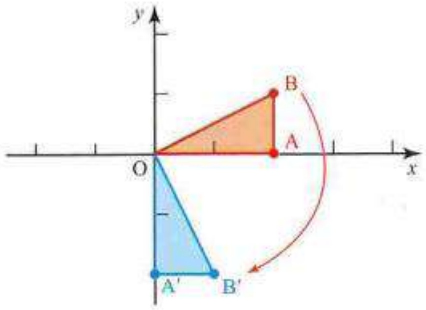


Figure 1.9 Rotation through 90° clockwise, centre O

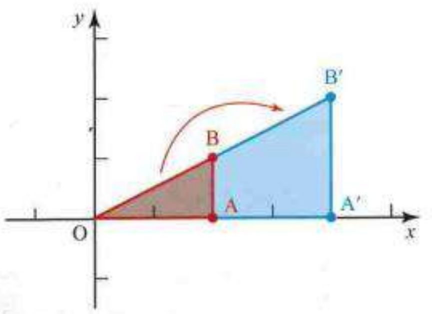


Figure 1.10 Enlargement centre O, scale factor 2

In this section, you will also meet the idea of

- a **stretch** parallel to the x-axis or y-axis

and three-dimensional transformations where

- a shape is reflected in the planes $x = 0$, $y = 0$ or $z = 0$
- a shape is rotated about one of the three coordinate axes.

A transformation maps an object according to a rule and can be represented by a matrix (see next section). The effect of a transformation on an object can be found by looking at the effect it has on the **position vector** of the point $\begin{pmatrix} x \\ y \end{pmatrix}$, i.e. the vector from the origin to the point (x, y). So, for example, to find the effect of a transformation on the point $(2, 3)$ you would look at the effect that the transformation matrix has on the position vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

Vectors that have length or **magnitude** of 1 are called **unit vectors**.

In two dimensions, two unit vectors that are of particular interest are

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ – a unit vector in the direction of the x-axis

$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ – a unit vector in the direction of the y-axis.

The equivalent unit vectors in three dimensions are

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ – a unit vector in the direction of the x-axis

$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ – a unit vector in the direction of the y-axis

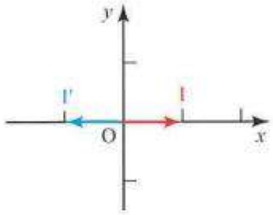
$\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ – a unit vector in the direction of the z-axis.

Finding the transformation represented by a given matrix

Start by looking at the effect of multiplying the unit vectors $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

The image of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ under this transformation is given by

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}.$$



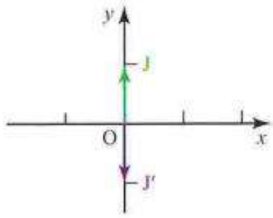
Note

The letter I is often used for the point (1, 0).

Figure 1.11

The image of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ under the transformation is given by

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$



Note

The letter J is often used for the point (0, 1).

Figure 1.12

You can see from this that the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ represents a rotation, centre the origin, through 180°.

Example 1.4

Describe the transformations represented by the following matrices.

(i) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Solution

(i) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

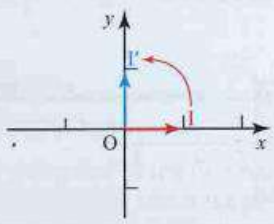


Figure 1.13

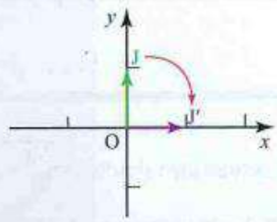


Figure 1.14

The matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$.

(ii) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

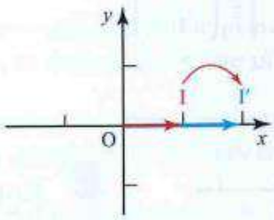


Figure 1.15

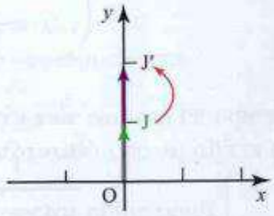


Figure 1.16

The matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ represents an enlargement, centre the origin, scale factor 2.

You can see that the images of $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the two columns of the transformation matrix.

Finding the matrix that represents a given transformation

Hint

You may find it easier to see what the transformation is when you use a shape, like the unit square, rather than points or lines.

The connection between the images of the unit vectors i and j and the matrix representing the transformation provides a quick method for finding the matrix representing a transformation.

It is common to use the unit square with coordinates O(0, 0), I(1, 0), P(1, 1) and J(0, 1).

You can think about the images of the points I and J, and from this you can write down the images of the unit vectors **i** and **j**.

This is done in the next example.

Example 1.5

By drawing a diagram to show the image of the unit square, find the matrices which represent each of the following transformations:

(i) a reflection in the x-axis

(ii) an enlargement of scale factor 3, centre the origin.

Solution

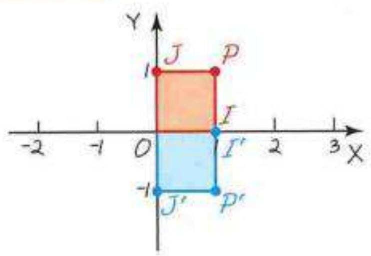


Figure 1.17

(i) You can see from Figure 1.17 that I (1, 0) is mapped to itself and J (0, 1) is mapped to J′ (0, −1).

So the image of **I** is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

and the image of **J** is $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$.

So the matrix which represents a reflection in the x-axis is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(ii)

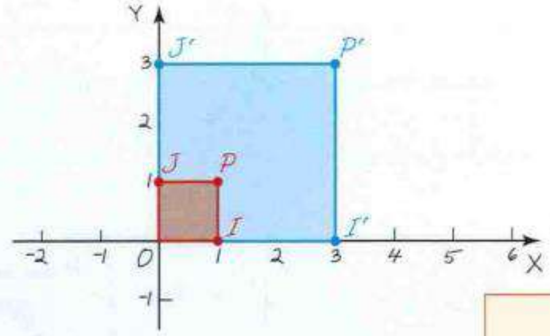


Figure 1.18

So the image of **I** is $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

You can see from Figure 1.18 that I (1, 0) is mapped to I′ (3, 0), and J (0, 1) is mapped to J′ (0, 3).

and the image of **J** is $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

So the matrix which represents an enlargement, centre the origin, scale factor 3 is $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

Discussion point

➜ For a general transformation represented by the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, what are the images of the unit vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$?

➜ What is the image of the origin (0, 0)?

ACTIVITY 1.3

Using the image of the unit square, find the matrix which represents a rotation of 45° anticlockwise about the origin.

Use your answer to write down the matrices which represent the following transformations:

(i) a rotation of 45° clockwise about the origin

(ii) a rotation of 135° anticlockwise about the origin.

Example 1.6

(i) Find the matrix which represents a rotation through angle θ anticlockwise about the origin.

(ii) Use your answer to find the matrix which represents a rotation of 60° anticlockwise about the origin.

Solution

(i) Figure 1.19 shows a rotation of angle θ anticlockwise about the origin.

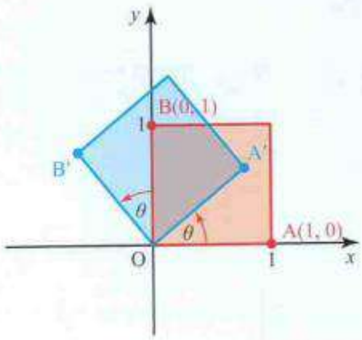


Figure 1.19

Call the coordinates of the point A' (p, q). Since the lines OA and OB are perpendicular, the coordinates of B' will be $(-q, p)$.

From the right-angled triangle with OA' as the hypotenuse, $\cos\theta = \frac{p}{1}$ and so $p = \cos\theta$.

Similarly $\sin\theta = \frac{q}{1}$ so $q = \sin\theta$.

So, the image point A' (p, q) has position vector $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ and the image point B' $(-q, p)$ has position vector $\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$.

Therefore, the matrix that represents a rotation of angle θ anticlockwise about the origin is $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

(ii) The matrix that represents an anticlockwise rotation of 60° about the origin is $\begin{pmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$.

Discussion point

→ What matrix would represent a rotation through angle θ clockwise about the origin?

TECHNOLOGY

You could use geometrical software to try different values of m and n.

ACTIVITY 1.4

Investigate the effect of the matrices:

(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$

Describe the general transformation represented by the matrices $\begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix}$.

Activity 1.4 illustrates two important general results.

- The matrix $\begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix}$ represents a stretch of scale factor m parallel to the x-axis.
- The matrix $\begin{pmatrix} 1 & 0 \\ 0 & n \end{pmatrix}$ represents a stretch of scale factor n parallel to the y-axis.

Summary of transformations in two dimensions

Note

All these transformations are examples of linear transformations. In a linear transformation, straight lines are mapped to straight lines, and the origin is mapped to itself.

Reflection in the x-axis	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Reflection in the y-axis	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
Reflection in the line $y = x$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Reflection in the line $y = -x$	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
Rotation anticlockwise about the origin through angle θ	$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$	Enlargement, centre the origin, scale factor k	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$
Stretch parallel to the x-axis, scale factor k	$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$	Stretch parallel to the y-axis, scale factor k	$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$

Transformations in three dimensions

When working with matrices, it is sometimes necessary to refer to a 'plane' – this is an infinite two-dimensional flat surface with no thickness. Figure 1.20 illustrates some common planes in three dimensions – the XY plane, the XZ plane and YZ plane. These three planes will be referred to when using matrices to represent some transformations in three dimensions. The plane XY can also be referred to as $z = 0$, since the z-coordinate would be zero for all points in the XY plane. Similarly, the XZ plane is referred to as $y = 0$ and the YZ plane as $x = 0$.

Chapter 1

Discussion point (Page 1)

3, 2, 1, 0

Discussion point (Page 4)

When subtracting numbers, the order in which the numbers appear is important – changing the order changes the answer, for example: $3 - 6 \neq 6 - 3$. So subtraction of numbers is not commutative.

The grouping of the numbers is also important, for example $(13 - 5) - 2 \neq 13 - (5 - 2)$. Therefore subtraction of numbers is not associative.

Matrices follow the same rules for commutativity and associativity under addition and subtraction as numbers. Matrix addition is both commutative and associative, but matrix subtraction is not commutative or associative. This is true because addition and subtraction of each of the individual elements will determine whether the matrices are commutative or associative overall.

You can use more formal methods to prove these properties. For example, to show that matrix addition is commutative:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix} = \begin{pmatrix} e+a & f+b \\ g+c & h+d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Addition of numbers is commutative

Exercise 1.1 (Page 4)

1 (i) 3×2 (ii) 3×3 (iii) 1×2 (iv) 5×1 (v) 2×4 (vi) 3×2

2 (i) $\begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$ (ii) $\begin{pmatrix} 3 & 1 & -4 \\ 4 & 2 & 12 \end{pmatrix}$

(iii) $\begin{pmatrix} -8 & 5 \\ -3 & 7 \end{pmatrix}$

(iv) Non-conformable (v) $\begin{pmatrix} -3 & -9 & 14 \\ 0 & 0 & 4 \end{pmatrix}$

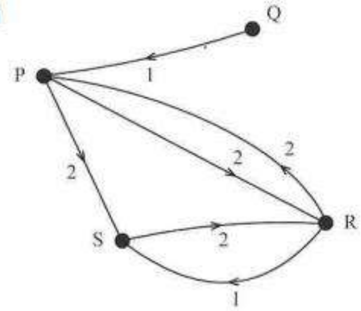
(vi) $\begin{pmatrix} 4 \\ 12 \\ 20 \end{pmatrix}$ (vii) $\begin{pmatrix} 9 & 7 & -17 \\ 10 & 5 & 28 \end{pmatrix}$

(viii) Non-conformable

(ix) $\begin{pmatrix} -15 & 8 \\ -4 & 3 \end{pmatrix}$

3 (i) $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & 1 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & 0 & 2 & 2 \\ 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix}$

(iii)



4 $w = 2, x = -6, y = -2, z = 2$

5 $p = -1$ or 6, $q = \pm\sqrt{5}$

6 (i) $\begin{pmatrix} 1 & 0 & 1 & 4 & 4 \\ 0 & 0 & 1 & 0 & 2 \\ 1 & 1 & 0 & 7 & 5 \\ 0 & 1 & 0 & 3 & 3 \end{pmatrix}$

$\begin{pmatrix} 3 & 1 & 1 & 10 & 7 \\ 0 & 0 & 4 & 2 & 10 \\ 3 & 1 & 1 & 11 & 8 \\ 1 & 2 & 1 & 8 & 6 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 & 3 \end{pmatrix}$

City 2 vs United 1
Rangers 2 vs Town 1
Rangers 1 vs /United 1

7 (i) $\begin{pmatrix} 15 & 3 & 7 & 15 \\ 5 & 9 & 15 & -3 \\ 19 & 10 & 9 & 3 \end{pmatrix}$

The matrix represents the number of jackets left in stock after all the orders have been dispatched. The negative element indicates there was not enough of that type of jacket in stock to fulfil the order.

(ii) $\begin{pmatrix} 20 & 13 & 17 & 20 \\ 15 & 19 & 20 & 12 \\ 19 & 10 & 14 & 8 \end{pmatrix}$

(iii) $\begin{pmatrix} 12 & 30 & 18 & 0 \\ 6 & 18 & 24 & 36 \\ 30 & 0 & 12 & 18 \end{pmatrix}$

Probably not very realistic, as a week is quite a short time.

Discussion point (Page 8)

The dimensions of the matrices are **A** (3 × 3), **B** (3 × 2) and **C** (2 × 2). The conformable products are **AB** and **BC**. Both of these products would have dimension (3 × 2), even though the original matrices are not the same sizes.

Activity 1.1 (Page 9)

$$\mathbf{AB} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -4 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -6 & -1 \\ -20 & 4 \end{pmatrix}$$

$$\mathbf{BA} = \begin{pmatrix} -4 & 0 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -8 & 4 \\ -1 & 6 \end{pmatrix}$$

These two matrices are not equal and so matrix multiplication is not usually commutative. There are some exceptions, for example if

$\mathbf{C} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix}$ then

$$\mathbf{CD} = \mathbf{DC} = \begin{pmatrix} 6 & 6 \\ -2 & -2 \end{pmatrix}.$$

Activity 1.2 (Page 10)

(i) $\mathbf{AB} = \begin{pmatrix} -6 & -1 \\ -20 & 4 \end{pmatrix}$

(ii) $\mathbf{BC} = \begin{pmatrix} -4 & -8 \\ 0 & -1 \end{pmatrix}$

(iii) $(\mathbf{AB})\mathbf{C} = \begin{pmatrix} -8 & -15 \\ -12 & -28 \end{pmatrix}$

(iv) $\mathbf{A}(\mathbf{BC}) = \begin{pmatrix} -8 & -15 \\ -12 & -28 \end{pmatrix}$

$(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$ so matrix multiplication is associative in this case

To produce a general proof, use general matrices such as

$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ and

$\mathbf{C} = \begin{pmatrix} i & j \\ k & l \end{pmatrix}$.

$$\mathbf{AB} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix},$$

$$\mathbf{BC} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}\begin{pmatrix} i & j \\ k & l \end{pmatrix} = \begin{pmatrix} ei + fk & ej + fl \\ gi + hk & gj + hl \end{pmatrix}$$

and so

$$(\mathbf{AB})\mathbf{C} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}\begin{pmatrix} i & j \\ k & l \end{pmatrix}$$

$$\begin{pmatrix} aei + bgi + afk + bhk & aej + bgj + afl + bhl \\ cei + cfk + dgi + dhk & cej + cfl + dgj + dhl \end{pmatrix}$$

and

$$\mathbf{A}(\mathbf{BC}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} ei + fk & ej + fl \\ gi + hk & gj + hl \end{pmatrix}$$

$$= \begin{pmatrix} aei + afk + bgi + bhk & aej + afl + bgj + bh \\ cei + dgi + cfk + dhk & cej + dgj + cfl + dh \end{pmatrix}$$

Since $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$ matrix multiplication is associative and the product can be written without brackets as **ABC**.

Exercise 1.2 (Page 10)

1 (i) (a) 3 × 3 (b) 1 × 3 (c) 2 × 3 (d) 2 × 4
(e) 2 × 1 (f) 3 × 5
(ii) (a) non-conformable
(b) 3 × 5
(c) non-conformable
(d) 2 × 3
(e) non-conformable

2 (i) $\begin{pmatrix} 21 & 6 \\ 31 & 13 \end{pmatrix}$ (ii) $\begin{pmatrix} -30 & -15 \end{pmatrix}$

(iii) $\begin{pmatrix} -54 \\ -1 \end{pmatrix}$

3 $\mathbf{AB} = \begin{pmatrix} 3 & -56 \\ 20 & -73 \end{pmatrix}$, $\mathbf{BA} = \begin{pmatrix} -25 & 8 \\ 28 & -45 \end{pmatrix}$

$\mathbf{AB} \neq \mathbf{BA}$ so matrix multiplication is non-commutative.

4 (i) $\begin{pmatrix} -7 & 26 \\ 2 & 34 \end{pmatrix}$ (ii) $\begin{pmatrix} 5 & 25 \\ 16 & 22 \end{pmatrix}$

(iii) $\begin{pmatrix} 31 & 0 \\ 65 & 18 \end{pmatrix}$ (iv) $\begin{pmatrix} 26 & 37 & 16 \\ 14 & 21 & 28 \\ -8 & -11 & 2 \end{pmatrix}$

(v) non-conformable (vi) $\begin{pmatrix} 28 & -18 \\ 26 & 2 \\ 16 & 25 \end{pmatrix}$

5 $\begin{pmatrix} -38 & -136 & -135 \\ 133 & 133 & 100 \\ 273 & 404 & 369 \end{pmatrix}$

6 (i) $\begin{pmatrix} 2x^2 + 12 & -9 \\ -4 & 3 \end{pmatrix}$ (ii) $x = 2$ or 3

(iii) $\mathbf{BA} = \begin{pmatrix} 8 & 12 \\ 8 & 15 \end{pmatrix}$ or $\begin{pmatrix} 18 & 18 \\ 12 & 15 \end{pmatrix}$

7 (i) (a) $\begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 8 & 7 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 16 & 15 \\ 0 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 2^n & 2^n - 1 \\ 0 & 1 \end{pmatrix}$

8 (i) $\begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

(ii) $\begin{pmatrix} 4 & 3 & 3 & 4 \\ 2 & 2 & 2 & 2 \\ 2 & 1 & 5 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}$ $\mathbf{M}^2$ represents the number of two-stage routes between each pair of resorts.

(iii) $\mathbf{M}^3$ would represent the number of three-stage routes between each pair of resorts.

9 (i) $\begin{pmatrix} 8 + 4x & -20 + x^2 \\ x - 8 & -3 - 3x \end{pmatrix}$

(ii) $x = -3$ or 4

(iii) $\begin{pmatrix} -4 & -11 \\ -11 & 6 \end{pmatrix}$ or $\begin{pmatrix} 24 & -4 \\ -4 & -15 \end{pmatrix}$

10 (i) $\mathbf{D} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$

$\mathbf{DA} = \begin{pmatrix} 299 & 199 & 270 & 175 & 114 \end{pmatrix}$

(ii) $\mathbf{F} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{AF} = \begin{pmatrix} 229 \\ 231 \\ 263 \\ 334 \end{pmatrix}$

(iii) $\mathbf{S} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{DAS} = (413)$,

$\mathbf{C} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{DAC} = (644)$

(iv) $\mathbf{P} = \begin{pmatrix} 0.95 \\ 0.95 \\ 1.05 \\ 1.15 \\ 1.15 \end{pmatrix}$,

$\mathbf{DAP} = (1088.95) = £1088.95$

11 (i) $\begin{pmatrix} b \\ a \\ c \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(iii) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} b \\ c \\ a \end{pmatrix}$

(iv) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} c \\ a \\ b \end{pmatrix}$

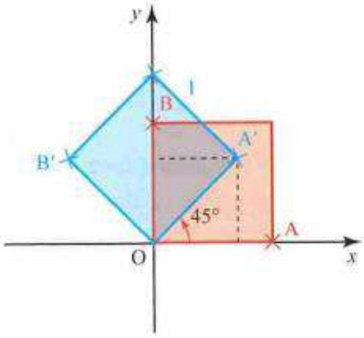
(v) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ The strands are back in the original order at the end of Stage 6.

Discussion point (Page 17)

The image of the unit vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is $\begin{pmatrix} a \\ c \end{pmatrix}$ and the image of the unit vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is $\begin{pmatrix} b \\ d \end{pmatrix}$.

Activity 1.3 (Page 17)

The diagram below shows the unit square with two of its sides along the unit vectors $\mathbf{i}$ and $\mathbf{j}$. It is rotated by 45° about the origin.



You can use trigonometry to find the images of the unit vectors $\mathbf{i}$ and $\mathbf{j}$.

For A′, the x-coordinate satisfies $\cos 45 = \frac{x}{1}$ so $x = \cos 45 = \frac{1}{\sqrt{2}}$.

In a similar way, the y-coordinate of A′ is $\frac{1}{\sqrt{2}}$.

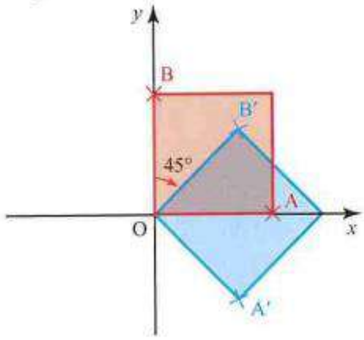
For B′, the symmetry of the diagram shows that the x-coordinate is $-\frac{1}{\sqrt{2}}$ and the y-coordinate is $\frac{1}{\sqrt{2}}$.

Hence, the image of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and the image of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and so the matrix representing an anticlockwise rotation of 45° about the origin is $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$.

Rotations of 45° clockwise about the origin and 135° anticlockwise about the origin are also represented by matrices involving $\pm\frac{1}{\sqrt{2}}$.

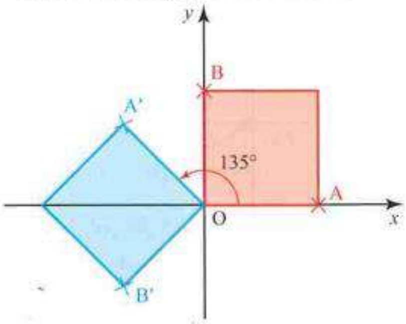
This is due to the symmetry about the origin.

(i) The diagram for a 45° clockwise rotation about the origin is shown below.



The image of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ and the image of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$, and so the matrix representing an anticlockwise rotation of 45° about the origin is $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$.

(ii) The diagram for a 135° anticlockwise rotation about the origin is shown below.



The image of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ and the image of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ and so the matrix representing an anticlockwise rotation of 45° about the origin is

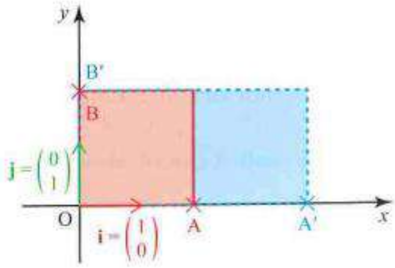
$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

Discussion point (Page 18)

The matrix for a rotation of θ° clockwise about the origin is $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

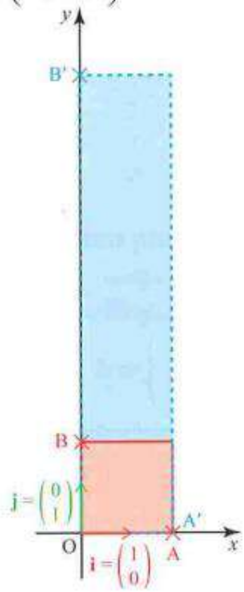
Activity 1.4 (Page 19)

(i) The diagram below shows the effect of the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ on the unit vectors **i** and **j**.



You can see that the vector **i** has image $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and the vector **j** is unchanged. Therefore this matrix represents a stretch of scale factor 2 parallel to the x-axis.

(ii) The diagram below shows the effect of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$ on the unit vectors **i** and **j**.



You can see that the vector **i** is unchanged and the vector **j** has image $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$. Therefore this matrix represents a stretch of scale factor 5 parallel to the y-axis.

The matrix $\begin{pmatrix} m & 0 \\ 0 & 1 \end{pmatrix}$ represents a stretch of scale factor m

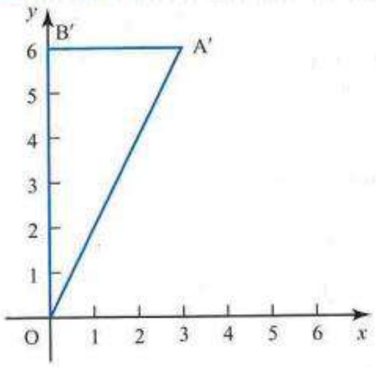
parallel to the x-axis.

The matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ represents a stretch of scale factor n

parallel to the y-axis.

Exercise 1.3 (Page 21)

1 (i) (a)

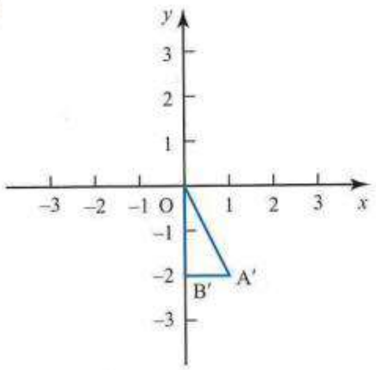


(b) $A' = (3, 6), B' = (0, 6)$

(c) $x' = 3x, y' = 3y$

(d) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

(ii) (a)

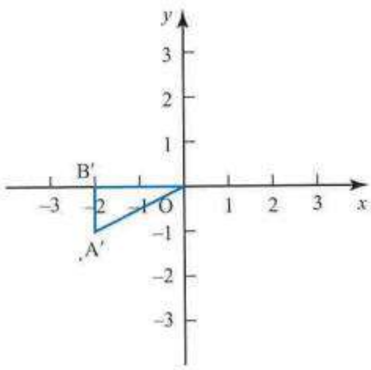


(b) $A' = (1, -2), B' = (0, -2)$

(c) $x' = x, y' = -y$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(iii) (a)

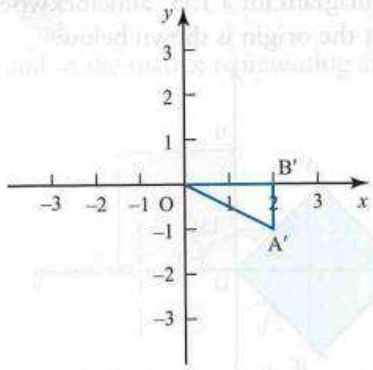


(b) $A' = (-2, -1), B' = (-2, 0)$

(c) $x' - y\ y' = -x$

(d) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(iv) (a)

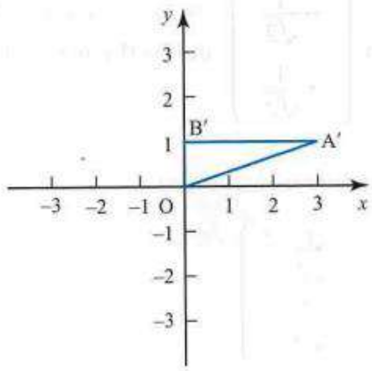


(b) $A' = (2, -1), B' = (2, 0)$

(c) $x' = y, y' = -x$

(d) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(v) (a)



(b) $A' = (3, 1), B' = (0, 1)$

(c) $x' = 3x, y' = \frac{1}{2}y$

(d) $\begin{pmatrix} 3 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

2 (i) Reflection in the x-axis

(ii) Reflection in the line $y = -x$

(iii) Stretch of factor 2 parallel to the x-axis and stretch factor 3 parallel to the y-axis

(iv) Enlargement, scale factor 4, centre the origin

(v) Rotation of 90° clockwise (or 270° anticlockwise) about the origin

3 (i) Rotation of 60° anticlockwise about the origin

(ii) Rotation of 55° anticlockwise about the origin

(iii) Rotation of 135° clockwise about the origin

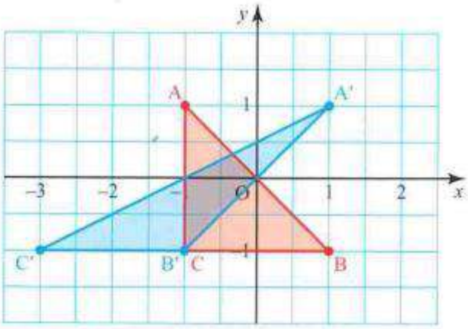
(iv) Rotation of 150° anticlockwise about the origin

4 (i) $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ (iv) $\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

5 $A'(4, 5), B'(7, 9), C'(3, 4)$. The original square and the image both have an area of one square unit.

6 (i)



(ii) The gradient of $A'C'$ is $\frac{1}{2}$, which is the reciprocal of the top right-hand entry of the matrix **M**.

7 (i) Rotation of 90° clockwise about the x-axis
(ii) Enlargement scale factor 3, centre (0, 0)
(iii) Reflection in the plane $z = 0$
(iv) Three-way stretch of factor 2 in the x-direction, factor 3 in the y-direction and factor 0.5 in the z-direction

8 (i) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ (ii) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

9 $(x, y) \rightarrow (x, x)$

The matrix for the transformation is $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$.

10 (i) Any matrix of the form $\begin{pmatrix} 5 & 0 \\ 0 & k \end{pmatrix}$ or $\begin{pmatrix} k & 0 \\ 0 & 5 \end{pmatrix}$.

If $k = 5$ the rectangle would be a square.

(ii) $\begin{pmatrix} \sqrt{2} & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & \sqrt{2} \end{pmatrix},$

$\begin{pmatrix} 1 & \sqrt{2} \\ 1 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 1 \\ \sqrt{2} & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 7 & \frac{3\sqrt{3}}{2} \\ 0 & \frac{3}{2} \end{pmatrix}, \begin{pmatrix} 0 & \frac{3}{2} \\ 7 & \frac{3\sqrt{3}}{2} \end{pmatrix},$

$\begin{pmatrix} \frac{3\sqrt{3}}{2} & 7 \\ \frac{3}{2} & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & \frac{3}{2} \\ 7 & \frac{3\sqrt{3}}{2} \end{pmatrix}$

Discussion point (Page 24)

(i) **BA** represents a reflection in the line $y = x$

(ii) The transformation A is represented by the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and the transformation B is represented by the matrix

$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The matrix product

$\mathbf{BA} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

This is the matrix which represents a reflection in the line $y = x$.

Activity 1.5 (Page 24)

(i) $\mathbf{P}' = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$

(ii) $\mathbf{P}'' = \begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$

$= \begin{pmatrix} pax + pby + qcx + qdy \\ rax + rby + scx + sdy \end{pmatrix}$

(iii)

$$\mathbf{U} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} pa + qc & pb + qd \\ ra + sc & rb + sd \end{pmatrix}$$

and so

$$\mathbf{UP} = \begin{pmatrix} pa + qc & pb + qd \\ ra + sc & rb + sd \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} pax + qcx + pby + rdy \\ rax + scx + rby + sdy \end{pmatrix}$$. Therefore $\mathbf{UP} = \mathbf{P}''$

Discussion point (Page 24)

AB represents 'carry out transformation B followed by transformation A.

(AB)C represents 'carry out transformation C followed by transformation AB, i.e. 'carry out C followed by B followed by A'.

BC represents 'carry out transformation C followed by transformation B'.

A(BC) represents 'carry out transformation BC followed by transformation A, i.e. carry out C followed by B followed by A'.

Activity 1.6 (Page 25)

(i) $\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$,

$\mathbf{B} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$

(ii)

$$\mathbf{BA} = \begin{pmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\sin\theta\cos\phi - \cos\theta\sin\phi \\ \sin\theta\cos\phi + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi \end{pmatrix}$$

(iii) $\mathbf{C} = \begin{pmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi) \end{pmatrix}$

(iv) $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

(v) A rotation through angle θ followed by rotation through angle ϕ has the same effect as a rotation through angle ϕ followed by angle θ.

Exercise 1.4 (Page 26)

1 (i) **A**: enlargement centre (0,0), scale factor 3
B: rotation 90° anticlockwise about (0,0)
C: reflection in the x-axis
D: reflection in the line $y = x$

(ii) $\mathbf{BC} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, reflection in the line $y = x$

$\mathbf{CB} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, reflection in the line $y = -x$

$\mathbf{DC} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, rotation 90° anticlockwise about (0, 0)

$\mathbf{A}^2 = \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}$, enlargement centre (0, 0), scale factor 9

$\mathbf{BCB} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, reflection in the x-axis

$\mathbf{DC^2D} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ returns the object to its original position

(iii) For example, $\mathbf{B}^4$, $\mathbf{C}^2$ or $\mathbf{D}^2$

2 (i) $\mathbf{X} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\mathbf{Y} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(ii) $\mathbf{XY} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, rotation of 180° about the origin

(iii) $\mathbf{YX} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(iv) When considering the effect on the unit vectors **i** and **j**, as each transformation only affects one of the unit vectors the order of the transformations is not important in this case.

3 (i) $\mathbf{P} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(ii) $\mathbf{PQ} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, reflection in the line $y = -x$

(iii) $\mathbf{QP} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(iv) The matrix **P** has the effect of making the coordinates of any point the negative of their original values,

i.e. $(x, y) \rightarrow (-x, -y)$

The matrix **Q** interchanges the coordinates,

i.e. $(x, y) \rightarrow (y, x)$

It does not matter what order these two transformations occur as the result will be the same

4 (i) $\mathbf{J} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ $\mathbf{K} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

$\mathbf{L} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\mathbf{M} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

(ii) (a) **LJ** (b) **MJ**
(c) $\mathbf{K}^2$ (d) **JLK**

5 (i) $\begin{pmatrix} 8 & -4 \\ -3 & 12 \end{pmatrix}$ (ii) $(32, -33)$

6 Possible transformations are $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, which is a rotation of 90° clockwise about the origin, followed by

$\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$, which is a stretch of scale factor 3 parallel to the x-axis. The order of these is important as performing **A** followed by **B** leads to the matrix $\begin{pmatrix} 0 & 1 \\ -3 & 0 \end{pmatrix}$. Could also have

$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$, which represents a stretch of factor 3 parallel to the y-axis, followed by

$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, which represents a rotation of 90° clockwise about the origin; again the order is important.

7 $\mathbf{X} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

A matrix representing a rotation about the origin has the form $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ and so the entries on the leading diagonal would be equal. That is not true for matrix **X** and so this cannot represent a rotation.

8 $\mathbf{Y} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

9 (i) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

(ii) A reflection in the x-axis and a stretch of scale factor 5 parallel to the x-axis

(iii) $\begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix}$

Reflection in the x-axis; stretch of scale factor 5 parallel to the x-axis; stretch of scale factor 2 parallel to the y-axis. The outcome of these three transformations would be the same regardless of the order in which they are applied. There are six different possible orders.

(iv) $\begin{pmatrix} \frac{1}{5} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$

10 (i) $\begin{pmatrix} 1 & -R_1 \\ 0 & 1 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & 0 \\ -\frac{1}{R_2} & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & -R_1 \\ -\frac{1}{R_2} & \frac{R_1}{R_2} + 1 \end{pmatrix}$

(iv) $\begin{pmatrix} 1+\frac{R_1}{R_2} & -R_1 \\ -\frac{1}{R_2} & 1 \end{pmatrix}$

The effect of Type B followed by Type A is different to that of Type A followed by Type B.

11 $a = \sqrt{\frac{\sqrt{2}+2}{4}}$ and $b = \sqrt{\frac{1}{2(\sqrt{2}+2)}}$

D represents an anticlockwise rotation of 22.5° about the origin.

By comparison to the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ for an anticlockwise rotation of θ about the origin, a and b are the exact values of $\cos 22.5°$ and $\sin 22.5°$ respectively.

Discussion point (Page 28)

In a reflection, all points on the mirror line map to themselves.

In a rotation, only the centre of rotation maps to itself.

Exercise 1.5 (Page 31)

1 (i) Points of the form $(\lambda, -2\lambda)$

(ii) (0, 0)

(iii) Points of the form $(\lambda, -3\lambda)$

(iv) Points of the form $(2\lambda, 3\lambda)$

2 (i) x-axis, y-axis, lines of the form $y = mx$

(ii) x-axis, y-axis, lines of the form $y = mx$

(iii) no invariant lines

(iv) $y = x$, lines of the form $y = -x + c$

(v) $y = -x$, lines of the form $y = x + c$

3 (i) Any points on the line $y = \frac{1}{2}x$, for example (0, 0), (2, 1) and (3, 1.5)

(ii) $y = \frac{1}{2}x$

(iii) Any line of the form $y = -2x + c$

(iv) Using the method of Example 1.11 leads to the equations

$$2m^2 + 3m - 2 = 0 \Rightarrow m = 0.5 \text{ or } -2$$

$$(4 + 2m)c = 0 \Rightarrow m = -2 \text{ or } c = 0$$

If $m = 0.5$ then $c = 0$ so $y = \frac{1}{2}x$ is invariant.

If $m = -2$ then c can take any value and so $y = -2x + c$ is an invariant line.

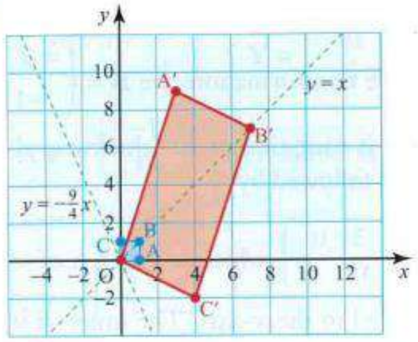
4 (i) Solving $\begin{pmatrix} 4 & 11 \\ 11 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ leads to the equations $y = -\frac{3x}{11}$ and $y = -\frac{11x}{3}$.

The only point that satisfies both of these is (0, 0).

(ii) $y = x$ and $y = -x$

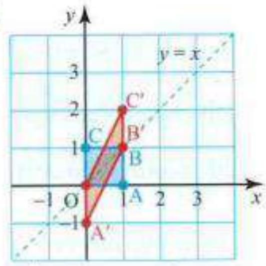
5 (i) $y = x$, $y = -\frac{9}{4}x$

(ii)



6 (i) $y = x$ (ii) $y = x$

(iii)



9 (i) $x' = x + a$, $y' = y + b$

(iii) (c) $a = -2b$

Transition Task: Geography

Subject	Qualification	Examination Board
Geography	A-Level	Edexcel
Additional Information:		

Task Overview:

Physical Geography:
Read the attached pdf document and answer the following questions
1. Assess the relative importance of the concept of vulnerability in understanding hazards impacts. (12)

2. Assess the usefulness of theoretical frameworks in understanding the prediction, impact and management of tectonic hazards. (12)

Human Geography:
Using your own research answer, the question 'Is the USA a developed country?'
Think about the influence of American brands, influence in global decisions, healthcare, education system, religion, gun laws.
You can present your answer in any format
Please see the additional resources on Schoology

Further wok will be available on Schoology. Please see log in details below

Success Criteria:

Extract information from a variety of sources and collate into a piece of extended writing.
You should use several sources in your answer

Resources:

Attached Pdf to help with the physical geography
https://app.schoology.com/ Class code: **MDBK-W9K5-F3C4B**
https://app.senecalearning.com/ Class code: **pmqiyw5auc**

TED talks are recommended for Human geography
See reading list for further sources

How the work produced will fit into subsequent work and the specification as a whole?

The first physical geography unit will be Tectonic hazards. The task will allow students to understand the underlying issues.
The first human geography unit will be Globalisation. We will be challenging stereotypes and investigating how countries are classified

How the work should be presented?

Physical geography tasks should be answered as exam questions. These can be hand written or typed.
The Human geography task can be presented in any format students think is suitable, essay, ppt etc

Who to contact if you should require further assistance with the work before the end of term?	Physical : c.dyer@gildredgehouse.org.uk Human: k.nicolle@gildredgehouse.org.uk
Length of time expected to complete tasks:	Research could take 3-4 hours in total Writing submissions should take no more than an hour per question
Submission Requirements:	To be submitted to your teacher during the first week of term

What equipment will be needed for the subject?

- 2 lever arched folders – one for human geography and one for physical
- A set of dividers and an ample supply of poly pockets would be useful

Optional Extension Task/Further Reading

In Geography, the news is important; it keeps you up to data with relevant information that can be used to make synoptic links with topic we study.
Over the summer, it would be useful to keep in touch with what is going on in the news nationally and internationally. Keep a notebook or folder containing news summaries. These could be from newspapers, news programs or documentaries.

Useful reading throughout your entire course:

- **Prisoners of Geography by Tim Marshall** – Splitting the world into 10 distinct regions suggest our key political driver continues to be our physical geography. It includes why China and India will never fall into conflict. One of the best books about geopolitics you could imagine!
- **Factfulness by Hans Rosling-** A radical explanation of why we systematically get the answers to questions about development wrong. Reveling 10 instinct that distort our perspective
- **The almighty dollar by Dharshini David**– Follows the $1 from a shopping trip in Texas, via China's central bank, Nigerian railroads, the oil fields of Iraq and beyond to reveal the complex relationships of our new globalized world.
- **Connectography by Parag Khanna** – A guide through the emerging global network civilization in which megacities compete over connectivity and boarders are increasingly irrelevant. Shows how a new foundation of connectivity is pulling together a world that appears to be falling apart
- **Divided by Tim Marshall** – There are many reasons why we put up walls. We are divided in many ways: wealth, religion, and politics. Understanding what has divided us, past and present, is essential to understanding much of what is going on in the world today.
- **Off the map by Alastair Bonnett** – From forgotten enclaves to floating islands, from hidden villages to New York gutter spaces, this book charts the hidden corners of our planet.
- **The silk roads by Peter Frankopan** – Our understanding of how the world is shaped by the narrow focus on Western Europe and the US. An antidote to Eurocentric accounts of the world, examining several continents and centuries and the factors that influenced the flow of goods and ideas
- **Worth dying for by Tim** Marshall **-** The histories, the power and the politics of the symbols that unite and divide us. We wave them and burn them and still in the 21st century, we die for them. We need to understand the symbols that people are rallying around.
- **Adventures in the Anthropocene by Gala Vince** – Our planet is said to be crossing in to the age of Humans. This book see what life is really like for people on the front line of the planet we've made, from artificial glaciers to electrified reefs.

- **10 billion by Stephen Emmott** – It's about failure: failure as individuals, the failure of businesses and the failure of politicians. It is about an unprecedented planetary emergency. It's about the future of us.
- **The Bottom Billion by Paul Collier** – Explains four traps that prevent the homelands of the world's billion poorest people from growing and receiving the benefits of globalization – civil war, natural resources, being landlocked and ineffective governance.
- **Peoplequake by Fred Pearce** – The population bomb is being defused. Half of the world's women are having two children or fewer and within a generation, the world's population will be falling, and we will all be getting very old.
- **This is the way the world ends by Jeff Nesbit** – Our world is in trouble right now. This book tells the real stories of the substantial impacts to Earth's systems unfolding across each continent from longer droughts in the Middle East to the monsoon season shrinking in India

Disaster Management Lifecycle

Disaster Resilience in the Built Environment

Learning Package 3: Disaster management lifecycle

Working in collaboration with ORBEE

Disaster management lifecycle

In this section you will:

- *Work to define the key stages of pre-disaster planning and post-disaster recovery*
- *Learn more about the role of built environment professionals in the different phases of disaster management*
- *Read about post-disaster reconstruction as a window of opportunity to address disaster risk*
- *Check what you have learned so far with reflective exercises*

Introduction

The process of disaster management is commonly visualised as a two-phase cycle, with post-disaster recovery informing pre-disaster risk reduction, and vice versa. The disaster management cycle illustrates the on-going process by which governments, businesses, and civil society plan for and reduce the impact of disasters, react during and immediately following a disaster, and take steps to recover after a disaster has occurred. The significance of this concept is its ability to promote the holistic approach to disaster management as well as to demonstrate the relationship between disasters and development.

Recovery and reconstruction are commonly identified within the post-disaster phase, the period that immediately follows after the occurrence of the disaster. Once a disaster has taken place, the first concern is effective recovery; helping all those affected to recover from the immediate effects of the disaster. Reconstruction involves helping to restore the basic infrastructure and services which the people need so that they can return to the pattern of life which they had before the disaster (Davis, 2005). The importance of the 'transitional phase', linking immediate recovery and long-term reconstruction, is also stressed by a number of publications (de Guzman, 2002; Max Lock Centre, 2006). With the recovery of social institutions, the economy and major infrastructure, efforts may shift to longer-term recovery and reconstruction.

Although the construction industry is traditionally associated with the long-term reconstruction phase of the management cycle, there is growing recognition that built environment professionals have a much broader role to anticipate, assess, prevent, prepare, respond, and recover from disruptive challenges. This learning package introduces the concept of a disaster management cycle and considers the role of the construction industry at different stages of the process, from pre-disaster planning and mitigation, through to longer term, sustainable reconstruction after the event.

Disaster management cycle

Disaster management aims to reduce, or avoid, the potential losses from hazards, assure prompt and appropriate assistance to victims of disaster, and achieve rapid and effective recovery (Warfield, 2004). The Disaster management cycle illustrates the on-going process by which governments, businesses, and civil society plan for and reduce the impact of disasters, react during and immediately following a disaster, and take steps to recover after a disaster has occurred. Appropriate actions at all points in the cycle lead to greater preparedness, better warnings, reduced vulnerability or the prevention of disasters during the next iteration of the cycle. The complete disaster management cycle includes the shaping of public policies and plans that either modify the causes of disasters or mitigate their effects on people, property, and infrastructure.

The mitigation and preparedness phases occur as disaster management improvements are made in anticipation of a disaster event. Developmental considerations play a key role in contributing to the mitigation and preparation of a community to effectively confront a disaster. As a disaster occurs, disaster management actors, in particular humanitarian organisations become involved in the immediate response and long-term recovery phases. The four disaster management phases illustrated in Figure 1 do not always, or even generally, occur in isolation or in this precise order. Often phases of the cycle overlap and the length of each phase greatly depends on the severity of the disaster.

Figure 1: Four phases of the disaster management cycle

Mitigation - Minimizing the effects of disaster.
Examples: building codes and zoning; vulnerability analyses; public education.

Preparedness - Planning how to respond.
Examples: preparedness plans; emergency exercises/training; warning systems.

Response - Efforts to minimize the hazards created by a disaster.
Examples: search and rescue; emergency relief.

Recovery - Returning the community to normal.
Examples: temporary housing; grants; medical care.

Mitigation

Mitigation activities actually eliminate or reduce the probability of disaster occurrence, or reduce the effects of unavoidable disasters. Mitigation measures include building codes; vulnerability analyses updates; zoning and land use management; building use regulations and safety codes; preventive health care; and public education.

Mitigation will depend on the incorporation of appropriate measures in national and regional development planning. Its effectiveness will also depend on the availability of information on hazards, emergency risks, and the countermeasures to be taken. The mitigation phase, and indeed the whole disaster management cycle, includes the shaping of public policies and plans that either modify the causes of disasters or mitigate their effects on people, property, and infrastructure.

Preparedness

The goal of emergency preparedness programs is to achieve a satisfactory level of readiness to respond to any emergency situation through programs that strengthen the technical and managerial capacity of governments, organisations, and communities. These measures can be described as logistical readiness to deal with disasters and can be enhanced by having response mechanisms and procedures, rehearsals, developing long-term and short-term strategies, public education and building early warning systems. Preparedness can also take the form of ensuring that strategic reserves of food, equipment, water, medicines and other essentials are maintained in cases of national or local catastrophes.

During the preparedness phase, governments, organisations, and individuals develop plans to save lives, minimise disaster damage, and enhance disaster response operations. Preparedness measures include preparedness plans; emergency exercises/training; warning systems; emergency communications systems; evacuations plans and training; resource inventories; emergency personnel/contact lists; mutual aid agreements; and public information/education. As with mitigations efforts, preparedness actions depend on the incorporation of appropriate measures in national and regional development plans. In addition, their effectiveness depends on the availability of information on hazards, emergency risks and the countermeasures to be taken, and on the degree to which government agencies, non-governmental organisations and the general public are able to make use of this information.

Response

The aim of emergency response is to provide immediate assistance to maintain life, improve health and support the morale of the affected population. Such assistance may range from providing specific but limited aid, such as assisting refugees with transport, temporary shelter, and food, to establishing semi-permanent settlement in camps and other locations. It also may involve initial repairs to damaged infrastructure. The focus in the response phase is on meeting the basic needs of the people until more permanent and sustainable solutions can be found. Humanitarian organisations are often strongly present in this phase of the disaster management cycle.

Recovery

As the emergency is brought under control, the affected population is capable of undertaking a growing number of activities aimed at restoring their lives and the infrastructure that supports them. There is no distinct point at which immediate relief changes into recovery and then into long-term sustainable development. There will be many opportunities during the recovery period to enhance prevention and increase preparedness, thus reducing vulnerability. Ideally, there should be a smooth transition from recovery to on-going development.

Recovery activities continue until all systems return to normal or better. Recovery measures, both short and long term, include returning vital life-support systems to minimum operating standards; temporary housing; public information; health and safety education; reconstruction; counselling programmes; and economic impact studies. Information resources and services include data collection related to rebuilding, and documentation of lessons learned.

Table 1 provides some examples of the type of activities or measures that might occur in each of the four disaster management phases, in respect of different types of disasters.

Table 1: Example of Measures in Each Disaster Risk Management Phase

Disaster / **Phase**	**Earthquake**	**Storm (cyclone, typhoon, hurricane)**	**Landslide**
Prevention / Mitigation	- Seismic design - Retrofitting of vulnerable buildings - Installation of seismic isolation/ seismic response control systems	- Construction of tide wall - Establishment of forests to protect against storms	- Construction of erosion control dams - Construction of retaining walls
Preparedness	- Construction and operation of earthquake observation systems	- Construction of shelter - Construction and operation of meteorological observation systems	- Construction and operation of meteorological observation systems
	- Preparation of hazard maps - Food & material stockpiling - Emergency drills - Construction of early warning systems - Preparation of emergency kits		
Response	- Rescue efforts - First aid treatment - Fire fighting - Monitoring of secondary disaster - Construction of temporary housing - Establishment of tent villages		
Recovery	- Disaster resistant reconstruction - Appropriate land use planning - Livelihood support - Industrial rehabilitation planning		

Activity

3.1 Expand Table 1by identifying examples of measures in each disaster risk management phase for:

(a) a flood

(b) a terrorist strike on a major retail centre

Humanitarian action

During a disaster, humanitarian agencies are often called upon to deal with immediate response and recovery. To be able to respond effectively, these agencies must have experienced leaders, trained personnel, adequate transport and logistic support, appropriate communications, and guidelines for working in emergencies. If the necessary preparations have not been made, the humanitarian agencies will not be able to meet the immediate needs of the people.

Sustainable development

Developmental considerations contribute to all aspects of the disaster management cycle. One of the main goals of disaster management, and one of its strongest links with development, is the promotion of sustainable livelihoods and their protection and recovery during disasters and emergencies. Where this goal is achieved, people have a greater capacity to deal with disasters and their recovery is more rapid and long lasting. In a development oriented disaster management approach, the objectives are to reduce hazards, prevent disasters, and prepare for emergencies. Therefore, developmental considerations are strongly represented in the mitigation and preparedness phases of the disaster management cycle. Inappropriate development processes can lead to increased vulnerability to disasters and loss of preparedness for emergency situations.

Post disaster reconstruction as a window of opportunity

Learning package 2 outlined the concept of a resilient built environment. If this concept is appealing, how can it be achieved? Despite the disaster management lifecycle's emphasis on pre-disaster planning, it frequently requires a major disaster to initiate a window of opportunity to address many of the vulnerabilities usually encountered in a community's built environment. There are several features of this post-disaster period that can be capitalised upon.

Firstly, the disaster has destroyed much of the built environment that was improperly designed and vulnerable, creating a fresh start from which to address disaster risk. Furthermore, the experience gained during the disaster typically generates new knowledge, which brings various stakeholders together around a shared awareness of the nature of risk. The mistakes of previous development policies and strategies are exposed and can be addressed. Next and perhaps even more significantly, the political will and desire to act is almost certainly stronger than usual. Any interest in disaster risk reduction that had been forgotten or side-lined before the disaster, will suddenly gain renewed prominence in the recovery period. In a similar vein, the lack of resourcing for risk reduction, any presence of corruption and otherwise weak institutional structures that allowed a vulnerable built environment to be constructed will have been highlighted. Finally, but perhaps most importantly, the post-disaster period often provides a level of resourcing, including considerable external funding, that would be otherwise unattainable. If properly utilised – something that is by no means certain – this additional resource does afford a major opportunity to reduce vulnerability.

The fact that this window of opportunity exists does not mean that the various actors involved in reconstruction will take advantage of it. Although many, if not all, of these features are usually present following a major disaster, even a cursory glance at the countless studies and evaluations of programming after disasters, provides evidence that it is frequently a missed opportunity.

There are a myriad of reasons as to why these failures occur. Humanitarian principles are primarily concerned with addressing acute human suffering. By necessity, a timely response is essential. Anything that slows this response is likely to be a problem. Unfortunately, the well-planned reconstruction of a more resilient built environment will take time. Likewise, humanitarian principles also tend to dictate maintaining independence, neutrality and impartiality. This can dissuade actors from highlighting previous failings, which would otherwise create the necessary political will for change.

Effective reconstruction of the built environment is also competing with many other priorities. Poverty alleviation, improved health, and good governance are a few of the many goals usually mainstreamed in the post-disaster recovery period. A more resilient built environment can certainly contribute to these goals, but there will inevitably be a time-lag; other recovery programmes can sometimes appear more appealing due to their ability to deliver short term results. If the window of opportunity is to be taken advantage of, then advocates of a more resilient built environment will need to demonstrate the vital role it plays in helping society achieve much broader development goals.

A further complication is the natural tension between the need for timely reconstruction and a desire to utilise and where necessary develop local capacity. Institutions and local enterprise to plan and construct the built environment may matter, but they are often simply not there. Government, both national and local, is usually called upon to make critical long term planning decisions, and to develop and enforce appropriate building regulations. This expectation is made of institutions that have usually failed to achieve this in far less challenging periods. The reality is that large scale reconstruction may have to be undertaken during a period soon after a major part of the civil service has perished, or at least been severely disrupted. At a time when even greater demands are being made of the civil service, its employees are sometimes being laid off, with the damage to the local tax base reducing available funding. At the same time, the local construction industry is suddenly called upon to increase its output to meet the needs of an unprecedented programme of reconstruction activity, while simultaneously familiarising itself with less vulnerable methods and materials. Building human resources and local capacity to address these shortfalls and support reconstruction, may take years.

The alternative, to make use of international agencies and private enterprises, understandably raises other concerns. International actors are often accused of poaching the most talented local civil servants and encroaching on a country’s independence, while the private sector is accused of disaster profiteering and leaves local industry unable to ‘benefit’ from the economic opportunities afforded by the disaster.

In summary, there is a window of opportunity, but it is beset with challenges. A pragmatic approach to the development of a resilient built environment needs to include an understanding of these difficulties and their implications for what can actually be done, at least in the short term. While the humanitarian efforts are frequently a rushed process, effective rebuilding for resilience will require reflection, discussion and consensus building. This should not undermine the importance of starting this process early in the recovery phase; indeed, a failure to consider long term reconstruction goals early in the recovery can lead to wasted or misguided effort, as well as undermine efforts for future resilience. Instead, it recognises the importance of a judicious approach that addresses the complexity of creating resilience.

Activity

3.2 Summarise the related challenges associated with reducing vulnerability:

(a) before a disaster occurs

(b) in the aftermath of a major disaster

Role of built environment professionals in disaster management

The recovery role of construction from both natural and human disasters is well documented. In particular, post-disaster reconstruction has been the subject of a significant body of research, with particular emphasis on developing countries that are less able to deal with the causes and impacts of disasters. The importance of improving the construction industries of developing nations is widely recognised, highlighting a need to equip them to manage recovery. Construction is typically engaged in a range of critical activities: temporary shelter before and after the disaster; restoration of public services such as hospitals, schools, water supply, power, communications, and environmental infrastructure, and state administration; and, securing income earning opportunities for vulnerable people in the affected areas. Similarly, disaster planners have begun to realise the link between disaster and development – a large and well-established field relating to social, economic, and significantly from a construction perspective, physical aspects of society.

Although more robust construction in and of itself will not eliminate the consequences of disruptive events, there is widespread recognition that the engineering community has a valuable role to play in finding and promoting rational, balanced solutions to what remains an unbounded threat. There has been considerable research aimed at developing knowledge that will enable the construction of a generation of buildings that are more resilient and safer, for example, through reduction of injury inducing blast debris, the development of glazing materials that do not contribute to the explosion-induced projectiles and have enhanced security application, as well as the integration of site and structure in a manner that minimises the opportunity for attackers to approach or enter a building.

The pre-disaster phase of the disaster management cycle includes both mitigation and preparedness. Disaster mitigation refers to any structural and non-structural measure undertaken to limit the adverse impacts of natural hazards, environmental degradation, and technological hazards. Mitigation measures may eliminate or reduce the probability of disaster occurrence, or reduce the effects of unavoidable disasters. These measures may include building codes; vulnerability analyses updates; zoning and land use management; building use regulations and safety codes. Mitigation seeks to eliminate the risk of future disasters by effective sharing of lessons learned through preparedness planning.

Construction managers have a key role to play because they are involved in the construction of the infrastructure, and therefore should also be involved when an event destroys that infrastructure. Construction management skill in getting equipment, scheduling a set of activities to accomplish a task, and knowing how to manage those activities can be very valuable when an extreme event occurs. Moreover, construction engineers possess valuable information about their projects, and that information can be critical in disaster preparedness, as well as response and recovery. The information they possess may be the difference between life and death. In a similar vein, the Max Lock Centre (2006) concluded that chartered surveyors, with appropriate training, have key roles to play during all disaster phases, from preparedness to immediate relief, traditional recovery and long-term reconstruction (see scenario 1).

Further reading

The details of some related articles on the disaster management lifecycle are provided in 'Reading Material'. Compare these authors' understanding of the lifecycle and built environment professionals' role within it, to those of the examples provided in the case study and scenario.

References

Davis, I. (2005) What Makes a Disaster, http://tilz.tearfund.org/Publications/Footsteps+11-20/Footsteps+18, accessed on 18th December 2006.

Guzman, M. de (2002) The total disaster risk management approach: an introduction, paper presented at the Regional Workshop on Networking and Collaboration among NGOs of Asian Countries in Disaster Reduction and Response, 20-22 February, Kobe, Japan.

Max Lock Centre (2006) Mind the Gap, Royal Institute of Chartered Surveyors, London.

Warfield, C. (2008) The Disaster Management Cycle. http://www.gdrc.org/uem/disasters/1-dm_cycle.html (accessed on March 25, 2008).

Subject	Qualification	Examination Board
History	A Level	AQA
Additional Information:		

Task Overview:

For 'The American Dream: Reality and Illusion 1945-80'

First and foremost, please buy a copy of this textbook ready for lesson 1: Oxford AQA History for A Level: The American Dream: Reality and Illusion 1945-1980 Paperback – 1 Oct 2015 by Mark Stacey (Author), Sally Waller (Series Editor) ISBN-10: 019835455X

We will be using this textbook in the majority of our lessons, so it is imperative you order it in enough time for the start of term.

1. Research and complete a Presidential timeline to include FDR, Truman, Eisenhower, JFK, LBJ, Nixon, Ford and Carter. Your timeline should include the following:
   - The dates and political party of each President
   - A comment on their background and character/popularity
   - A minimum of four key events from their Presidency. For an extra challenge, think about how you can colour code your events into different categories to show themes across the timeline, e.g. Civil Rights, economy etc.)
2. Research the US political system and produce an annotated diagram to show the key components and how these relate to one another.
3. Watch the documentary series 'Eyes on the Prize', episodes 1-5. Make notes on each of the key events that are covered, and think about the success of each. All episodes are available on YouTube here: https://www.youtube.com/watch?v=Ts10IVzUDVw&t=42s
4. Use the following playlist on Youtube to research the beginning of the Cold War. Episode 1 in mandatory, and anything up to episode 5 is desirable – particularly if the Cold War is going to be completely new to you.
   https://www.youtube.com/playlist?list=PLygA1_PUbd9dbrhm9h7a_ZCMna4j7GsZC

For **'The Tudors: England, 1485-1603'**

Please complete the below tasks. I have also attached reading to help you complete these tasks.

Alongside the America textbook, please purchase a copy of the following book: Oxford AQA History for A Level: The Tudors: England 1485-1603 Paperback – 3 Sep 2015 by Michael Tillbrook (Author), Sally Waller (Series Editor) ISBN-10: 0198354606

1. Produce one A4 page of relevant and interesting information on each of the 4 Tudor monarchs. You may want to focus around the following areas: Social structures & society, the economy, religion and politics. Your information pages should include visual images and 5-10 bullet points on each monarch (4 pages in total).
2. Write a mini-essay (1.5 pages) to evaluate who you think was the most significant Tudor monarch from your research. Do not panic over this – we are not expecting you to be experts on this topic (yet!). It is more a chance for us to see your writing style for the first time.
3. This is an optional extra… Read and/or watch "Wolf Hall" by Hilary Mantel. Or look up and watch any of the many films on Elizabeth or on the Tudor period – there is so much. Just a short google, scroll through Netflix or flick though a book store will throw up lots of interesting dramas to get stuck in to… just be wary of their historical accuracy!

Success Criteria:
Displaying good research and writing skills.

Resources:
- Eyes on the Prize, 1-5: https://www.youtube.com/watch?v=Ts10IVzUDVw&t=42s - Cold War Crash Course, 1-5: https://www.youtube.com/playlist?list=PLygA1_PUbd9dbrhm9h7a_ZCMna4j7GsZC

How will the work produced will fit into subsequent work and the specification as a whole?
This work will form a base level of understanding to allow us to delve deeper into topics from the offset.

How should the work should be presented?
A physical copy of all work should be brought to the first lesson of the academic year.

Who should you contact if you should require further assistance with the work before the end of term?	Please contact Miss Ince, the Head of History and Politics: g.ince@gildredgehouse.org.uk
Length of time expected to complete tasks:	20 hours
Submission Requirements:	

What equipment will be needed for the subject?
• Two lever arch folders with dividers • Notepad and pen • Aforementioned AQA America and Tudor textbooks
Optional Extension Task/Further Reading

Subject	Qualification	Examination Board
Mathematics	A Level	Edexcel
Additional Information:		

Task Overview:
Starting with confidence booklet. A hard copy will be provided on transition day. Copies can be emailed on request.
Success Criteria:
Students should complete the booklet, following the instructions carefully.
Resources:
There are hints, tips and worked examples within the booklet. We have also given links to videos and question banks if additional help is required.
How the work produced will fit into subsequent work and the specification as a whole?
The booklet covers some of the key skills from GCSE Maths that we will build upon during the A Level course. The content will be tested during the first week of school in September.
How the work should be presented?
Students should write in the booklet and follow the instructions carefully.

Who to contact if you should require further assistance with the work before the end of term?	Mrs Dowle j.dowle@gildredgehouse.org.uk
Length of time expected to complete tasks:	Varies depending on current fluency in the subject
Submission Requirements:	Bring completed booklet to your first lesson in September

What equipment will be needed for the subject?
Year 1 Text books Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS Textbook • ISBN-10: 129218339X / ISBN-13: 978-1292183398 Edexcel AS and A level Mathematics Statistics & Mechanics Year 1/AS Textbook • ISBN-10: 1292232536 / ISBN-13: 978-1292232539 A4 Binder with dividers and lined paper Blue or black pen, different colour pen, 30cm ruler, pencil, eraser, sharpener An appropriate scientific calculator. The minimum requirement is for the **'Classwiz'** Casio fx-991EX (£25-£30)
Optional Extension Task/Further Reading

Gildredge House

Yr12 Summer Transition Task

Gildredge House

Mathematics

STARTING WITH CONFIDENCE

Name:……………………………………………………

This booklet has been designed to help you to bridge the gap between GCSE Maths and starting A level Maths.

Please read the information below carefully before you start this booklet.

Things to do:

1. Complete this booklet (use flowchart on page 5) and bring it with you to your **first lesson** in September. Your teacher will expect this to be **100% complete and correct.**
   Please print out the booklet if you have received it via email.

   Compulsory sections: **Mini Test A** p16, **Mini Test B** p27, **Applied Section** p35-39 and **Are you ready for A Level Test** p40.
   Remember to use the **flow chart on page 5** to help you decide whether on not you should do more than this.

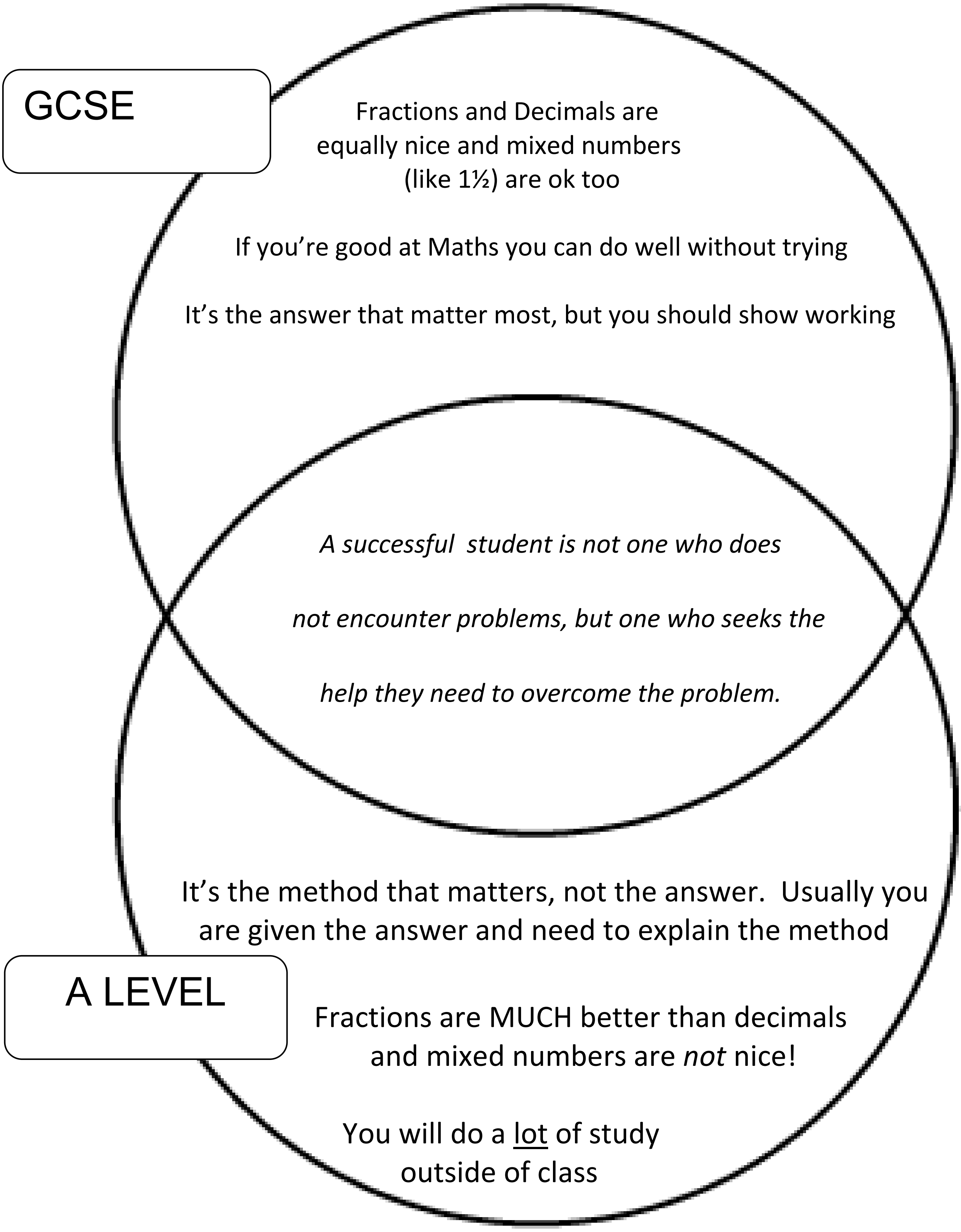
2. Mark all questions as you go along, using the answers from p41 onwards. **But don't look at the answers to help you work out the question**! Marking only.

3. Complete, mark and record your score for **'Are you ready for A level?'** test on page 40 along with the **Applied Maths Section** on page 35 . Bring this to your first maths lesson. Your teacher will ask to see these.

4. Make sure you are ready for the test in your second maths lesson. It will be on the topics in section A and B of this booklet to ensure you have the skills needed to be successful.

5. Make sure that you have an appropriate scientific calculator. A level is a big step up from GCSE and you will need a more advanced calculator. The minimum requirement is for the **'Classwiz'** Casio fx-991EX (£25-£30).

SIMILARITIES AND DIFFERENCES BETWEEN GCSE AND A LEVEL MATHS



When (not if) you get stuck…..

Studying Mathematics at advanced level is about learning how to solve problems. The first stage of solving a problem is being stuck so you should expect to get stuck while working through this booklet. Some of these topics may seem unfamiliar to you but they are all GCSE level topics and you need to be able to do all these techniques **before** you start A Level Mathematics.

So, when you get stuck…

- Look up the topic https://www.mathsgenie.co.uk/gcse.html.
- Look again at the examples. Maybe there is one which shows you how to solve your problem?
- Have you made a mistake? It might be that your method is correct but you've made an error in your working somewhere.
- Try looking up the topic in a GCSE higher tier textbook or revision guide (you can get these from your local library) or look online
- **Email Mrs Dowle** j.dowle@gildredgehouse.org.uk . Or call a friend who is also starting A Level Mathematics in September - you might be able to work it out between you.
- During term time we offer support most days of the week for A level students – there are plenty of opportunities to get help!

CONTENTS

Part A – Learning to Avoid common algebraic 'Mistakes'.

We all make occasional mistakes when manipulating algebra and learning to make fewer mistakes (and finding the ones you have made!) is an important part of the study of Maths at advanced level. However, there are also mistakes that aren't mistakes at all but are actually the result of a deeply held misunderstanding about the laws of algebra. These misunderstandings need to be exterminated as soon as possible. Do you understand why these examples are wrong?

$$\frac{2+x^2}{3x} = \frac{2+x}{3}$$

WRONG!! Exterminate!!

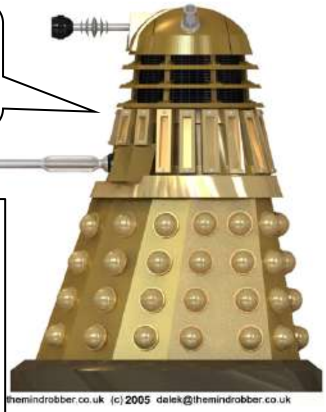
$$(a+b)^2 = a^2 + b^2$$

WRONG!! Exterminate!!

$$\frac{2-x}{3x^2} = \left(\frac{2}{3}\right)\left(\frac{-x}{x^2}\right)$$

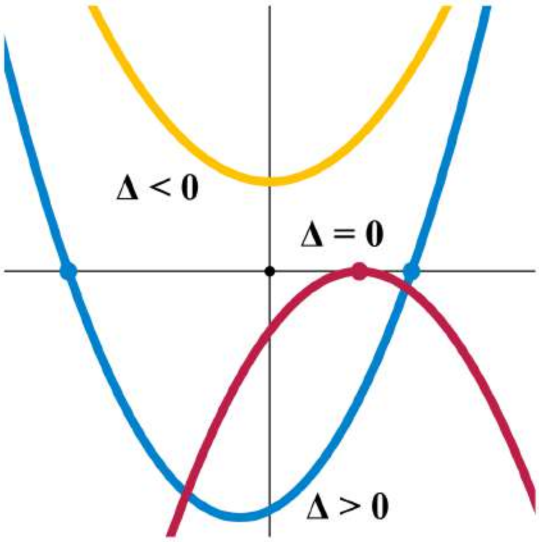
WRONG!! Exterminate!!

EXTERMINATE!!
Exterminate!!



Part B – Developing Confidence with Quadratics

A quadratic is any algebraic expression with some x^2 bits and some x bits and a number i.e. $ax^2 + bx + c$. In GCSE maths you will have met and learned to solve quadratic equations. In order to cope with the demands of AS Maths you need to be <u>confident</u> working with quadratics and this is something we have found to cause a lot of problems in the transition from GCSE to A'Level Maths. This part of the booklet will outline everything you need to remember about quadratics and give you a chance to practise building your confidence with these important equations.



You should recognise these curves as quadratic curves.

On page 18 of this booklet you will learn what Δ is and how to measure and interpret it for any quadratic ☺

PS. You should be able to complete this entire booklet WITHOUT using a calculator (with the exception of the applied section)

SUGGESTED STUDY PLAN

Do you feel really confident with all of the grade 7-9 techniques that you learnt at GCSE?

No

We will not have time to cover these techniques in class next year, but you ARE required to know them when you start the A level course. Therefore you need to practise over the Summer. The exercises in this booklet are designed to help you do that. It would be better if you practise little and often, rather than a lot all at once.

Look at the breakdown of topics and approximate timings on the back page and plan your time appropriately.

Work through the exercises in Part A until you are confident with all of the techniques.

On a different day, do the section A mini-test. Did you pass (and do better than last time)?

No

Yes

Work through the exercises in Part B until you are confident with all of the techniques.

On a different day, do the section B mini-test. Did you pass (and do better than last time)?

No

Yes

Yes

Do the mini-tests at the end of Part A (pg 16) and Part B (pg 27). Did you score …?

Less than 60%

You need to be much more confident with these techniques before September. Work through the whole booklet carefully (again) and use the "When You Get Stuck" tips on page 2 to help you make progress.

60-90%

Pretty good but there are obviously some areas you still need to work on. Identify these sections in the booklet. Go through the examples carefully and do the exercises (again!)

More than 90%

This is a really good score – **well done!** Go over your mistakes. What mistakes did you make? How could you avoid making them in the future? Use the examples and exercises in the booklet to help you.

Finally, make sure you have gone through the booklet and collected together the common mistakes (indicated by daleks). Add them to your table at the back of the booklet together with a correction / explanation.

In the last week of the holidays, do the "Are you Ready for A level Test?" (pg 38). Did you score…?

Less than 60%

Go through the exercises again where you are having problems. Consider attending the support sessions available.

60-80%

Identify the areas where you are making mistakes. Go through the relevant exercises again.

More than 80%

Well done – you have the necessary building blocks in place in order to start A level Maths with confidence.

Part A – Section 1 - FRACTIONS

TOP TIP! Never use a slanted line like this $1/2x$ because the *x* will try to escape by moving right a bit and growing

$\frac{1}{2x} \ldots 1/2x \rightarrow 1/2\,x \rightarrow 1/2\;x = \frac{1}{2}x = \left(\frac{1}{2}\right)\left(\frac{x}{1}\right) = \frac{x}{2}$.

It is much harder for the *x* to escape if you use a horizontal line.

TOP TIP! You will make fewer mistakes if you write things next to each other like 3*x* rather than $3 \times x$ and $\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)$ rather than $\frac{2}{3} \times \frac{4}{5}$.

TOP TIP! If you want to multiply a fraction by a number, you can write the number as a fraction by putting it over 1: $5 \times \frac{x}{2} = \left(\frac{5}{1}\right)\left(\frac{x}{2}\right) = \frac{5x}{2}$. This avoids the possibility of making the **common mistake** that $5 \times \frac{x}{2} = \frac{5x}{10}$

Exercise 1a) In the spaces available, carry out the following, leaving your answer as a single fraction.

(1) $\frac{3x}{4} \times 5$ (hint: look at top tip 3 if you are not sure about this)	(2) $\frac{2}{x} + \frac{3}{x^2}$ (hint: make the denominators the same by multiplying top and bottom of $\frac{2}{x}$ by *x*, then add the numerators)	(3) $\frac{3x}{2} \div 5$ (hint: use top tip 3 then remember that dividing by a fraction is the same as multiplying by its reciprocal)
Answers at the back Tick when you're correct..... ☐	Tick when correct………… ☐	Tick when correct………… ☐

<u>Exercise 1b)</u> In the spaces available, carry out the following (Q1-4 WITHOUT A CALCULATOR!), leaving your answer as a single fraction (Answers are at the bottom of this page)

(1) $\frac{3}{2} \div \frac{1}{4} \div 3$	(2) $\left(\frac{3}{2} \times \frac{1}{4}\right) + 3$	(3) $\left(\frac{12}{11} - \frac{4}{3}\right) \div \frac{1}{3}$
Answers at the bottom Tick when you're correct..... ☐	Tick when correct............ ☐	Tick when correct............ ☐
(4) $\frac{-2}{x} + \frac{3}{2}$	(5) $\frac{5}{x} + \frac{2x}{5}$	(6) $\frac{2x+7}{2} - \frac{3}{5}$
Tick when you're correct..... ☐	Tick when correct............ ☐	Tick when correct............ ☐

ANSWERS to part A section 1

1) 2 2) $\frac{27}{8}$ 3) $-\frac{8}{11}$ 4) $\frac{3x-4}{2x}$ 5) $\frac{(25+2x^2)}{5x}$ 6) $\frac{10x+29}{10}$

Part A – Section 2 - INDICES

$$2^{3x} = 2^3 2^x$$ WRONG!!!

Students often think that if there is multiplication in the powers it must correspond to multiplication.

In fact, $2^{3x} = (2^x)^3$ or $2^{3x} = (2^3)^x = 8^x$.

$$2^{x+1} = 2^x + 2^1$$ WRONG!!!

Students often think that if there is addition in the power it must correspond to addition. In fact, $2^{x+1} = 2^x 2^1 = 2(2^x)$.

Answers are on p41

Exercise 2 Evaluate the following, tick the boxes when they are correct:

THE RULES OF INDICES

Rules: $a^m a^n = a^{m+n}$ $\qquad \frac{a^m}{a^n} = a^{m-n}$ $\qquad (a^m)^n = a^{mn}$

Also: $(ab)^n = a^n b^n$ $\qquad a^0 = 1$ $\qquad a^1 = a$

A negative power indicates a reciprocal e.g 6^{-2} means $\frac{1}{6^2}$ = $\frac{1}{36}$ and 5^{-3} means $\frac{1}{5^3}$ = $\frac{1}{125}$	(1) 2^{-6}	☐
	(2) $9^{-\frac{1}{2}}$	☐
Example: $4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = \left(\sqrt{4}\right)^3 = 2^3 = 8$ Tick the box when you understand! ☐	(3) $81^{-\frac{1}{4}}$	☐
A fractional power indicates a root a power of $\frac{1}{2}$ means 'square root'. $25^{\frac{1}{2}} = \sqrt{25} = 5$ a power of $\frac{1}{3}$ means 'cube root'. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$	(4) $4^{\frac{5}{2}}$	☐
	(5) $32^{\frac{3}{5}}$	☐
Example: $144^{-\frac{1}{2}} = \frac{1}{144^{\frac{1}{2}}} = \frac{1}{\sqrt{144}} = \frac{1}{12}$ Tick the box when you understand. ☐	(6) $16^{-\frac{7}{4}}$	☐

Indices continued (What you need for A level)

It is very useful to mathematicians to be able to write algebraic expressions in different ways and one of the most important ways is in the form (number) $^{x \text{ power}}$

$$\frac{2}{3x^2} = 6x^{-2}$$

WRONG!! Actually,

$$\frac{2}{3x^2} = \left(\frac{2}{3}\right)\left(\frac{1}{x^2}\right) = \frac{2}{3}x^{-2}$$

$$\frac{3}{x} = 3^{-x}$$

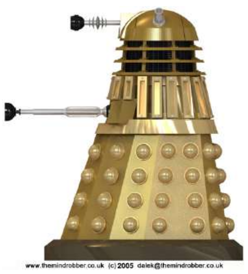
WRONG!! Actually,

$$\frac{3}{x} = \left(\frac{3}{1}\right)\left(\frac{1}{x}\right) = 3x^{-1}$$

$$\frac{1}{2x} = 2x^{-1}$$

WRONG!! Actually,

$$\frac{1}{2x} = \left(\frac{1}{2}\right)\left(\frac{1}{x}\right) = \frac{1}{2}x^{-1}$$

Examples of writing things in the form αx^n. Tick the box when you understand.	**Now try Exercise 3:** Write these in the form αx^n. Tick when correct.
$\frac{2x}{3} = \left(\frac{2}{3}\right)\left(\frac{x}{1}\right)$ $= \frac{2}{3}x$ ☐	(1) $\frac{x}{5} =$ ☐
$\frac{2}{5x} = \left(\frac{2}{5}\right)\left(\frac{1}{x}\right)$ $= \frac{2}{5}x^{-1}$ ☐	(2) $\frac{3}{2\sqrt{x}} =$ ☐
$\frac{x}{3\sqrt{x}} = \left(\frac{1}{3}\right)\left(\frac{x}{\sqrt{x}}\right)$ $= \frac{1}{3}x^{1-\frac{1}{2}}$ ☐ $= \frac{1}{3}x^{\frac{1}{2}}$	(3) $\frac{\sqrt{x}}{3x^2} =$ ☐
$2\sqrt{16x^3} = 2\sqrt{16}\sqrt{x^3}$ $= 8\left(x^3\right)^{\frac{1}{2}}$ $= 8x^{\frac{3}{2}}$ ☐	(4) $\sqrt[3]{8x^2} =$ ☐
$\frac{2+x}{\sqrt{x}} = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{x}}$ $= \left(\frac{2}{1}\right)\left(\frac{1}{x^{\frac{1}{2}}}\right) + x^{\left(1-\frac{1}{2}\right)}$ $= 2x^{-\frac{1}{2}} + x^{\frac{1}{2}}$ ☐	(5) $\frac{2\sqrt{x}+4}{x^2} =$  ☐

Note: This one has two terms so is written in the form $\alpha x^n + \beta x^m$

More practice of **the most important type of indices…** Write these in the form $\alpha x^n + \beta x^m$. Tick the boxes when they are correct.

(6) $\dfrac{2x-4}{3x^2} = \dfrac{2x}{3x^2} - \dfrac{4}{3x^2}$

$= \left(\dfrac{2}{3}\right)\left(\dfrac{x}{x^2}\right) - \left(\dfrac{4}{3}\right)\left(\dfrac{1}{x^2}\right)$

$= \dfrac{2}{3}x^{-1} - \dfrac{4}{3}x^{-2}$

Understand?............. ☐

(7) $\dfrac{1-4x}{4x^3} =$ ☐

(8) $\dfrac{1-4\sqrt{x}}{x} =$ ☐

(9) $\dfrac{x^2-3}{\sqrt{x}} =$ ☐

(10) $\dfrac{x-2}{x^2} =$ ☐

(11) $\dfrac{2+\sqrt{x}}{\sqrt{x}} =$ ☐

(12) $\dfrac{2x+4}{4x} =$ ☐

(13) $\dfrac{\sqrt{x}+6}{3x^2} =$ ☐

(14) $\dfrac{2x-1}{x^2} =$ ☐

<table>
<tr><td>Examples of solving index equations by doing the same thing to both sides. Tick when understood.</td><td colspan="2">Exercise 3 continued:
Solve each of the following equations for x. Tick when correct.</td></tr>
<tr><td>$x^{-\frac{1}{2}}=3$
$\left(x^{-\frac{1}{2}}\right)^{-1}=3^{-1}$
$x^{\frac{1}{2}}=\frac{1}{3}$
$\left(x^{\frac{1}{2}}\right)^2=\left(\frac{1}{3}\right)^2$
$x=\frac{1^2}{3^3}=\frac{1}{9}$ ☐</td><td>(15) $x^{-\frac{2}{3}}=9$
☐</td><td>(16) $x^{-\frac{1}{2}}=5$
☐</td></tr>
<tr><td>$x^{\frac{2}{5}}=2$
$\left(x^{\frac{2}{5}}\right)^5=2^5$
$x^2=32$
$x=\sqrt{32}$
$x=\sqrt{16}\sqrt{2}$
$x=4\sqrt{2}$ ☐</td><td>(17) $x^{\frac{2}{5}}=4$
☐</td><td>(18) $x^{\frac{2}{3}}=16$
☐</td></tr>
<tr><td>$x^{\frac{2}{3}}=\frac{4}{9}$
$\left(x^{\frac{2}{3}}\right)^{\frac{1}{2}}=\left(\frac{4}{9}\right)^{\frac{1}{2}}$
$x^{\frac{1}{3}}=\frac{\sqrt{4}}{\sqrt{9}}$
$\left(x^{\frac{1}{3}}\right)^3=\left(\frac{2}{3}\right)^3$
$x=\frac{2^3}{3^3}$
$x=\frac{8}{27}$ ☐
Note: think about how much harder this would have been if we had started by cubing both sides rather than square rooting. It would still work but it would have been more difficult.</td><td>(19) $x^{\frac{3}{4}}=\frac{1}{27}$
☐
With this question, is it easiest to start by cube rooting each side or by raising each side to the
power 4?</td><td>(20) $x^{\frac{2}{3}}=\frac{1}{25}$
☐</td></tr>
</table>

Part A – Section 3 - SURDS

A surd is an IRRATIONAL ROOT e.g., $\sqrt{2}$, $\sqrt{3}$, etc., but not $\sqrt[3]{8}$ because $\sqrt[3]{8}=2$

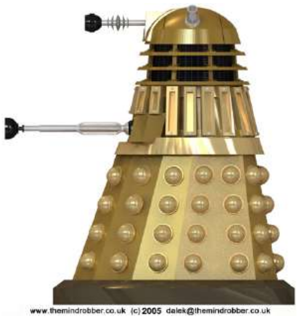
ANOTHER TOP TIP! When you write a root, make sure that it has a top which goes over everything in the root otherwise things can jump out without you noticing. √4*x* could mean $\sqrt{4x}$ which is 2*x* or it might mean $\sqrt{4}x$ which is $2\sqrt{x}$.

$$\frac{\sqrt{12}}{2}=\sqrt{6}$$ NO!!

$$\frac{\sqrt{12}}{2}=\frac{\sqrt{12}}{\sqrt{4}}=\sqrt{\frac{12}{4}}=\sqrt{3}$$

$$\sqrt{16+x^2}=4+x$$ WRONG!!!

Students often make up the rule that a power can be applied to the two terms of a sum separately. Actually, nothing can be done to simplify this expression.



Examples. Tick when you understand.	**Now try exercise 4:** Simplify into the form $a\sqrt{b}$. Tick when correct.
Multiplication and roots: $\sqrt{ab}=\sqrt{a}\sqrt{b}$ $\sqrt{80}=\sqrt{16}\sqrt{5}$ $=4\sqrt{5}$ ☐	(1) √27 = ☐ (2) √45 = ☐ (3) √12 = ☐ (4) √48 = ☐ (5) √75 = ☐
Division and roots: $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ $\sqrt{5\frac{4}{9}}=\sqrt{\frac{49}{9}}$ $=\frac{\sqrt{49}}{\sqrt{9}}$ $=\frac{7}{3}$ ☐	(6) $\frac{\sqrt{12}}{2}=$ ☐ (7) $\frac{\sqrt{98}}{7}=$ ☐ (8) $\frac{\sqrt{18}}{\sqrt{2}}=$ ☐ (9) $\frac{\sqrt{27}}{\sqrt{3}}=$ ☐

Can you see that top heavy fractions are much nicer than mixed numbers or decimals?!

Simplifying surds

Example

Simplifying and collecting like terms. Tick the box when you understand.

$$\begin{aligned}\sqrt{75}+2\sqrt{48}-5\sqrt{12} &= \sqrt{(25)(3)}+2\sqrt{(16)(3)}-5\sqrt{(4)(3)} \\ &= \sqrt{25}\sqrt{3}+2\sqrt{16}\sqrt{3}-5\sqrt{4}\sqrt{3} \\ &= 5\sqrt{3}+2(4)\sqrt{3}-5(2)\sqrt{3} \\ &= 5\sqrt{3}+8\sqrt{3}-10\sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$$

Exercise 4 continued. Tick when correct.

(10) $\sqrt{12}+3\sqrt{75}=$

(11) $\sqrt{200}+\sqrt{18}-2\sqrt{72}=$

(12) $\sqrt{20}+2\sqrt{45}-3\sqrt{80}=$

RATIONALISING THE DENOMINATOR

This means write the fraction differently, so there is no surd on the bottom.

TYPE 1 Examples: Multiplying the top and bottom by the surd on the bottom. Tick when understood.	**Exercise 5:** Rationalise the denominators and write in the form $a\sqrt{b}$ (where *a* is usually a fraction). Tick when correct.
$\frac{1}{\sqrt{3}}$ $=\frac{\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $=\frac{\sqrt{3}}{3}$ $=\left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{1}\right)$ $=\frac{1}{3}\sqrt{3}$ ☐	(1) $\frac{1}{\sqrt{2}}$ = ☐ (2) $\frac{1}{\sqrt{7}}$ = ☐
$\frac{1}{4\sqrt{2}}$ $=\frac{\sqrt{2}}{4\sqrt{2}\sqrt{2}}$ $=\frac{\sqrt{2}}{4(2)}$ $=\left(\frac{1}{8}\right)\left(\frac{\sqrt{2}}{1}\right)$ $=\frac{1}{8}\sqrt{2}$ ☐	(3) $\frac{7}{\sqrt{5}}$ = ☐ (4) $\frac{\sqrt{2}}{3\sqrt{3}}$ = ☐

If the denominator is a sum or difference you can use the clever technique of multiplying top and bottom by the 'opposite' of the denominator to create a difference of two squares on the bottom:

$$(a-b)(a+b)=a^2-b^2$$

TYPE 2 Examples. Multiply top and bottom by the 'opposite' of the bottom. Follow the example carefully then try to do it yourself. Tick when understood.	**Now try Exercise 6:** Rationalise the denominators and write in the form $a+b\sqrt{c}$. Tick when correct.	
$\frac{1}{1+\sqrt{3}}=\frac{(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$ The bottom is 1+√3 so we multiply top and bottom by 1-√3 $=\frac{1-\sqrt{3}}{(1)^2-(\sqrt{3})^2}$ $=\frac{1-\sqrt{3}}{1-3}$ $=\frac{1-\sqrt{3}}{-2}$ Do you recognise this step from exercise 3? $=\frac{1}{-2}-\frac{\sqrt{3}}{-2}$ $=-\frac{1}{2}+\frac{\sqrt{3}}{2}$ $=-\frac{1}{2}+\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{1}\right)$ $=-\frac{1}{2}+\frac{1}{2}\sqrt{3}$	(1) $\frac{1}{1+\sqrt{2}}=$ ☐	(2) $\frac{5}{1-\sqrt{3}}$ ☐
Important step! $\frac{3}{4-\sqrt{2}}=\frac{3(4+\sqrt{2})}{(4-\sqrt{2})(4+\sqrt{2})}$ The bottom is 4 -√2 so we multiply top and bottom by 4+√2 $=\frac{3(4+\sqrt{2})}{(4)^2-(\sqrt{2})^2}$ $=\frac{3(4+\sqrt{2})}{16-2}$ $=\frac{12+3\sqrt{2}}{14}$ $=\frac{12}{14}+\frac{3\sqrt{2}}{14}$ $=\frac{6}{7}+\left(\frac{3}{14}\right)\left(\frac{\sqrt{2}}{1}\right)$ $=\frac{6}{7}+\frac{3}{14}\sqrt{2}$	(3) $\frac{1}{1-\sqrt{5}}=$ ☐	(4) $\frac{4}{5+\sqrt{3}}$ ☐

Part A Mini-Test

So, you've completed all the exercises in part A. Well done!

The important question now is whether your brain has really learned the techniques in part A. To find out, use this mini-test in exam conditions then mark it yourself using the answers at the back of the booklet and give yourself a score. You should aim for $\frac{27}{27}$ of course but certainly anything less than $\frac{15}{27}$ should be a worry. Each question number comes from that number exercise. **Go back to the exercises containing the questions you got wrong** then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 30 minutes. No Calculator allowed.
Good Luck!

1 (a) Write $\frac{3x}{4} \times 5$ as a single fraction

(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction

(c) Solve algebraically

$$\frac{2x+7}{2} - \frac{3(4x+1)}{5} = 5$$

2 (a) Evaluate $32^{\frac{3}{5}}$

(b) Evaluate $9^{-\frac{1}{2}}$

3 (a) Write $\frac{3}{2\sqrt{x}}$ in the form αx^n

(b) Write $\frac{2\sqrt{x}+4}{x^2}$ in the form $\alpha x^n + \beta x^m$

(c) Solve the equation $x^{-\frac{2}{3}} = 9$

4 (a) Simplify $\sqrt{45}$

(b) Simplify $\frac{\sqrt{12}}{2}$

(c) Simplify $\sqrt{200} + \sqrt{18} - 2\sqrt{72}$

5 Rationalise the denominator of $\frac{7}{\sqrt{5}}$ leaving your answer in the form $a\sqrt{5}$

6 Rationalise the denominator of $\frac{1}{1+\sqrt{2}}$

Mark your test using the solutions at the back of the booklet (p43) and put your score here

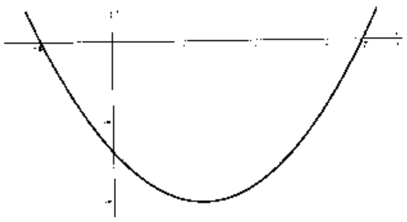
$\frac{\quad}{27}$

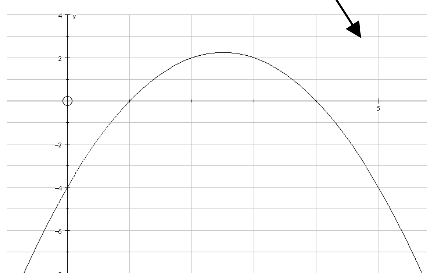
PART B - QUADRATICS

You should know what a 'quadratic' is but in order to start A level you need to REALLY understand and be able to use quadratics. You need to be able to manipulate quadratic expressions by factorising and completing the square and you need to be able to solve quadratic equations using 3 different methods.

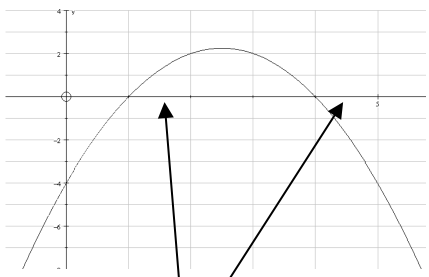
A **QUADRATIC EXPRESSION** is just some algebra written in the form $ax^2 + bx + c$. The numbers a, b and c can be anything you like (b and c could even be zero!). It is usually given the name y or $f(x)$.

A **QUADRATIC GRAPH** looks like this depending on whether a is positive or negative:





A **QUADRATIC EQUATION** can always be rearranged to make the right hand side equal to zero, i.e., so that it is in the form $ax^2 + bx + c = 0$. The solutions can be <u>seen</u> (where the graph crosses the x-axis). Normally, you would expect there to be two possible answers, as in the graphs above.



Solutions to the equation $ax^2 + bx + c = 0$

Of course, if the quadratic graph is totally above or below the x axis then it will never cross the x axis. In these cases, the quadratic equation has no solutions. Or possibly the quadratic graph might just sit on the x axis rather than crossing it, in which case the quadratic equation will only have one solution (called a repeated root).

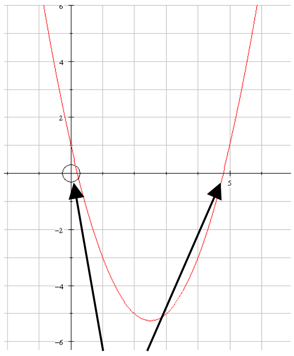
Can we solve the equation $2x^2 + 6x = 8$?!

First get everything on the left hand side so it equals zero…… $2x^2 + 6x - 8 = 0$.

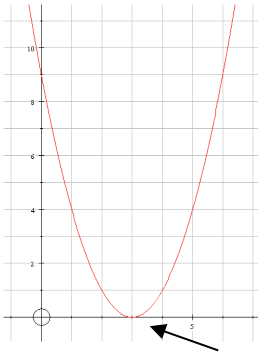
You are now ready to solve the equation – if it can be solved….. This quadratic might have 2 solutions like in the picture above, it might have one solution or it might have no solutions. Over the next few pages, you will first practise working out whether it has none, one or two solutions. Then you will practise finding the solutions (if they exist!) by three different methods.

Part B – Section 1 – THE DISCRIMINANT (Δ)

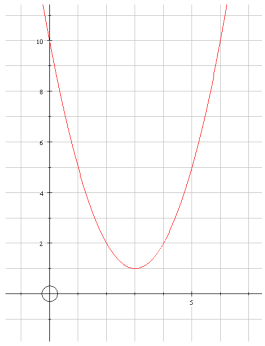
All quadratic graphs cross the *y*-axis. The *y*-intercept is the value of the quadratic when $x = 0$. The behaviour on the *x*-axis is a bit more complicated. Some quadratic graphs cross the *x*-axis twice, giving two solutions to the equation $ax^2 + bx + c = 0$. Other quadratics simply 'sit' on the *x* axis, so they only have one solution to the equation $ax^2 + bx + c = 0$. There are also some quadratics which don't cross the *x* axis at all so these quadratics have no solutions to the equation $ax^2 + bx + c = 0$.



Two distinct roots



One repeated root
(two equal roots)



No real roots

The solutions of an equation, i.e., the places where the graph crosses the *x*-axis, are called the **roots of the equation**.

We know that the solutions to a quadratic equation are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What could go wrong? Why do we sometimes get two solutions, sometimes one solution and sometimes no solutions?! The answer lies inside the square root sign.

$b^2 - 4ac > 0$ (positive)

Everything is fine. We square root $b^2 - 4ac$ and get two solutions using the quadratic formula.

$b^2 - 4ac = 0$

If $b^2 - 4ac = 0$ then $\sqrt{b^2 - 4ac} = 0$ so in this case $x = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$. Just one (repeated) solution.

$b^2 - 4ac < 0$ (negative)

If $b^2 - 4ac < 0$ have a problem. We can't square root a negative number so we are stuck. That is why, in this situation, there are no solutions.

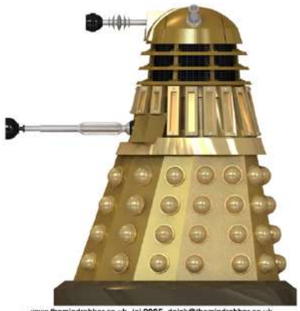
$b^2 - 4ac$ is called the DISCRIMINANT (which is the Δ referred to on page 4) of the quadratic because it helps us to discriminate between the quadratics with no roots, quadratics with one repeated root and quadratics with two roots.

Sometimes the symbol Δ is used to refer to the discriminant of a quadratic. Now go back to page 4 of this booklet and look at the quadratics at the bottom of the page. Does it make sense to you that you can see whether the discriminant is positive, negative or zero by looking at the graph of the quadratic?

It is important to remember that in the discriminant, $b^2 - 4ac$, a represents the amount of x^2 in your quadratic, b represents the amount of x in your quadratic and c represents everything else in your quadratic (ie the numbers). **Don't let yourself get muddled if the quadratic is written in a funny order!**

Exercise 7 Write down the discriminant of each of these quadratics and hence state whether each one has two roots, one repeated root or no roots. Tick when correct.

> Note: make sure that you square all of b!
>
> If b is – 6 then b^2 is $(-6)^2 = 36$ NOT -36
>
> If b is $2k$ then $b^2 = (2k)^2 = 4k^2$ NOT $2k^2$



Quadratic	Value of Discriminant	Circle the number of roots
EXAMPLE (1) $x^2 + 8x + 7$	$(8)^2 - 4(1)(7) = 36$ $36 > 0$	None One Repeated (Two) ☐
(2) $3x + x^2 - 2$	$(\ \ )^2 - 4(\ \ )(\ \ ) =$	None One Repeated Two ☐
(3) $x^2 + 3$	$(\ \ )^2 - 4(\ \ )(\ \ ) =$	None One Repeated Two ☐
(4) $2x^2 + 3 - 6x$	$(\ \ )^2 - 4(\ \ )(\ \ ) =$	None One Repeated Two ☐
(5) $x - x^2$	$(\ \ )^2 - 4(\ \ )(\ \ ) =$	None One Repeated Two ☐
(6) $x^2 - 6x + 9$	$(\ \ )^2 - 4(\ \ )(\ \ ) =$	None One Repeated Two ☐

Part B – Section 2- FACTORISING QUADRATICS

… Using the difference of two squares $(a)^2 - (b)^2 = (a - b)(a + b)$

	Exercise 8 Factorise the following	Tick when correct
Example 1 $x^2 - 9 = (x - 3)(x + 3)$	(1) $x^2 - 1$	
	(2) $4x^2 - 9$	
Example 2 $9x^2 - 16 = (3x)^2 - (4)^2$ $= (3x - 4)(3x + 4)$	(3) $49 - x^2$	
	(4) $2x^2 - 8$	
	(5) $x^2 - 16$	
Example 3 $8x^2 - 2 = 2(4x^2 - 1)$ $= 2((2x)^2 - 1^2)$ $= 2(2x - 1)(2x + 1)$	(6) $9x^2 - 1$	
	(7) $36 - 25x^2$	
	(8) $9x^2 - 36$	

Exercise 9 Factorise the following quadratics. Remember to expand out to check your answers. The first one has been completed for you. Tick when correct!

(1) $x^2 - 2x - 15$	(2) $6x^2 - 3x$	(3) $x^2 - 5x - 6$
$= (x - 3)(x + 5)$ Check: $(x - 3)(x + 5)$ $= x^2 - 3x + 5x - 15$ $= x^2 - 2x - 15$		
(4) $x^2 + x - 6$	(5) $2x^2 + 6x$	(6) $x^2 - 6x - 16$

Exercise 10 Factorise the following. Don't forget to expand out to check your answers. Tick when correct.

(1) $2x^2 + 5x + 2$	(2) $3x^2 - 8x + 4$
(3) $2x^2 + 7x + 6$	(4) $3x^2 - 13x - 10$
(5) $2x^2 + 9x - 5$	(6) $2x^2 - 11x + 12$

Part B – Section 3 - COMPLETING THE SQUARE

Completing the square is a bit like factorising. It doesn't change the quadratic but it changes the way the quadratic expression is written.

When we factorise, we change	$x^2 + bx + c$	into	$(x - p)(x - q)$	by finding p and q
When we complete the square, we change	$x^2 + bx + c$	into	$(x + B)^2 + C$	by finding B and C

$$x^2 + bx + c = (x + \text{half of } b)^2 - (\text{half of } b)^2 + c$$

	Exercise 11 Complete the square of the following quadratics.	
<u>Example</u> Express $x^2 + 6x + 11$ in the completed square form $(x+B)^2 + C$. $x^2 + 6x + 11 = (x+3)^2 - (3)^2 + 11$ $= (x+3)^2 - 9 + 11$ $= (x+3)^2 + 2$ Tick when understood. ☐	**(1) $x^2 + 8x + 7$** ☐	(2) $x^2 - 2x - 15$ ☐
<u>Example 2</u> Express $x^2 - 10x + 13$ in the completed square form $(x+B)^2 + C$. $x^2 - 10x + 13 = (x + -5)^2 - (-5)^2 + 13$ $= (x-5)^2 - 25 + 13$ $= (x-5)^2 -`12$ ☐	(3) $x^2 + 6x + 10$ ☐	(4) $x^2 - 10x + 9$ ☐
(5) $x^2 + 12x + 100$ ☐	(6) $x^2 + 2x - 6$ ☐	(7) $x^2 + 6x - 5$ ☐

Part B – Section 4 - SOLVING QUADRATICS

There are 3 ways to solve a quadratic equation:- by factorising, by using the quadratic formula or by completing the square.

- Factorising uses the fact that if 2 things multiply together to make zero then one of them MUST be zero. You can't always factorise a quadratic even if it has solutions.
- The quadratic formula will always give you the solutions, so long as there are some!
- Completing the square allows you to simply rearrange the quadratic to find *x*. If there are solutions to the quadratic equation then completing the square will always work.

<u>Example - Factorising</u>

$$2x^2 - 5x + 3 = 0$$

Factorising gives: $(2x - 3)(x - 1) = 0$

so either $2x - 3 = 0$ or $x - 1 = 0$

$2x = 3$

$\therefore x = \frac{3}{2}$ or $x = 1$

This means that the graph of the quadratic function $f(x) = 2x^2 - 5x + 3$ crosses the *x* axis at $\frac{3}{2}$ and 1. Tick when understood ☐

Factorising

<u>Exercise 12</u> Solve the following quadratic equations by factorising. Tick when correct.

(1) $x^2 + 11x + 28 = 0$	(2) $x^2 + 3x = 0$	(3) $2x^2 + 3x - 14 = 0$
☐	☐	☐

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

To solve the quadratic equation $ax^2 + bx + c = 0$ you can use the quadratic formula above.

Then you will need to rearrange these answers into the form

$$x = A \pm B\sqrt{C}$$

<u>Example – Using the formula</u>

Solve $x^2 + 3x + 1 = 0$

$$x = \frac{-3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{9 - 4}}{2}$$

$$= \frac{-3 \pm \sqrt{5}}{2}$$

Important step!

$$= -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$$

$$= -\frac{3}{2} \pm \left(\frac{1}{2}\right)\left(\frac{\sqrt{5}}{1}\right)$$

$$= -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$$

Tick when understood ☐

Now try Exercise 13: Solve the following quadratic equations using the quadratic formula and leave your answers in the form $x = A \pm B\sqrt{C}$ as in the example on the left.

(1) $2x^2 + 4x + 1 = 0$

(2) $x^2 - 7x + 9 = 0$

To solve the quadratic equation $ax^2 + bx + c = 0$ you can complete the square then simply rearrange the algebra. The answers will come out nicely in the form you want: $x = p \pm q\sqrt{r}$. In the examples below we have shown every tiny step to help you to follow what is happening.

Example.	**Now try Exercise 14** Solve this quadratic by completing the square. Tick when correct.
Solve $x^2 + 3x + 1 = 0$ by completing the square $\left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 1 = 0$ $\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{4}{4} = 0$ $\left(x+\frac{3}{2}\right)^2 - \frac{5}{4} = 0$ First complete the square, then expand out the (half b)^2 bit (remember to square the top AND bottom of the fraction) then add it to c $\left(x+\frac{3}{2}\right)^2 = \frac{5}{4}$ $x+\frac{3}{2} = \pm\sqrt{\frac{5}{4}}$ $x+\frac{3}{2} = \pm\frac{\sqrt{5}}{\sqrt{4}}$ Put the number on the right hand side then square root both sides, remembering to add the ± sign! $x+\frac{3}{2} = \pm\left(\frac{1}{2}\right)\left(\frac{\sqrt{5}}{1}\right)$ $x = -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$ Finally move the 'half of b' to the other side so it says x = $x = -\frac{3}{2} + \frac{1}{2}\sqrt{5}$ *or* $x = -\frac{3}{2} - \frac{1}{2}\sqrt{5}$ Tick when understood ☐	(1) $x^2 + 2x - 6 = 0$ ☐

Example – Using the Completed Square to solve a quadratic.

Solve $x^2 - x = 0$ by completing the square

$$\left(x-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 = 0$$

First complete the square, then expand out the (half b)^2 bit. In this question, c = 0

$$\left(x-\frac{1}{2}\right)^2 - \frac{1}{4} = 0$$

$$\left(x-\frac{1}{2}\right)^2 = \frac{1}{4}$$

Put the number on the right hand side then square root both sides, remembering to add the ± sign!

$$x-\frac{1}{2} = \pm\sqrt{\frac{1}{4}}$$

$$x-\frac{1}{2} = \pm\frac{\sqrt{1}}{\sqrt{4}}$$

Remember that the square root of a fraction is the square root of the top over the square root of the bottom.

$$x-\frac{1}{2} = \pm\frac{1}{2}$$

$$x = \frac{1}{2}-\frac{1}{2} \quad or \quad x = \frac{1}{2}+\frac{1}{2}$$

$$x = 0 \qquad or \quad x = 1$$

Finally move the 'half of b' to the other side so it says x =

Tick when understood ☐

Exercise 14 continued Solve this quadratic by completing the square. Tick when correct.

(2) $x^2 + 6x - 5 = 0$

(pages 21-27)

☐

Part B Mini-Test

So, you've completed all the exercises in part B. Well done! The important question is whether your brain has really learned these techniques. To find out, use this mini test in exam conditions then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly anything less than 60% should be a worry. Go back to the exercises containing the questions you got wrong then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Time: 30 minutes. No Calculator allowed.
Good Luck!

7 Evaluate the discriminant of the quadratic $y = 2x^2 + 3 - 6x$ and hence state the number of roots of the equation $2x^2 + 3 - 6x = 0$

8 Factorise the quadratic $y = 4x^2 - 9$ using the difference of two squares.

9 Factorise the quadratic $y = 2x^2 + 6x$

10 Factorise the quadratic $y = 3x^2 - 13x - 10$

11 Write the quadratic $y = x^2 + 8x + 7$ in completed square form.

12 Solve the equation $x^2 + 3x = 0$ by factorising.

13 Solve the equation $2x^2 + 4x + 1 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.

14 Solve the equation $x^2 + 6x - 5 = 0$ by rearranging the completed square, leaving the answer(s) in surd form.

Quadratic formula: $$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Completed square: $$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Mark your test using the solutions at the back of the booklet and put your score here

$\overline{}$
20

GCSE HIGHER reminder

Now that you have completed your GCSE and this booklet, you should be able to complete all of these questions. If you have forgotten any areas, please look at your GCSE revision guide, use https://www.mathsgenie.co.uk/gcse.html, or use the support available in the first week of September. Lots of these questions are similar to the start of your A Level, so make sure you can do these, and this will help when you start the course.

You may use a Calculator

1. (a) Expand

$(2a + 3)^2$

....

(b) Factorise fully

$20xy + 30x^2$

....

---

2. (a) Factorise $x^2 + 2x – 15$

....

(b) Use your answer to solve the equation $x^2 + 2x – 15 = 0$

....

---

3. Expand and simplify $(t + 4)(t − 2)$

....

---

4. (a) Simplify $6^5 \times 6^8$

....

(b) Simplify $\frac{6^7 \times 6^3}{6^2}$

....

5. Write as a single fraction in its simplest form.

$$\frac{3}{x+1}+\frac{5x}{2x^2+7x+5}$$

……………………………

6. Solve the equation $3x^2 + 2x - 15 = 0$

……………………………

7. (a) Solve the equation $7x + 2 = 3x - 2$

$x = ...$

(3)

(b) Solve $2x + 1 = \frac{5x}{3}$

$x = ...$

8. A straight line has equation $y = \frac{1}{2}x + 1$

The point P lies on the straight line.
P has a y-coordinate of 5.

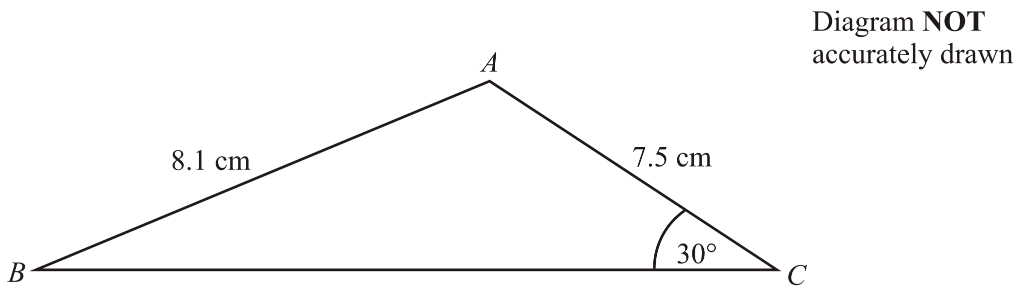
(a) Find the x-coordinate of P.

.... (2)

(b) Rearrange $y = \frac{1}{2}x + 1$ to make x the subject.

....

9.



In triangle *ABC*,
AB = 8.1 cm,
AC = 7.5 cm,
angle *ACB* = 30°.

(a) Calculate the size of angle *ABC*.

Give your answer correct to 3 significant figures.

……………………°

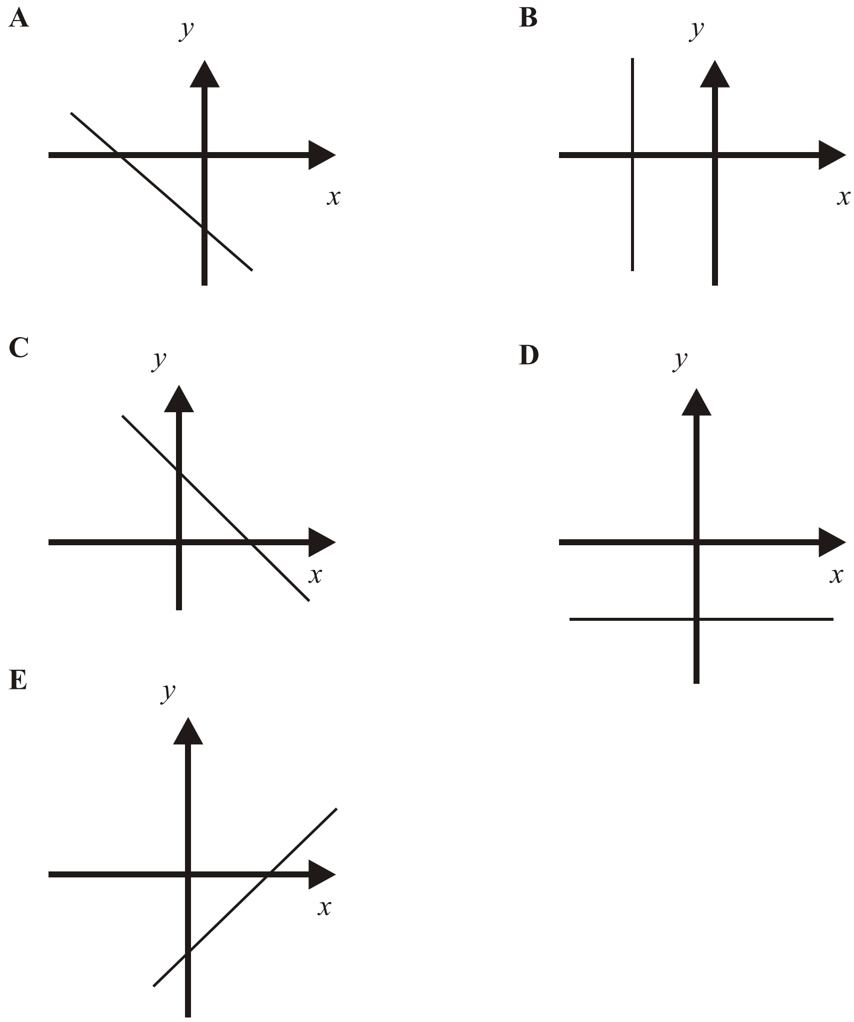
(3)

(b) Calculate the area of triangle *ABC*.

Give your answer correct to 3 significant figures.

…………………..cm^2

10. Here are five graphs labelled **A**, **B**, **C**, **D** and **E**.



Each of the equations in the table represents one of the graphs **A** to **E**.

Write the letter of each graph in the correct place in the table.

Equation	**Graph**
$x + y = 5$	
$y = x - 5$	
$y = -5 - x$	
$y = -5$	
$x = -5$	

(Total 3 marks)

11. Rearrange to make **t** the subject of the formula:

a) $k = 5t^3 - 3$

b) $h = \frac{3t+2}{5t-4}$

(4 marks)

12. a) Find the distance between the points (0,4) and (2,7)

(2 marks)

b) Find the gradient of the line between (2,7) and (5,-4)

(2 marks)

c) Find the equation of the line through the points (0,5) and (2,-3)

(2 marks)

GCSE question Answers

1)

a. $4a^2$+12a + 9

b. $10x(2y+3x)$

2)

a. $(x-3)(x+5)$

b. $x = 3\ or\ x = -5$

3) $t^2 + 2t - 8$

4)

a. 6^{13}

b. 6^5

5) $\frac{11x+15}{(x+1)(2x+5)}$

6) $-\frac{1}{3} \pm \frac{1}{3}\sqrt{46}$

7)

a. $x = -1$

b. $x = -3$

8)

a. $x = 8$

b. $x = 2y - 2$

9)

a. $\theta = 27.6$

b. $Area = 25.6$

10)

Equation	Graph
$x + y = 5$	C
$y = x - 5$	E
$y = -5 - x$	A
$y = -5$	D
$x = -5$	B

11) a) t = $\sqrt[3]{\frac{k+3}{5}}$ b) t = $\frac{4h+2}{5h-3}$

12) a) $\sqrt{13}$ b) $-\frac{11}{3}$ c) y = 2 – 4x (or y = -4x + 2)

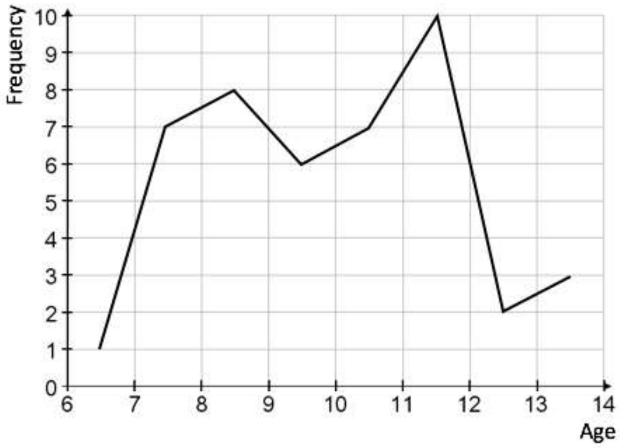
Starting with Confidence: Applied Maths Section

As part of your A Level Mathematics course you will study Mechanics and Statistics, alongside Pure Maths. Although much of these applied topics will be new to you, we will be building on some core skills that you studied in your GCSE course.

Try the 14 questions below and mark your work with the answers provided. If you score 90% or less you will need to study the topics that you found difficult (The associated https://www.mathsgenie.co.uk/gcse.html link is given next to each question to help you with this) and then try these questions again until you achieve 90% or more.

Applied Mathematics Questions (You may use a calculator):

1) The frequency polygon drawn below shows how the ages of some people.



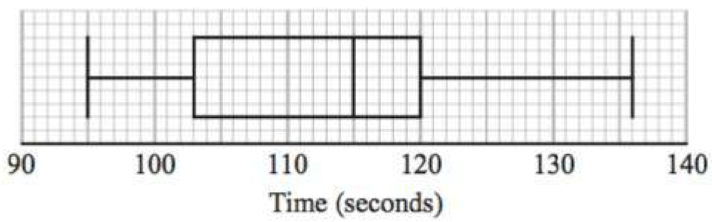
a. Find the proportion of students that are less than 10 years old

b. Find an estimate for the mean age.

Give your answer correct to 2 decimal places.

Maths Genie ref: **Frequency Polygons, Mean of Grouped Data 1**

2) Tom recorded the times, in seconds, some boys took to complete an obstacle course. He drew this box plot for his results.



Tom also recorded the times some girls took to complete the obstacle course. Here are the times, in seconds, for the girls.

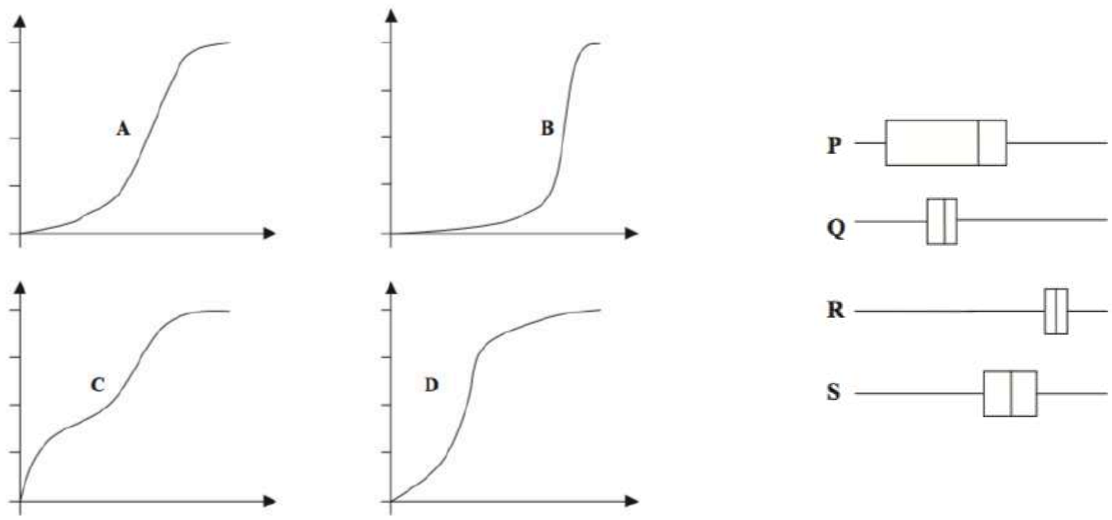
99 101 103 106 108 109 110 110 111 112

113 114 115 115 117 120 124 125 132

Compare the distribution of the times for the boys with the distribution of the times for the girls.

Maths Genie ref: **Box and Whisker Plots**

3) Here are four cumulative frequency diagrams and four box plots.



For each box plot, write down the letter of the appropriate cumulative frequency diagram.
Maths Genie: **Box Plots, Cumulative Frequency**

4) A garage keeps records of the costs of repairs to its customers' cars.
The table gives information about the costs of all repairs which were less than £250 in one week.

Cost, (£C)	Frequency
$0 < C \leqslant 50$	4
$50 < C \leqslant 100$	8
$100 < C \leqslant 150$	7
$150 < C \leqslant 200$	10
$200 < C \leqslant 250$	11

a. Write down the modal class for these repairs.

b. Estimate the mean cost of these repairs.

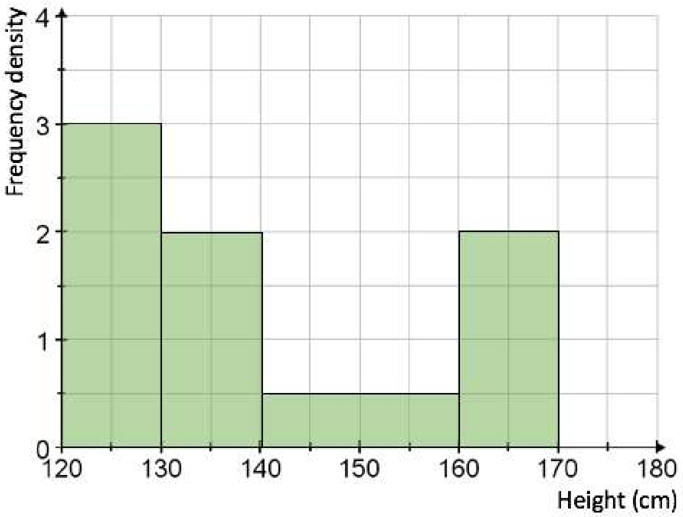
c. In which class interval does the median cost of these repairs lie?

d. The median and mean costs are different, which would you choose to represent the 'average' cost for repairs under £250 and why?

e. There was only one further repair that week, not included in the table. That repair cost £1000. Dave says 'The class interval in which the median lies will change.' Is Dave correct?

Maths Genie ref: **Median, Averages from a Frequency Table**

5) Use the histogram below to find an estimate for the median height.



Maths Genie ref: **Histograms, Averages, Averages from a Frequency Table**

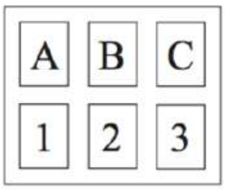
6) The data in the grouped frequency table below represents the weight in kilograms of some animals in a zoo.

Weight (x kg)	Frequency
$3 \leq x < 6$	w
$6 \leq x < 10$	3
$10 \leq x < 12$	5
$12 \leq x < 20$	4

Given that an estimate for the mean is 8.95, work out the value of w.
Maths Genie ref: **Averages from frequency table**

7) Laura has 3 red pens, 7 blue pens and 5 black pens in her pencil case. Kyle adds some red pens in her pencil case and the probability to pick a red pen becomes equal to $\frac{1}{2}$. How many red pens are now in Laura's pencil case?
Maths Genie ref: **Probability**

8) The diagram shows a security lock.

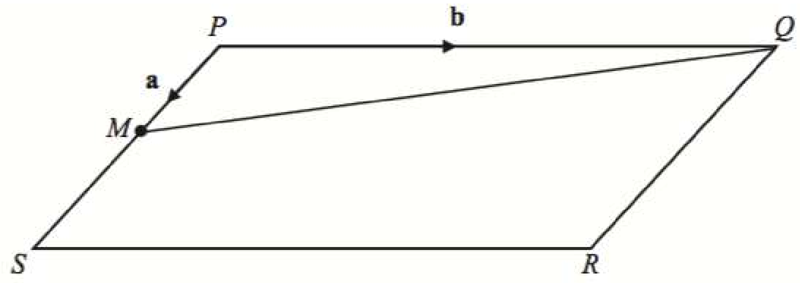


You have to enter the correct code to open the lock. The correct code is B3. Dan does **not** know the code.
He enters at random one of the letters. He then enters at random one of the numbers. Work out the probability that Dan enters the correct code.
Maths Genie ref: **Listing Outcomes**

9) Owen has 3 red sweets, 2 purple sweets and 5 green sweets in a bag. Owen picks a sweet at random from the bag, eats it, then picks another sweet. Find the probability that Owen will pick two sweets of the same colour.
Mathsgenie ref: **Conditional Probability**

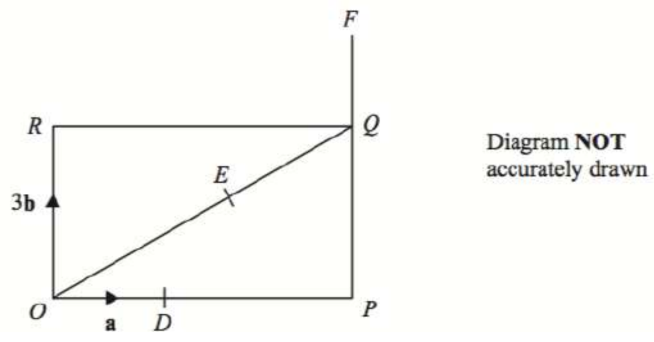
10) Carolyn has 20 biscuits in a tin.
She has
 12 plain biscuits
 5 chocolate biscuits
 3 ginger biscuits
Carolyn takes at random two biscuits from the tin. Work out the probability that the two biscuits were **not** the same type.
Mathsgenie ref: **Conditional Probability**

11) The diagram shows a parallelogram, *PQRS*.
M is the midpoint of *PS*. $\overrightarrow{PM} = \mathbf{a}$ $\overrightarrow{PQ} = \mathbf{b}$



Find $\overrightarrow{PR}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$.
Maths Genie: **Vectors Proof**

12)



$OPQR$ is a rectangle.

D is the point on OP such that $OD = \frac{1}{3}OP$

E is the point on OQ such that $OE = \frac{2}{3}OQ$

PQF is the straight line such that $QF = \frac{1}{3}PQ$

$\overrightarrow{OD} = \mathbf{a}$ $\overrightarrow{OR} = 3\mathbf{b}$

Find $\overrightarrow{DE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Mathsgenie ref: **Vectors Proof**

13) There are 31 students in a class.
The only languages available for the class to study are French and Spanish.
17 students study French.
15 students study Spanish.
6 students study neither French nor Spanish.

Using a Venn diagram, or otherwise, work out how many students study only one language.
Mathsgenie ref: **Venn Diagrams**

14) Of 30 students, 20 could play the piano, 5 the flute and 6 the violin. 3 violinists could also play the piano but only one of these 3 could also play the flute. 5 students could play both the piano and the flute. None of these students could play any other type of musical instrument. How many students played only the piano?
Mathsgenie ref: **Venn Diagrams**

Applied Maths – Solutions

Remember, these are for marking only. Do not be tempted to use them to help you work out the question. Each question has a link that you can use to help you if you are stuck or need to do more work on that topic.

1) a. 50% b. 9.95 years
2) On **average** the boys' times were longer (median for boys is **115 minutes** versus **112 minutes** for the girls). However, the boys' times were **more spread out,** the **range** was **41 minute**s for the boys and **33 minutes** for the girls. *You could also compare the IQR. Words in bold must be seen.*
3) PC, QD, RB, SA
*4)*a. Modal group $200 < x \leq 250$ b. Mean cost is £145, c. Median cost is in group $150 < x \leq 200$ d. The median value would be better because it isn't influenced by the small number of very low costs. e. The median would still be in group $150 < x \leq 200$
5) 135cm
6) 8
7) 12
8) $\frac{1}{9}$
9) $\frac{28}{90}$
10) $\frac{222}{380}$
11) 2**a**+**b**
12) 3**a**+3**b**
13) 18
14) 12

ARE YOU READY FOR A LEVEL TEST?

In order to be confident starting A level mathematics you need to be confident with the techniques in this booklet. When you start the course we will give you a test like this one to check that you are ready to start the A Level. Try this test in exam conditions then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly anything less than 60% should be a worry. Go back to the exercises containing the questions you got wrong then try this test again in a few days time. If you feel you need help, follow the tips on the second page of this booklet.

Quadratic formula:

$$x = \frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2a}$$

Completed square:

Staple your completed test into your booklet so that you with your teacher in September.

Time: 1 hour. No Calculator allowed.
Good Luck!

1(a) Write $\frac{3x}{2} \div 5$ as a single fraction

(b) Write $\frac{2}{x} + \frac{3}{x^2}$ as a single fraction

2(a) Evaluate $16^{-\frac{7}{4}}$

(b) Evaluate $4^{\frac{5}{2}}$

3(a) Write $\frac{2+\sqrt{x}}{\sqrt{x}}$ in the form $\alpha x^n + \beta x^m$

(b) Solve the equation $x^{\frac{3}{4}} = \frac{1}{27}$

4(a) Simplify $\sqrt{48}$

(b) Simplify $\frac{\sqrt{18}}{\sqrt{2}}$

(c) Simplify $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80}$

5(a) Rationalise the denominator of $\frac{\sqrt{2}}{3\sqrt{3}}$

leaving your answer in the form $a\sqrt{6}$

(b) Rationalise the denominator of $\frac{1}{\sqrt{2}}$

6 Rationalise the denominator of

$\frac{5}{1-\sqrt{3}}$

7 Evaluate the discriminant of the quadratic $y = x - x^2$ and hence state the number of roots of the equation $x - x^2 = 0$

8 Factorise the quadratic $y = 2x^2 - 8$ using the difference of two squares

9 Factorise the quadratic $y = 6x^2 - 3x$

10 Factorise the quadratic $y = 2x^2 - 11x + 12$

11 Write the quadratic $y = x^2 - 6x - 16$ in completed square form.

12 Solve the equation $2x^2 + 3x - 14 = 0$ by factorising.

13 Solve the equation $x^2 - 7x + 9 = 0$ by using the quadratic formula, leaving the answer(s) in surd form.

14 Solve the equation $x^2 + 2x - 6 = 0$ by rearranging the completed square, leaving the answer(s) in surd form.

Mark your test using the solutions at the back of the booklet and put your score here /40

ANSWERS. These answers are for marking your completed solution only. Do not be tempted to use them to help you answer the question. If you do need help, use the 'when you get stuck' tips on page 3.

Exercise 1

(1) $\frac{15x}{4}$	(2) $\frac{2x+3}{x^2}$	(3) $\frac{3x}{10}$

Exercise 2

(1) $\frac{1}{64}$	(2) $\frac{1}{3}$	(3) $\frac{1}{3}$	(4) 32	(5) 8	(6) $\frac{1}{128}$

Exercise 3

(1) $\frac{1}{5}x$	(2) $\frac{3}{2}x^{-\frac{1}{2}}$	(3) $\frac{1}{3}x^{-\frac{3}{2}}$	(4) $2x^{\frac{2}{3}}$
(5) $2x^{-\frac{3}{2}}+4x^{-2}$	(6) $\frac{2}{3}x^{-1}+\frac{4}{3}x^{-2}$	(7) $\frac{1}{4}x^{-3}-x^{-2}$	(8) $x^{-1}-4x^{-\frac{1}{2}}$
(9) $x^{\frac{3}{2}}-3x^{-\frac{1}{2}}$	(10) $x^{-1}-2x^{-2}$	(11) $2x^{-\frac{1}{2}}+1$	(12) $\frac{1}{2}+x^{-1}$
(13) $\frac{1}{3}x^{-\frac{3}{2}}+2x^{-2}$	(14) $2x^{-1}-x^{-2}$	(15) $x=\pm\frac{1}{27}$	(16) $\frac{1}{25}$
(17) $x=\pm 32$	(18) $x=\pm 64$	(19) $x=\frac{1}{81}$	(20) $\pm\frac{1}{125}$

Exercise 4

(1) 3√3	(2) 3√5	(3) 2√3	(4) 4√3	(5) 5√3	(6) √3
(7) √2	(8) 3	(9) 3	(10) 17√3	(11) √2	(12) -4√5

Exercise 5

(1) $\frac{1}{2}\sqrt{2}$	(2) $\frac{1}{7}\sqrt{7}$	(3) $\frac{7}{5}\sqrt{5}$	(4) $\frac{1}{9}\sqrt{6}$

Exercise 6

(1) $-1+\sqrt{2}$	(2) $-\frac{5}{2}-\frac{5}{2}\sqrt{3}$	(3) $-\frac{1}{4}-\frac{\sqrt{5}}{4}$	(4) $\frac{10}{11}-\frac{2\sqrt{3}}{11}$

Exercise 7

(1) 36, two	(2) 17, two	(3) – 12, none
(4) 12, two	(5) 1, two	(6) 0, one repeated

Exercise 8

(1) $(x-1)(x+1)$	(2) $(2x-3)(2x+3)$	(3) $(7-x)(7+x)$
(4) $2(x-2)(x+2)$	(5) $(x-4)(x+4)$	(6) $(3x-1)(3x+1)$
(7) $(6-5x)(6+5x)$	(8) $9(x-2)(x+2)$	

Exercise 9

(1) $(x+3)(x-5)$	(2) $3x(2x-1)$	(3) $(x-6)(x+1)$
(4) $(x-2)(x+3)$	(5) $2x(x+3)$	(6) $(x-8)(x+2)$

Exercise 10

(1) $(2x+1)(x+2)$	(2) $(3x-2)(x-2)$	(3) $(2x+3)(x+2)$
(4) $(3x+2)(x-5)$	(5) $(2x-1)(x+5)$	(6) $(2x-3)(x-4)$

Exercise 11

(1) $(x+4)^2 - 9$	(2) $(x-1)^2 - 16$	(3) $(x+3)^2 + 1$
(4) $(x-5)^2 - 16$	(5) $(x+6)^2 + 64$	(6) $(x+1)^2 - 7$
(7) $(x+3)^2 - 14$		

Exercise 12

(1) $x = -7$ or -4	(2) $x = 0$ or -3	(3) $x = -^7/_2$ or 2

Exercise 13

(1) $x = -1 \pm \frac{1}{2}\sqrt{2}$	(2) $x = \frac{7}{2} \pm \frac{1}{2}\sqrt{13}$

Exercise 14

(1) $x = -1 + \sqrt{7}$ or $x = -1 - \sqrt{7}$	(2) $x = -3 + \sqrt{14}$ or $x = -3 - \sqrt{14}$

Part A Mini Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!) , 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! Give yourself a bonus mark if you got (6b) correct ☺. The total test is out of 27 and **anything below 15/27 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

1 (a) $\frac{3x}{4}\times 5 = \left(\frac{3x}{4}\right)\left(\frac{5}{1}\right) = \frac{15x}{4}$

2 (a) $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

1 (b) $\frac{2}{x}+\frac{3}{x^2} = \frac{2x}{x^2}+\frac{3}{x^2} = \frac{2x+3}{x^2}$

2 (b) $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

1 (c) $\frac{2x+7}{2} - \frac{3(4x+1)}{5} = 5 \rightarrow \frac{10x+35-24x-6}{10} = 5 \rightarrow -14x = 21 \rightarrow x = -\frac{3}{2}$

3 (a) $\frac{3}{2\sqrt{x}} = \left(\frac{3}{2}\right)\left(\frac{1}{\sqrt{x}}\right) = \frac{3}{2}x^{-\frac{1}{2}}$

(b) $\frac{2\sqrt{x}+4}{x^2} = \frac{2\sqrt{x}}{x^2}+\frac{4}{x^2} = \left(\frac{2}{1}\right)\left(\frac{\sqrt{x}}{x^2}\right)+\left(\frac{4}{1}\right)\left(\frac{1}{x^2}\right) = 2x^{\frac{1}{2}-2}+4x^{-2} = 2x^{-\frac{3}{2}}+4x^{-2}$

(c)

$$x^{-\frac{2}{3}} = 9 \quad x^{\frac{2}{3}} = \frac{1}{9}$$

$$x^{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{9}} \quad x^{\frac{1}{3}} = \pm\frac{1}{3}$$

$$x = \left(\pm\frac{1}{3}\right)^3 \quad x = \pm\frac{1}{27}$$

4 (a) $\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

(b) $\frac{\sqrt{12}}{2} = \frac{\sqrt{4}\sqrt{3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

(c) $\sqrt{200}+\sqrt{18}-2\sqrt{72} = \sqrt{100}\sqrt{2}+\sqrt{9}\sqrt{2}-2\sqrt{36}\sqrt{2} = 10\sqrt{2}+3\sqrt{2}-12\sqrt{2} = \sqrt{2}$

5 $\frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}}{5} = \left(\frac{7}{5}\right)\left(\frac{\sqrt{5}}{1}\right) = \frac{7}{5}\sqrt{5}$

6 $\frac{1}{1+\sqrt{2}} = \frac{1\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)} = \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = \frac{1}{-1}-\frac{\sqrt{2}}{-1} = -1+\sqrt{2}$

Part B Mini Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!), 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! Give yourself 2 bonus marks if you got (7) correct and 2 bonus marks if you got (8) correct ☺.
The total test is out of 20 and **anything below 12/20 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

7 $b^2-4ac=(-6)^2-4(2)(3)=36-24=12$. 12 > 0 hence the equation has 2 distinct roots.

8 $4x^2-9=(2x-3)(2x+3)$

9 $2x^2+6x=2x(x+3)$

10 $3x^2-13x-10=(3x+2)(x-5)$

11 $x^2+8x+7=(x+4)^2-(4)^2+7=(x+4)^2-9$

12

$$x^2+3x=0$$
$$x(x+3)=0$$
$$x=0,\ \ x+3=0$$
$$x=0,\ \ x=-3$$

13

$$x=\frac{-4\pm\sqrt{(4)^2-4(2)(1)}}{2(2)}=\frac{-4\pm\sqrt{16-8}}{4}=\frac{-4\pm\sqrt{8}}{4}=\frac{-4}{4}\pm\frac{\sqrt{8}}{4}$$
$$=-1\pm\frac{\sqrt{4}\sqrt{2}}{4}=-1\pm\left(\frac{2}{4}\right)\left(\frac{\sqrt{2}}{1}\right)=-1\pm\frac{1}{2}\sqrt{2}$$

14

$$x^2+6x-5=0$$
$$(x+3)^2-(3)^2-5=0$$
$$(x+3)^2-14=0$$
$$(x+3)^2=14$$
$$x+3=\pm\sqrt{14}$$
$$x=-3\pm\sqrt{14}$$

Are you ready for A level? Test Solutions.

For each part, give yourself 2 marks for a perfect answer (including working!), 1 mark for correct method but made a mistake and 0 marks for doing it totally wrong! The total test is out of 40 and **anything below 24/40 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

1a) $\frac{3x}{2} \div 5 = \frac{3x}{2} \times \frac{1}{5} = \frac{3x}{10}$

b) $\frac{2}{x} + \frac{3}{x^2} = \frac{2x}{x^2} + \frac{3}{x^2} = \frac{2x+3}{x^2}$

2a) $16^{-\frac{7}{4}} = \frac{1}{16^{\frac{7}{4}}} = \frac{1}{(16^{\frac{1}{4}})^7} = \frac{1}{2^7} = \frac{1}{128}$

b) $4^{\frac{5}{2}} = (4^{\frac{1}{2}})^5 = (\sqrt{4})^5 = 2^5 = 32$

3a) $\frac{2+\sqrt{x}}{\sqrt{x}} = \frac{2}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}} = 2x^{-\frac{1}{2}} + 1$

b) $x^{\frac{3}{4}} = \frac{1}{27}$

$(x^{\frac{1}{4}})^3 = \frac{1}{27}$

$x^{\frac{1}{4}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}}$

$x^{\frac{1}{4}} = \frac{1}{3}$

$x = \frac{1^4}{3^4}$

$x = \frac{1}{81}$

4a) $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

b) $\frac{\sqrt{18}}{\sqrt{2}} = \frac{\sqrt{9\times2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}} = 3$

c) $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80} = \sqrt{4}\sqrt{5} + 2\sqrt{9}\sqrt{5} - 3\sqrt{16}\sqrt{5} = 2\sqrt{5} + 6\sqrt{5} - 12\sqrt{5} = -4\sqrt{5}$

5a) $\frac{\sqrt{2}}{3\sqrt{3}} = \frac{\sqrt{2}}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{2}\sqrt{3}}{3 \times 3} = \frac{1}{9}\sqrt{6}$

b) $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

6) $\frac{5}{1-\sqrt{3}} = \frac{5}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \frac{5(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$

$= \frac{5+5\sqrt{3}}{1-\sqrt{3}\sqrt{3}}$

$= \frac{5+5\sqrt{3}}{-2}$

$= -\frac{5}{2} - \frac{5}{2}\sqrt{3}$

7) $(1)^2 - 4(-1)(0) = 1$, two roots

8) $2x^2 - 8 = 2(x^2 - 4) = 2(x+2)(x-2)$

9) $6x^2 - 3x = 3x(2x-1)$

10) $2x^2 - 11x + 12 = (2x-3)(x-4)$

11) $x^2 - 6x - 16 = (x-3)^2 - (-3)^2 - 16$

$= (x-3)^2 - 25$

12) $2x^2 + 3x - 14 = 0$

$(2x+7)(x-2) = 0$

$2x + 7 = 0$ or $x - 2 = 0$

$x = -\frac{7}{2}$, or $x = 2$

13) $x^2 - 7x + 9 = 0$

$x = \frac{-(-7)\pm\sqrt{(-7)^2-4(1)(9)}}{2(1)}$

$x = \frac{7\pm\sqrt{13}}{2}$

$x = \frac{7}{2} \pm \frac{1}{2}\sqrt{13}$

14) $x^2 + 2x - 6 = 0$

$(x+1)^2 - (1)^2 - 6 = 0$

$(x+1)^2 = 7$

$x + 1 = \pm\sqrt{7}$, $x = -1 \pm \sqrt{7}$

Subject	Qualification	Examination Board
Physics	A Level	AQA
Additional Information:		

Task Overview:
1. Experiment Design 2. Data Analysis 3. Presenting Data 4. Maths Skills 5. YouTube Physics video links 6. Physics GCSE-standard Exam Questions 7. Physics in the News 8. Physics Key Vocabulary 9. Astrophysics Research Project 10. Applications of Physics Case Study 11. Chernobyl Newspaper Article 12. Physics Popular Literature
Success Criteria:
See further task details below
Resources:
Internet, GCSE revision materials, calculator, ruler, pencil, graph paper.
How will the work produced will fit into subsequent work and the specification as a whole?
The range of activities allow you to demonstrate your knowledge in various key areas vital to the study of Physics. Some will also give you the opportunity to discover more about the content you will learn whilst studying A Level Physics as well as developing other skills.
How should the work should be presented?
Once each assignment has been completed, you should email (if possible) to Mr Alker (see address below). For assignments that can only be completed on paper, these should be filed so that they can be handed in when you are next in school.

Who should you contact if you should require further assistance with the work before the end of term?	Mr Alker (Head of Science/Physics) m.alker@gildredgehouse.org.uk
Length of time expected to complete tasks:	2 – 3 hours per task
Submission Requirements:	Email work when completed, or store in a file.

What equipment will be needed for the subject?
This is our course text book: AQA Physics A Level Second Edition Year 1 Student Book Author: Jim Breithaupt, ISBN: 978 0 19 835186 3, Publisher: Oxford University Press, www.global.oup.com
Optional Extension Task/Further Reading

http://www.physics.org/
http://www.iop.org/education/student/youth_membership/page_41684.html
https://www.iop.org/

Task Details

1. Experiment Design

On Earth, the acceleration due to gravity, ‘g’, is 9.81 ms^{-2}. You may be more familiar with using g = 10 ms^{-2}.

How did scientists work this out?

Design an experiment to determine the value of the acceleration due to gravity, ‘g’.
- Research the required Physics to be able to calculate ‘g’.
- Design an experiment that could be carried out using standard laboratory equipment to experimentally determine the value of ‘g’.
- Draw a clear diagram of your apparatus.
- Write a detailed set of instructions in your plan, ensuring you describe the exact measurements to take.
- Explain how you would make your results as ACCURATE as possible.
- Explain how you would make your results as RELIABLE as possible.
- Explain how you would process your data to determine the value of ‘g’.

2. Data Analysis

Using the provided data, write a report analysing the patterns and trends of Nuclear Power generation in the UK.
- Ensure you analyse the graphs carefully to fully describe the patterns shown
- Comment on the sources provided – how reliable do you think they are?
- Provide a context to your analysis by discussing any global trends

3. Presenting Data

For each set of data provided, display the information on an appropriate graph.

4. Maths Skills

Complete the exercises provided covering a range of essential maths skills.

5. YouTube Physics Video Links

Using the provided list of website URLs, watch each of the short YouTube video links to review the key topics from GCSE that form the foundations of the A Level course. If you would like the links emailed to you, please contact m.alker@gildredgehouse.org.uk

6. Physics GCSE-standard Exam Questions

Complete the GCSE-standard exam style questions provided in your pack.

7. Physics in the News

Read the provided articles of examples of recent advances of Physics and Engineering.

8. Physics Key Vocabulary

Learn the definitions of the list of key Physics vocabulary, provided in your pack.

9. Astrophysics Research Project

Pick **one** of the following research questions to form the basis of an in-depth research project. Present your research in a format of your choosing (eg: written report, powerpoint presentation, etc):

1. How do Astrophysicists measure vast distances, such as those between stars?
2. What is a black hole?
3. How is our Universe changing and what evidence do we have?
4. How have telescopes evolved over time?

10. Application of Physics Case Study

Choose one mode of transport (aeroplane/boat/car/bike/rocket). Research the Physics involved in making your chosen mode of transport move. Summarise your findings into a short case study report.

11. Chernobyl Newspaper Article

Watch a (factual!) documentary on the Chernobyl nuclear disaster (such as National Geographic's: https://www.youtube.com/watch?v=AZ4qOMN527s)

Then write a newspaper article about the incident.

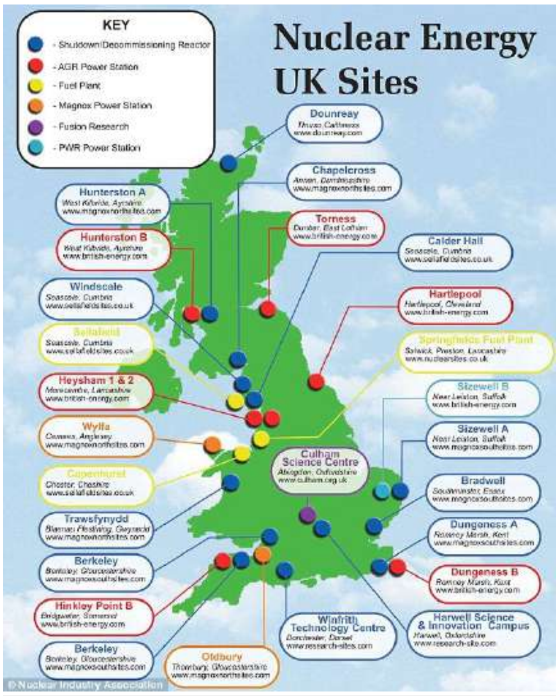
12. Physics Popular Literature

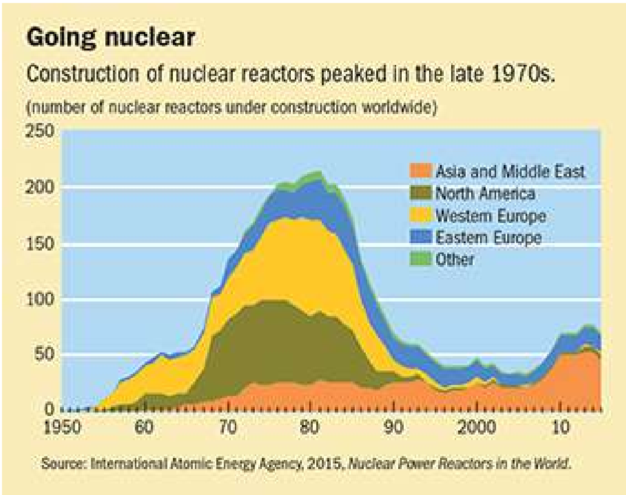
Find a copy of at least one of the following books in your local library and read it.
- Cosmos, by Carl Sagan
- A Brief History of Time, by Stephen Hawking
- The Road to Reality, by Roger Penrose
- The Tragedy of the Moon, by Isaac Asimov
- The Elegant Universe, by Brian Greene
- Introduction to Quantum Mechanics, by David J Griffiths
- 13 Things That Don't Make Sense: The Most Intriguing Scientific Mysteries of Our Time, by Michael Brooks
- Goldilocks and the Water Bears: The Search for Life in the Universe, by Louisa Preston

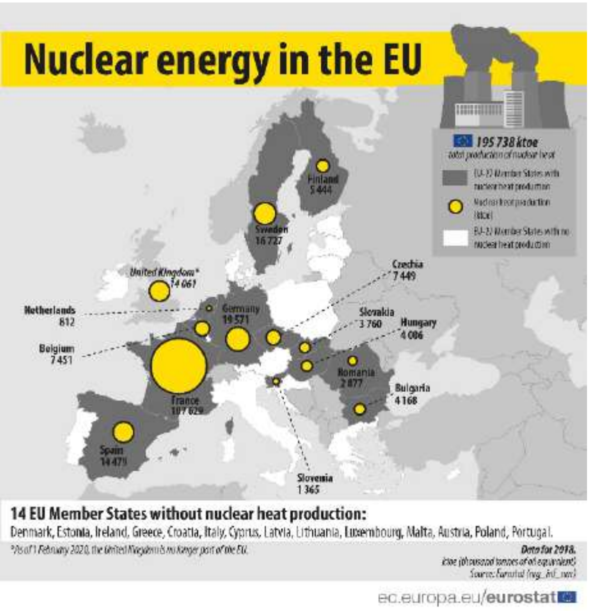
Physics Task 2: Data Analysis

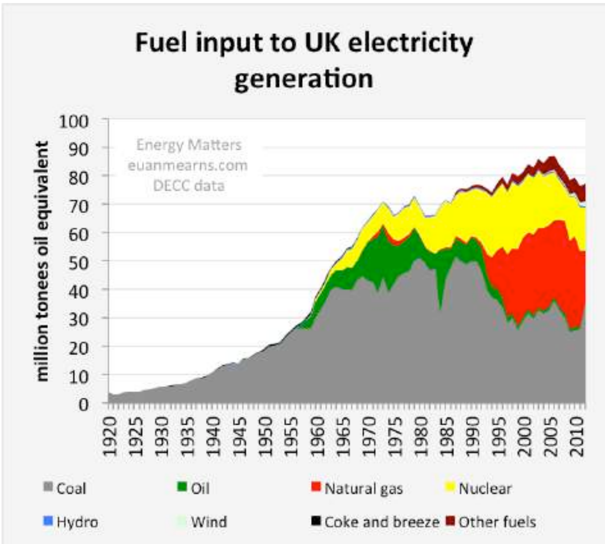
Using the provided data, write a report analysing the patterns and trends of Nuclear Power generation in the UK.

- Ensure you analyse the graphs carefully to fully describe the patterns shown
- Comment on the sources provided – how reliable do you think they are?
- Provide a context to your analysis by discussing any global trends









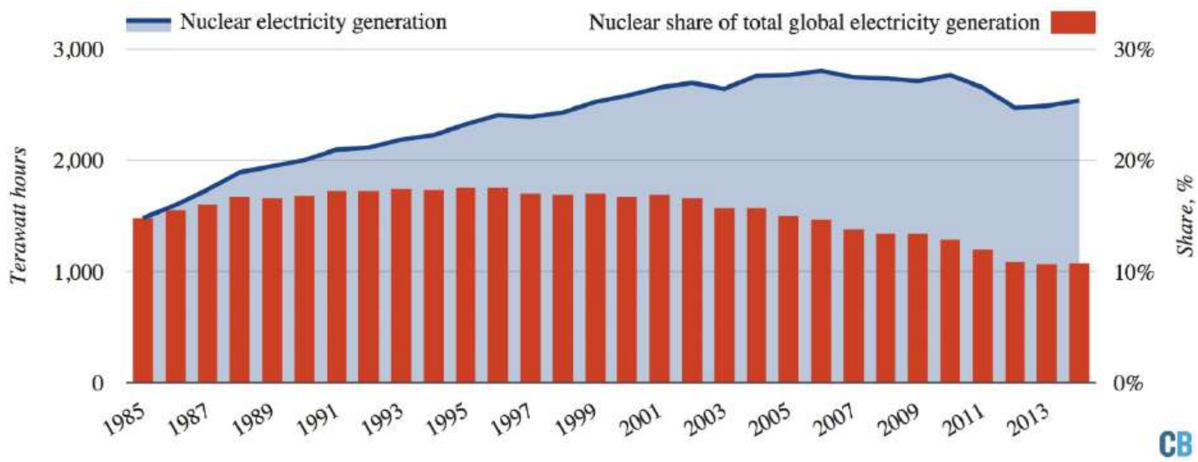
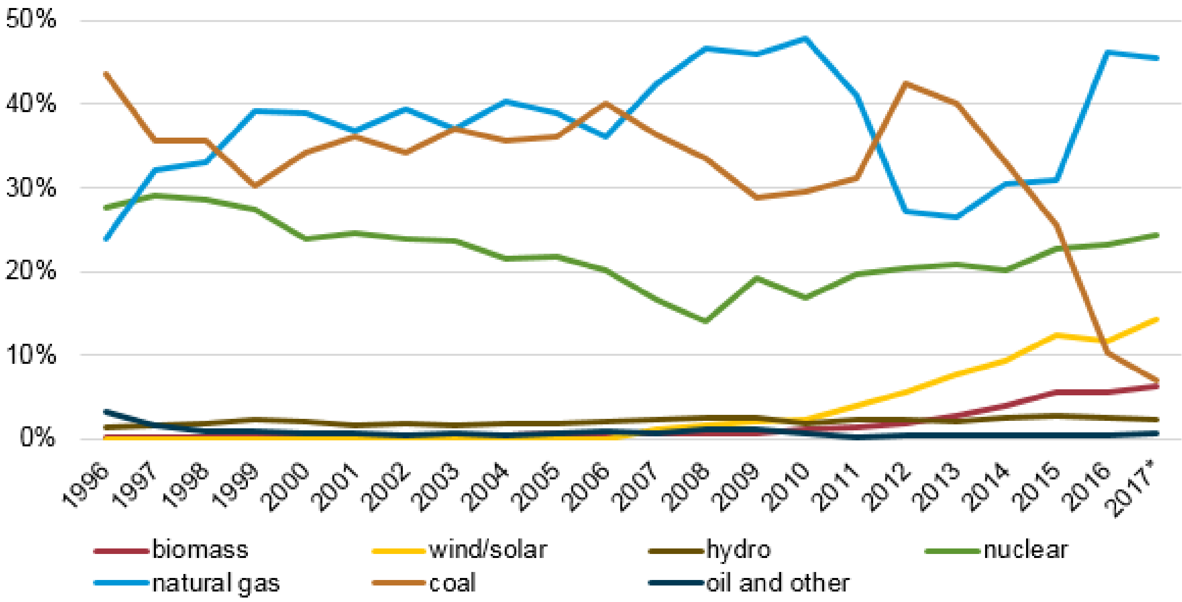


Figure 12. United Kingdom electricity generation share by fuel source



* note 2017 values are estimates based on data through September

Source: U.S. Energy Information Administration based on Digest of UK Energy Statistics and National Statistics: Energy Trends

Task 3: Presenting Data

For each set of data below, display the information on an appropriate graph.

Data set 1:

Table 2: Mass of balls and diameters of crater formed

	mass(±0.01g)	Diameter of crater(±0.1cm)					Average diameter(±0.2cm)
ball1	27.92	8.0	8.0	8.3	8.4	8.3	8.2
ball2	46.53	9.3	9.4	9.3	9.1	9.4	9.3
ball3	65.37	9.7	9.4	9.5	9.5	9.7	9.6
ball4	105.44	9.9	10.5	10.2	9.9	10.4	10.2
ball5	112.01	10.6	10.6	10.8	10.5	10.6	10.6
ball6	136.74	10.7	10.7	10.9	10.9	11.4	10.9
ball7	174.45	11.2	11.4	11.6	11.2	11.2	11.3

Table 2: The largest average deviation of the trials was used as the uncertainty of the average diameter.

Data set 2:

Length of wire (cm)	Volts (Volts)			Current (amps)			Resistance (ohms)		
	1	2	3	1	2	3	1	2	3
10	0.47	0.47	0.47	0.24	0.23	0.23	1.96	2.04	2.04
20	0.6	0.59	0.58	0.16	0.17	0.17	3.75	3.47	3.41
30	0.65	0.64	0.64	0.14	0.13	0.13	4.64	4.92	4.92
40	0.69	0.69	0.68	0.11	0.11	0.11	6.27	6.27	6.18
50	0.72	0.72	0.72	0.1	0.09	0.08	7.2	8	9.13
60	0.76	0.76	0.76	0.07	0.07	0.07	10.9	10.9	10.9
70	0.82	0.82	0.82	0.06	0.06	0.06	13.67	13.67	13.67

Data set 3:

Angle between the spring gun and the horizon (degrees)	Range (meters)
20	6.4
30	8.6
40	9.8
50	9.6
60	8.7
70	6.3
80	3.4

Data set 4:

Table 1		
Velocity of Sound in Various Media		
Media	Density (kg per cubic meter)	Velocity (m/s)
Air	1.0	343.0
Pure Water	1,000.0	1,493.0
Sea Water	1,020.0	1,533.0
Glass	2,600.0	4,540.0
Iron	7,870.0	5,130.0
Lead	11,350.0	1,158.0

Data set 5:

Table 2		
Velocity of Sound in Water		
Temperature (°C)	Density (kg per cubic meter)	Velocity (m/s)
0	999.8395	1,402.39
10	999.7026	1,447.28
20	998.2071	1,482.36
30	995.6502	1,509.14
40	992.2	1,528.88
50	988.04	1,542.57
60	983.2	1,552.00
80	971.8	1,555.00
100	958.37	1,543.05

In questions 17 to 32, $a = 4, b = -2, c = -6$

17 $\frac{1}{4}bc$	18 $3b$	19 c^2	20 $\frac{2}{9}c^2$
21 $5a^2$	22 $(4b)^2$	23 $9b^2$	24 $a^2 + c^2$
25 $ab + c$	26 $\frac{1}{3}bc - \frac{1}{2}a$	27 $\frac{1}{8}abc$	28 $\frac{3a}{2b}$
29 $\frac{c}{b} + \frac{5a}{2b}$	30 $(2a)^2 - \frac{1}{2}a^2$	31 $b^2(2a - c)$	32 $\frac{1}{2}(a - b) - \frac{1}{3}c$

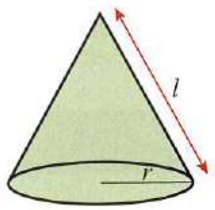
In questions 33 to 48, $x = -3, y = 5, z = -1$

33 $z(2x + y)$	34 $x^2 - \frac{1}{4}z$	35 $\frac{1}{3}x^2 + 2x$	36 $\frac{3}{5}y^2 - 2x^2$
37 z^3	38 $x^2(2y + 3z)$	39 $\frac{4y + 2x^2}{z^2}$	40 $x^2(y^2 - z^2)$
41 $z^2 - 3z + 2$	42 $\frac{2}{3}x^3 + z^3$	43 $\frac{2y - 5z}{5x}$	44 $\frac{x(x^2 - z)}{3y}$
45 $\frac{(2x)^2 + z}{y}$	46 $(2x + 4z)(3y + z)$	47 $x^3z^3(x^2 - z)$	48 $\frac{4}{5}xy + \frac{2}{3}xz$

M Formulas

M4.2

1



The surface area A of a cone is roughly given by the formula

$$A = 3rl$$

Find the value of A when

(a) $r = 2, l = 10$ (b) $r = 5, l = 3$ (c) $r = 8, l = 5$

2 The position P of the middle value of some numbers is found from the formula

$$P = \frac{1}{2}(n + 1)$$

where n is how many numbers there are.

Find P when (a) $n = 7$ (b) $n = 99$

3 The interest I made by some money P is given by the formula

$$I = \frac{PTR}{100}$$

where T is the time and R is the rate of interest. Find the value of I when

(a) $P = 800, T = 2, R = 5$ (b) $P = 40, T = 5, R = 8$

4

The volume V of an orange is roughly given by the formula

$$V = 4r^3$$

where r is the radius of the orange.

Find the value of V when (a) $r = 5$ (b) $r = 10$

5 A ball is dropped. The distance s it travels is given by the formula

$$s = 4.9t^2$$

where t is the time taken.

Find the value of s when

(a) $t = 10$ (b) $t = 2$

6 The temperature in degrees Fahrenheit F can be changed into degrees Centigrade C by using the formula

$$C = \frac{5}{9}(F - 32)$$

Find the value of C when

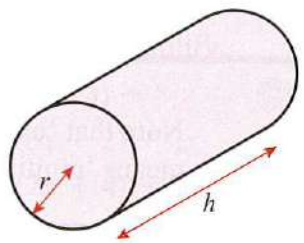
(a) $F = 50$ (b) $F = 68$

7 Using the formula $h = \sqrt{(a^2 - b^2)}$ find the value of h when

(a) $a = 13, b = 12$

(b) $a = 25, b = 24$

8



The surface area A of a cylinder is roughly given by the formula

$$A = 6rh + 3r^2$$

Find the value of A when

(a) $r = 2, h = 5$

(b) $r = 4, h = 20$

Can you still?

Mixed

1. Some chocolates are shared between Gianna and Vitali in the ratio 3 : 7. Gianna gets 12 chocolates less than Vitali. Vitali eats two-thirds of his share. How many chocolates does Vitali now have?

2. Work out

   (a) $\frac{(-7)^2 + 11}{1 - (-4)}$

   (b) $\frac{(-2 - 3)^3 - 5^2}{2{\cdot}5^2 + 3{\cdot}75}$

3. An exterior angle of a regular polygon is 18°. Work out the sum of the interior angles.

4. Simplify

   (a) $\frac{(4m^2)^2}{8m^3}$

   (b) $\sqrt[3]{(8x^{12}y^9)}$

   (c) $\frac{(2y^3)^2}{(2y^2)^3}$

In questions 9 to 12, you may *use a calculator* if you wish.

9 The formula $v = \sqrt{u^2 + 2as}$ gives the final speed v of a car whose initial speed is u, acceleration is a and displacement is s.
Find the value of v (to 3 significant figures if necessary) when
(a) $u = 10, a = 3, s = 7$
(b) $u = -3{\cdot}6, a = 9{\cdot}8, s = 15{\cdot}3$

10 The formula $s = vt - \frac{1}{2}at^2$ gives the displacement s of a particle after time t.
The final speed is v and acceleration is a.
Find s (to 3 significant figures if necessary) when
(a) $v = 5, a = 3, t = 2$ (b) $v = 0, a = 9{\cdot}81, t = 1{\cdot}03$

11 The formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ gives the area A of a triangle with sides a, b and c where s is half the perimeter.
Find the area A (to 2 significant figures if necessary) of the following triangles (by first finding and stating the value of s).
(a) $a = 3, b = 4, c = 5$ (b) $a = 7.8,\ b = 18.72,\ c = 20{\cdot}28$

12 The total resistance R in a circuit with resistors R_1 and R_2 in parallel is given by the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Find R in the following cases (to 3 significant figures if necessary).
(a) $R_1 = 2$ ohms and $R_2 = 2$ ohms
(b) $R_1 = 3$ ohms and $R_2 = 4$ ohms

M Multiplying out brackets

Multiply out $2(a + b)$ means $2 \times a$ add $2 \times b = 2a + 2b$

Expand $5n(2n + 3)$ means $5n \times 2n$ add $5n \times 3 = 10n^2 + 15n$

Note that 'expand' means 'multiply out'.

M4.3

1 Simplify:

(a) $4y \times 2$	(b) $6 \times 8x$	(c) $2a \times 4b$	(d) $c \times 5c$
(e) $7a \times 2a$	(f) $24x \div 4$	(g) $42n \div 6$	(h) $64p \div 8$
(i) $6c \times 9c$	(j) $3a \times 2b \times 2c$	(k) $-9y \times 4$	(l) $-4c \times 5d$
(m) $-6c \times -3d$	(n) $-9y \div 3$	(o) $28q \div -4$	(p) $-7p \times -5p$

In questions 2 to 10, answer 'true or false'.

2 $3 \times a = a \times 3$	3 $3a - a = 3$	4 $8n \times 4n = 32n^2$
5 $4n + 4n = 8n^2$	6 $2 \times 3n = 5n^2$	7 $10a \div 2 = 5a$
8 $a \times 3a = 3a^2$	9 $a + a^2 = a^3$	10 $12p \div 3 = 9p$

Multiply out

11 $2(a + 3)$	12 $6(4x - 2)$	13 $7(3x - 5)$	14 $5(a - b)$
15 $7(3x + y)$	16 $6(3x + 2)$	17 $4(p + 2q)$	18 $9(4c + 8d)$
19 $x(2x + y)$	20 $a(b + a)$	21 $c(2c + d)$	22 $a(a - 7)$
23 $a(3a - 4)$	24 $5x(y + 2)$	25 $4a(a + 2b)$	26 $6a(4b - 8c)$

Expand

27 $-2(x + 6)$	28 $-3(a - 2)$	29 $-5(c + 10)$	30 $-4(3p - 5)$
31 $-3(2c + 4)$	32 $-a(b + c)$	33 $-p(2p + q)$	34 $-a(a + b)$
35 $-x(2x - y)$	36 $-m(m - n)$	37 $-(2p + 5q)$	38 $-6a(4 - 2b)$

Expand

39 $a(a^2 - 2b)$	40 $x^2(4x + y)$	41 $5b^2(2b + 3)$	42 $3p(pq + 2p^2)$

M Expanding and simplifying brackets

(a) Simplify $2(3n + 1) + 3n$
multiply out brackets first
$= 6n + 2 + 3n$
now collect like terms
$= 9n + 2$

(b) Simplify $5(2a + 1) - 3(a - 2)$
$= 10a + 5 - 3a + 6$
⇧ Note
$= 7a + 11$

M4.4

Expand and simplify:

1 $2(x + 3) + 5$	2 $5(2x + 1) + 3$
3 $4(3x + 2) + 2x$	4 $9(2x + 3) - 14$
5 $3(2a + 4) - 2a$	6 $9(3y + 2) - 6$

Expand and simplify:

7 $5(a + 2) + 2(2a + 1)$
8 $3(x + 4) + 6(x + 2)$
9 $6(x + 1) + 3(2x + 4)$
10 $3(4a + 8) + 2(a - 3)$
11 $7(2x + 3) + 4(3x + 1)$
12 $4(2d + 2) + 6(3d + 4)$

Can you still?

Mixed

1 Simplify
(a) $\frac{4}{\sqrt{2}}$ (b) $\frac{(5 + \sqrt{2})(5 - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})}$

2 A population of 900 increases by 10% then decreases by 10% of its current size. How large is the population now?

3 Evaluate (a) 6^{-2} (b) $9^{-\frac{1}{2}}$ (c) $64^{\frac{1}{3}}$

Simplify

13 $3(4a + 2) - 2(a - 2)$ 14 $5(3x + 1) - 3(3x + 2)$ 15 $6(2x + 3) - 4(2x + 2)$

16 $5(3a + 2) - 3(2a + 1)$ 17 $3(4d + 1) - 2(6d - 5)$ 18 $7(2c + 6) - 4(c + 5)$

Expand and simplify:

19 $9y - 5(y + 2) - 3$ 20 $11x + 2 - 3(2x - 5)$ 21 $6a + 2(3a + 1) - 7 + 2a$

22 $15(n + m) - 6(2n - m)$ 23 $8(2a + b) - 3(3a + 2b)$ 24 $8x + 6(2 - x) + 2(3x + 5)$

Simplify:

25 $a(3a + 2) + 5a(2a + 1)$ 26 $4x(3x + 2) - 3x(x - 3)$

27 $2y(4y + 5) - 5y(y + 2)$ 28 $3a(a + 2b) + 2b(5a + 2b)$

29 $4m(3m + 2p) - 3m(p - 2m)$ 30 $6x(2x + 3y) - 3x(x - 4y)$

31 $5a(a + 2b) + 4b(3a - c)$ 32 $7x(2x + 3y + z) - 3y(3x - 2z)$

M Multiplying out 2 brackets

Key Facts

Each term in one bracket must be multiplied by each term in the other bracket.

Consider $(a + b)(c + d)$.

- F: $(a + b)(c + d)$ multiply the First terms in each bracket → ac
- O: $(a + b)(c + d)$ multiply the Outer terms in each bracket → $+ ad$
- I: $(a + b)(c + d)$ multiply the Inner terms in each bracket → $+ bc$
- L: $(a + b)(c + d)$ multiply the Last terms in each bracket → $+ bd$

First
Outer
Inner
Last

Follow this order each time to make sure you do not miss any terms → $(a + b)(c + d) = ac + ad + bc + bd$

(a) Expand $(x + 5)(x - 2)$

$(x + 5)(x - 2)$

F O I L

$= x^2 - 2x + 5x - 10$

these middle 2 terms can be collected together

$= x^2 + 3x - 10$

(b) Expand $(2x + 3)(5x - 1)$

$(2x + 3)(5x - 1)$

$= 10x^2 - 2x + 15x - 3$

$= 10x^2 + 13x - 3$

(c) Expand $(x - 5)^2$

$(x - 5)(x - 5)$

$= x^2 - 5x - 5x + 25$

$= x^2 - 10x + 25$

WARNING!

$(x - 5)^2$ is **not** $x^2 - 25$ or $x^2 + 25$

Similarly

$(x + 4)^2$ is **not** $x^2 + 16$

M4.5

1 Copy and complete the following.

(a) $(x + 3)(x + 4)$
$= x^2 + 4x + \square + 12$
$= x^2 + \square + 12$

(b) $(a - 5)(a + 3)$
$= \square + 3a - 5a - \square$
$= \square - 2a - \square$

(c) $(m - 7)^2$ $(m - 7)(m - 7)$
$= m^2 - \square - \square + \square$
$= m^2 - \square + \square$

Expand the following:

2 $(x + 2)(x + 6)$ 3 $(p + 1)(p + 5)$ 4 $(a + 3)(a + 7)$

5 $(m + 3)(m - 1)$ 6 $(y + 2)(y - 6)$ 7 $(n - 5)(n + 2)$

8 $(b - 8)(b + 3)$ 9 $(x - 6)(x + 8)$ 10 $(c - 8)(c - 3)$

11 $(q - 2)(q - 7)$ 12 $(f - 2)(f - 10)$ 13 $(a + 9)(a - 4)$

Multiply out the following:

14 $(2x + 1)(3x + 4)$ 15 $(5y + 2)(y + 3)$ 16 $(4p + 2)(2p + 7)$

17 $(3a - 4)(5a + 2)$ 18 $(6f - 1)(2f + 3)$ 19 $(9y - 4)(4y - 2)$

20 $(3x - 4)(7x + 2)$ 21 $(3q - 4)(7q - 1)$ 22 $(4b - 3)(5b - 3)$

23 $(4z - 6)(2z - 3)$ 24 $(x + 7)(4x - 9)$ 25 $(6a - 5)(5a + 4)$

Expand the following:

26 $(x + 6)(x + 6)$ 27 $(x + 5)^2$ 28 $(a + 10)^2$

29 $(y - 1)^2$ 30 $(p - 3)^2$ 31 $(b - 9)^2$

32 $(2a - 1)^2$ 33 $(5x + 4)^2$ 34 $(3y - 5)^2$

13 Copy and complete

$3(3x + 4) = 4(2x - 2)$

$9x + 12 = \square - 8$

$9x - \square = -8 - 12$

$x = \square$

14 Copy and complete

$5(2n - 1) - 6(n + 1) = 1$

$10n - \square - 6n - \square = 1$

$10n - 6n = 1 + \square + \square$

$\square = \square$

$n = \square$

Solve the following equations:

15 $7(x - 1) = 2(2x + 4)$

16 $8(w - 3) = 4(3 - w)$

17 $3(a + 2) = 4(1 - a)$

18 $3(2n + 1) = 4(7 - n)$

19 $5(2 - p) = 2(4 + 2p)$

20 $2(3 - 2m) = 5(2 - m)$

21 $4(a - 2) = 3(a + 3) - 4$

22 $3(2q + 3) + 4(q - 2) = 8$

23 $6(2x + 5) + 3(2 - 3x) = x$

24 $5(c - 2) + 1 = 3(c - 1)$

25 $7(2h + 1) - 1 = 5(3h + 2)$

26 $2(4w - 3) - 7(2w - 7) = 1$

27 $4(2y + 1) - 9(y - 1) = 3$

28 $3(x + 1) + 1 = 2(2x + 1) - 3$

29 $2(b + 1) + 1 = 3(3b - 5) - 10$

30 $\frac{1}{6}(x - 14) = 2(x - 3)$

31 $\frac{1}{2}(4y + 1) - \frac{3}{4}(2y + 3) = \frac{3}{4}$

32 $\frac{1}{3}(2w - 5) + \frac{1}{6}(w - 4) = 1$

Mixed linear equations

Solve the following equations:

(a) $\frac{2x - 1}{3} = 5$

$2x - 1 = 5 \times 3$ [multiply both sides by 3]

$2x - 1 = 15$

$2x = 16$

$x = 8$

(b) $\frac{5}{x} = -2$

$5 = -2x$ [multiply both sides by x]

$x = \frac{5}{-2}$ [divide both sides by -2]

$x = -2{\cdot}5$

(c) $\frac{x - 3}{x + 2} = 7$ [multiply both sides by $(x + 2)$]

$x - 3 = 7(x + 2)$ [multiply out brackets]

$x - 3 = 7x + 14$

$-3 - 14 = 7x - x$

$-17 = 6x$

$x = -2\frac{5}{6}$

M6.3

Solve the following equations:

1 $\frac{x + 5}{4} = 5$

2 $\frac{n + 9}{3} = 7$

3 $\frac{1}{3}a + 2 = 7$

4 $\frac{x - 8}{2} = 5$

5 $\frac{1}{6}w - 8 = 1$

6 $2 = \frac{m - 10}{3}$

7 $5 - \frac{1}{4}z = 2$

8 $\frac{3m - 1}{5} = 4$

9 $8 = \frac{1}{7}(6b + 2)$

Copy and complete:

10 $\frac{16}{x} = 3$

$16 = \square$

$x = \frac{\square}{\square}$

$x = \square\frac{\square}{\square}$

11 $\frac{12}{x - 2} = 4$

$12 = \square(x - 2)$

$12 = \square - \square$

$12 + \square = \square$

$\square = \square$

$x = \square$

12 $\frac{3x + 2}{x + 4} = 5$

$3x + 2 = \square(x + 4)$

$3x + 2 = \square + \square$

$2 - \square = \square - 3x$

$-\square = \square$

$x = \square$

Solve the following equations:

13 $\frac{15}{a} = 3$

14 $\frac{24}{x + 1} = 6$

15 $\frac{36}{x - 2} = 4$

16 $\frac{18}{m - 8} = 3$

17 $4 = \frac{5}{a}$

18 $\frac{13}{m} = 6$

19 $-5 = \frac{9}{n}$

20 $\frac{3c + 1}{5} = 2$

21 $\frac{1}{3}(4x - 3) = 7$

22 $\frac{x + 1}{x - 3} = 3$

23 $\frac{2a + 1}{a - 4} = 5$

24 $\frac{3m - 2}{m - 6} = 7$

25 $\frac{5w - 3}{w - 2} = 6$

26 $\frac{4b + 1}{b + 3} = 3$

27 $\frac{5}{n} - 4 = 4$

28 $\frac{3}{f} + 7 = 9$

29 $6(a - 2) = 4(2a + 1)$

30 $\frac{1}{9}x + 7 = 10$

31 $5(2p + 3) + 2(3p + 5) = 33$

32 $\frac{7}{3m} = 1$

33 $8(a - 2) = 5(2a + 3) + 3(a - 4)$

M Setting up linear equations

Many problems can be solved by writing them as linear equations first.
The unknown quantity is often chosen to be x.

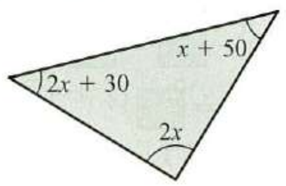
The sum of four consecutive numbers is 42. Let the first number be x and write down the other three numbers in terms of x. Find the four numbers.
Other three numbers are $(x + 1)$, $(x + 2)$ and $(x + 3)$.

Sum is 42 so $x + (x + 1) + (x + 2) + (x + 3) = 42$

$$4x + 6 = 42$$
$$4x = 36$$
$$x = 9$$

The four numbers are 9, 10, 11 and 12.

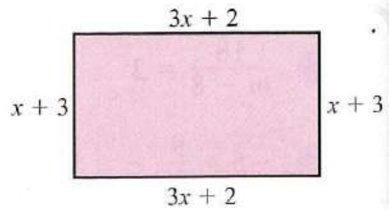
M6.4

1 

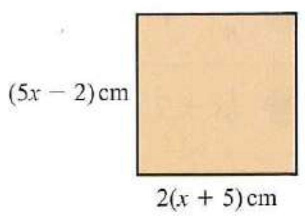
(a) Write down an equation using the angles.
(b) Find x.
(c) Write down the actual value of each angle in this triangle.

2 The sum of four consecutive numbers is 78. Let the first number be x. Set up an equation to find x then find the four numbers.

3 The perimeter of this rectangle is 58 cm.
(a) Write down an equation using the perimeter.
(b) Find x.
(c) Write down the actual length and width of the rectangle.

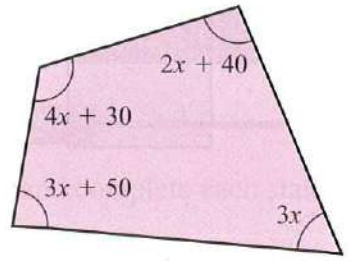


4 A rectangle has its length twice its width. If its perimeter is 42 cm, find the width of the rectangle.

5 

Work out the actual perimeter of this square.

6 £190 is divided between Jack and Halle so that Jack receives £72 more than Halle. How much does each person get? (Hint: Let x = Halle's money.)

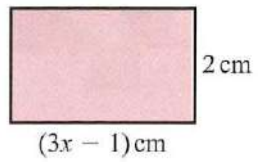
7 

(a) Write down an equation using the angles.
(b) Find x.
(c) Write down the actual value of each angle in this quadrilateral.

8 The length of a rectangle is 3 times its width. If the perimeter of the rectangle is 32 cm, find its length and width.

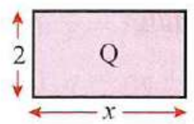
9 A triangle has 2 angles which are each 4 times the size of the third angle. Find the size of each angle.

10 The sum of four consecutive odd numbers is 216.
(a) If x is the smallest number, write down the other numbers in terms of x.
(b) Find the actual numbers.

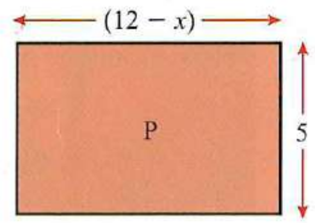
11 

The area of the rectangle is 46 cm^2.
Find the perimeter of the rectangle.

12 You have three consecutive *even* numbers so that the sum of twice the smallest number plus three times the middle number is four times the largest number. Find the three numbers.

13 

The area of rectangle P is five times the area of rectangle Q. Find x.



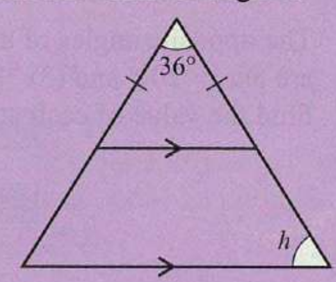
Can you still?

Mixed

1 Evaluate
(a) 2^{-1} (b) 3^{-2} (c) $\left(\frac{1}{9}\right)^{-\frac{1}{2}}$

2 Factorise
(a) $ac - bc - ad + bd$
(b) $12x^2 - x - 6$

3 Find the value of angle h.

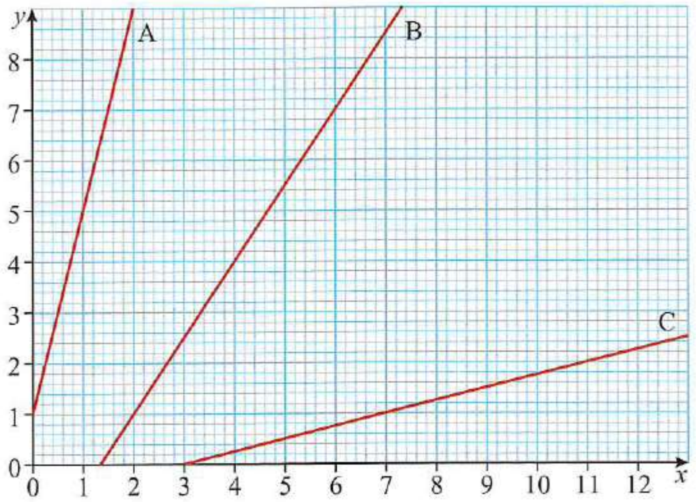


4 AB = $\frac{1}{4}$ cm and CD = $\frac{2}{3}$ cm.
Write down the ratio AB:CD in the form m:n where m and n are integers.

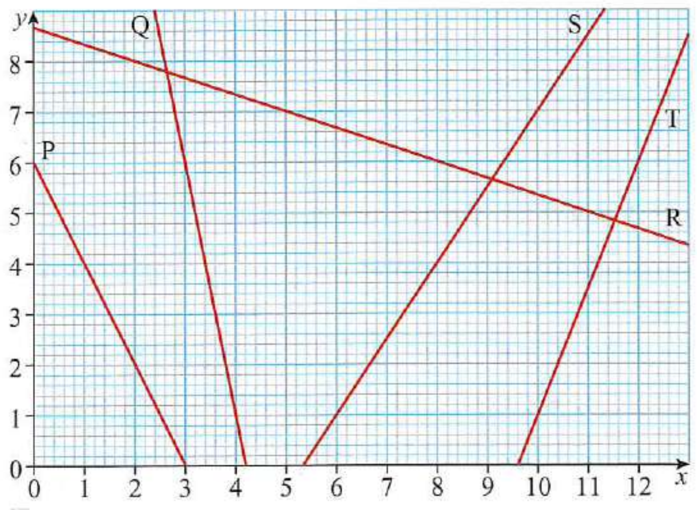
5 Solve $8^{\frac{x}{2}} = 32$

M6.12

1 Find the gradient of each line below:



2 For each line below, find the rate of change of y as x varies.



3 Find the gradient of the line joining each pair of points below:

(a) (1, 1) and (3, 5) (b) (2, 4) and (3, 7) (c) (3, 6) and (5, 2)

(d) (3, 1) and (5, 4) (e) (1, 4) and (3, 2) (f) (0, 5) and (2, 4)

(g) (5, 2) and (7, 3) (h) (−1, 2) and (2, 7) (i) (5, −2) and (9, −2)

(j) (−2, −3) and (1, −5) (k) (−4, 6) and (2, 5) (l) (1, 2) and (1, −5)

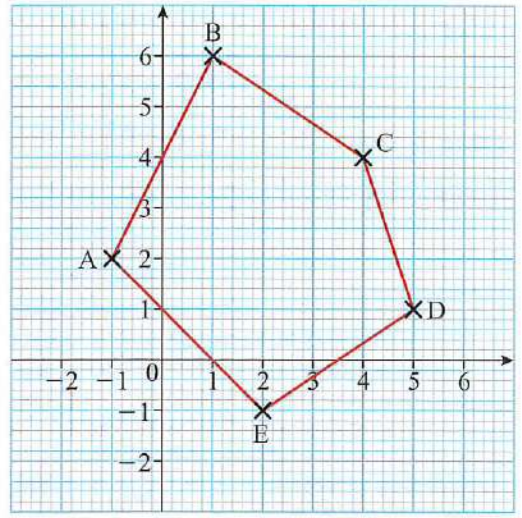
(m) $\left(\frac{1}{4}, 2\right)$ and $\left(\frac{2}{3}, 4\right)$ (n) $\left(-\frac{1}{3}, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, -\frac{1}{4}\right)$ (o) $\left(\frac{2}{5}, -\frac{1}{3}\right)$ and $\left(-\frac{1}{4}, \frac{1}{5}\right)$.

4 Find the gradient of the line joining:

(a) B and C

(b) C and D

(c) A and E.



5 A line has a gradient of 6. One point on the line is (−2, 5). A point P on the line has an x-value of 1. What is the y-value at P?

6 A line shows that the rate of change of y as x varies is −4. One point on the line is (3, 7). A point Q on the line has a y-value of −5. What is the x-value at Q?

M $y = mx + c$

M6.13

Use a graphical calculator or computer if possible.

1 (a) Complete the table opposite then draw the straight line $y = 2x + 3$.

x	1	2	3
y			

(b) Use another table to draw $y = 2x$ on the same grid.

(c) Draw $y = 2x + 1$ on the same grid.

(d) Draw $y = 2x - 1$ on the same grid.

(e) Find the gradient of each line.

(f) What do you notice about the gradient of each line and its equation?

(g) Look at where each line cuts the y-axis. For each line what do you notice about this value and its equation?

2 Draw the following lines using the same set of axes and repeat parts (e), (f) and (g) from question **1**:

$y = -3x + 1$ $y = -3x + 4$ $y = -3x$

$y = -3x - 2$ $y = -3x - 5$

Key Facts

The equation of a straight line may be written in the form

$$y = mx + c$$

m is the gradient of the line

c is the *y*-value at the point where the line cuts the *y*-axis. This is known as the *'y-intercept'*

$y = mx + c$ is sometimes known as the 'gradient–intercept' form of the straight line.

(a) Write down the gradient and *y*-intercept of $y = \frac{1}{2}x - 3$

Gradient $= \frac{1}{2}$ $\quad$ *y*-intercept $= -3$

(b) Write down the gradient of $2x + 3y = 1$ and write down the co-ordinates of the point where the line cuts the *y*-axis.

rearrange into form $y = mx + c$

$$2x + 3y = 1$$
$$3y = -2x + 1$$

write the *x*'s first

$$y = -\frac{2}{3}x + \frac{1}{3}$$

Gradient $= -\frac{2}{3}$ and the line cuts the *y*-axis at $\left(0, \frac{1}{3}\right)$.

(c) Find the equation of the line which passes through (3, 1) and (6, 13).

Find gradient *m* first $\quad m = \frac{13-1}{6-3} = \frac{12}{3} = 4$

We know $y = mx + c$ so $y = 4x + c$

$x = 3$ when $y = 1$

To find *c*, substitute one pair of *x* and *y* values into the equation of the line

so $1 = 4 \times 3 + c$

$1 = 12 + c$

$c = -11$

Equation of line is $y = 4x - 11$.

M6.14

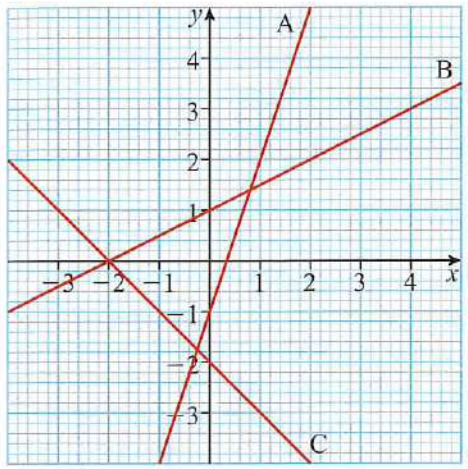
Write down the gradient and *y*-intercept of each of the following lines:

1 $y = 3x + 4$	**2** $y = 2x - 5$	**3** $y = 8x - 1$	**4** $y = x + 6$
5 $y = -4x - 2$	**6** $y = -4x + 3$	**7** $y = -x - 2$	**8** $y = -5x + 2$
9 $y = 3 - x$	**10** $y = 4 - 2x$	**11** $y = \frac{1}{3}x - 7$	**12** $y - 5x = 1$
13 $y + 4x = 5$	**14** $6x - y = 3$	**15** $2x + 5y = 3$	**16** $3x - 4y = 6$
17 $5x - 3y = 3$	**18** $4y - 2 = 5x$	**19** $4x + y - 6 = 0$	**20** $5x - 7y - 2 = 0$

Use your knowledge of $y = mx + c$ to *sketch* each of the following lines:

21 $y = 2x + 2$	**22** $y = 5 - x$	**23** $y - 3x = 1$	**24** $2x + 4y = 3$

25 Write down the equation of each of the 3 lines below:



26 Which of the following lines are parallel?

(a) $y = 4x + 1$ $\quad$ (b) $y = 2 - 4x$ $\quad$ (c) $y = 2x + 4$

(d) $y - 4x = 2$ $\quad$ (e) $4x - y = 2$ $\quad$ (f) $y = 4 - 3x$

Find the equation of each line in questions **27** to **36**.

27 The line passes through (0, 4) with gradient = 5.

28 The line passes through (0, 2) with gradient = −4.

29 The line passes through (3, 5) with gradient = 3.

30 The line passes through (5, −1) with gradient = 1.

Find the equation of the line passing through each pair of points below, giving the answer in the form $y = mx + c$.

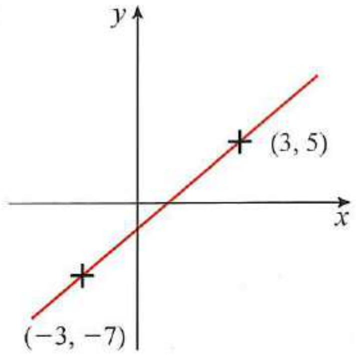
31 (3, 2) and (5, 8)	**32** (6, 1) and (8, 9)	**33** (−3, 4) and (−1, 10)
34 (5, −3) and (8, −9)	**35** (−2, −4) and (−5, −25)	**36** (1, −7) and (−3, 5)

37 Find the equation of the line that is parallel to the line $y = 4x - 3$ and passes through (3, 2).

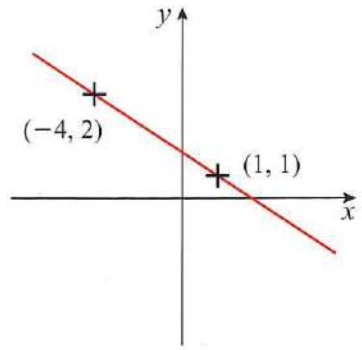
38 Find the equation of the line that is parallel to the line $2x + y = 1$ and passes through (1, −4).

39 Find the equation of each line below:

(a)



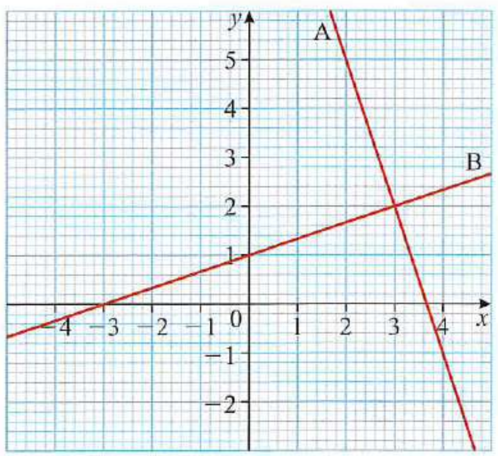
(b)



E Gradients of perpendicular lines

E6.7

1



Line A and Line B above are perpendicular (at right angles).

(a) Find the gradient of line A.

(b) Find the gradient of line B.

(c) *Multiply* together the gradient of line A and the gradient of line B.

Can you still?

Mixed

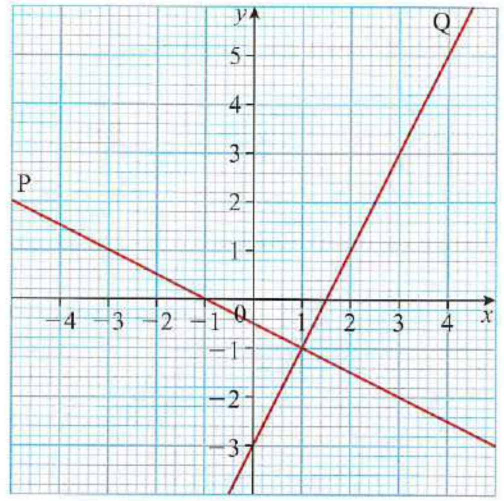
1. Carl and Dan earn the same amount each week. Carl gets a 5% pay rise and now earns £477·75.
   Dan only gets a 3% pay increase. How much does Dan earn now?
2. Simplify $\frac{4}{\sqrt{2}} + \sqrt{18}$
3. P is directly proportional to the square root of Q.
   $P = 12$ when $Q = 9$.
   Find the value of P when $Q = 36$.
4. Expand $(x + 4)^2$
5. Truncate 7·81932 to two decimal places.

2 Line P and line Q are perpendicular (at right angles).

(a) Find the gradient of line P.

(b) Find the gradient of line Q.

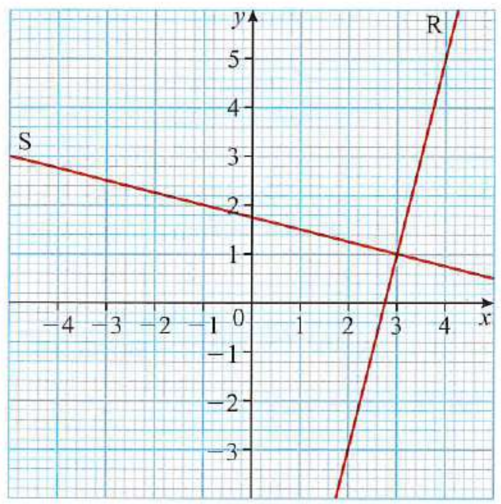
(c) Find the *product* of the gradient of line P and the gradient of line Q.



3 What do you notice about your answers to part (c) in both questions **1** and **2**?

4 If a line has a gradient of 4, what is the gradient of a line perpendicular to this one?

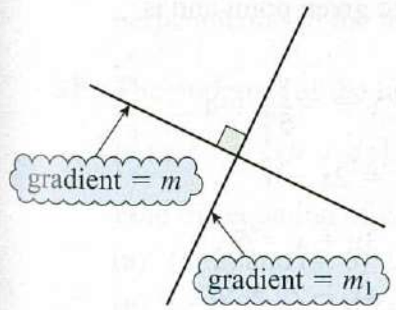
5



Line R has a gradient of 4.
Line S is perpendicular to line R.

Find the gradient of line S to check if your answer to question **4** was correct.

Key Facts



The product of the gradients of two perpendicular lines is −1

$m\,m_1 = -1 \quad \text{so} \quad m = -\frac{1}{m_1}$

Given a line with gradient $= m_1$, to find the gradient of a perpendicular line, find the reciprocal of m_1 $\left(\text{i.e. } \frac{1}{m_1}\right)$ then change its sign $\left(\text{i.e. } -\frac{1}{m_1}\right)$.

Write down the gradient of a line which is perpendicular to a line of gradient (a) -5, (b) $\frac{2}{3}$

(a) $-\frac{1}{m_1} = \frac{-1}{-5} = \frac{1}{5}$ (b) $-\frac{1}{m_1} = \frac{-1}{2/3}\,\frac{(\times 3)}{(\times 3)} = -\frac{3}{2}$ (the simplest way is to turn the fraction upside down and change the sign)

E6.8

1 Find the gradient of the line which is perpendicular to a line with each gradient given below:

(a) 7 (b) 1 (c) -4 (d) -8 (e) $\frac{1}{3}$ (f) $\frac{2}{5}$

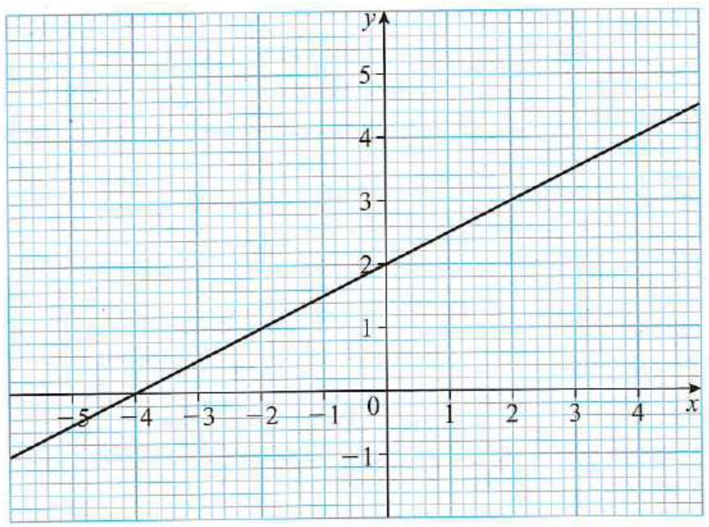
(g) $-\frac{1}{6}$ (h) $-\frac{3}{4}$ (i) $-\frac{9}{2}$ (j) $-0{\cdot}5$ (k) $0{\cdot}2$ (l) 0

2 Write down the gradient of any line which is perpendicular to:

(a) $y = 3x - 2$ (b) $y = -\frac{2}{3}x + 7$ (c) $5x + 8y = 3$

(d) $4y = x + 7$ (e) $6x - 2y = 3$ (f) $3x + 5y - 1 = 0$

3



Find the equation of the line which passes through (2, 1) and is perpendicular to the line shown.

4 Find the equation of the line which passes through the given point and is perpendicular to the given line.

(a) (0, 3) $y = \frac{1}{3}x + 6$ (b) (0, -2) $y = -\frac{1}{5}x + 4$

(c) (1, 1) $y = 8 - \frac{1}{4}x$ (d) (2, 5) $y = 2x - 1$

(e) (1, 4) $2y - x = 3$ (f) (-6, 2) $3y + x = 5$

(g) (-3, -3) $3x + y = 7$ (h) (4, -1) $4x - 2y = 9$

(i) (-1, 6) $x + y - 6 = 0$ (j) (-4, -3) $6x + 3y - 5 = 0$

5 A line passes through (3, 0) and is parallel to the line $y = 5x - 3$. Find the equation of the line.

6 Line P has equation $5y - 2x = 13$.
Line Q has equation $2y + 5x = 7$.
Show that line P is perpendicular to line Q.

7 Without drawing any of these lines, put them into pairs of lines which are perpendicular to one another.

(a) $3y - 2x = 7$ (b) $y = -\frac{1}{3}x + 5$

(c) $2y + x = 9$ (d) $5y + x = 9$

(e) $3x + 2y = 11$ (f) $y = 2x + 1$

(g) $y = x + 3$ (h) $y - 3x = 10$

(i) $y = 5x - 1$ (j) $y = -x + 5$

8 A line passes through (2, 5) and is *parallel* to the line $x + 2y = 1$. Find the equation of the line.

9 Line A has equation $7y = 3x - 4$.
Line B has equation $3y = 5 - 7x$.
Show that line A is perpendicular to line B.

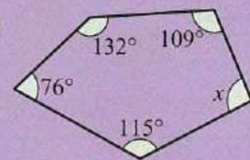
Can you still?

Mixed
Do not use a calculator

1 Draw a Venn diagram to show the prime factors of 42 and 112. Use the Venn diagram to find the HCF and LCM of 42 and 112.

2 Work out $(4 \times 10^{-19}) \times (8 \times 10^{7})$ leaving the answer in standard form.

3



Find the value of angle x.

4 Solve $x^2 - 6x + 8 = 0$

5 Solve $\frac{1}{3}(x - 1) = \frac{1}{6}x + 2$

6 Expand $(x + 2)^3$

10 Find the equation of the line which passes through (3, 2) and is perpendicular to the line which joins (-1, 0) to (3, 2).

11 The midpoint of the line joining (a, b) to (c, d) has co-ordinates given by $\left(\frac{1}{2}(a + c), \frac{1}{2}(b + d)\right)$.

Find the equation of the perpendicular bisector of the line joining:

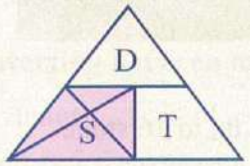
(a) (6, 2) and (4, 6)

(b) (-1, 3) and (4, 2)

(c) (2, 5) and (-4, 3)

(a) A car travels 100 miles in 2 hours 30 minutes. Find the speed in m.p.h.

Time must be in hours only: 2 hours 30 minutes = 2·5 hours.

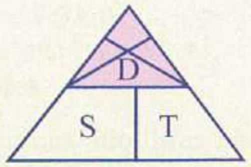


$$S = \frac{D}{T} = \frac{100}{2{\cdot}5} = 40 \text{ m.p.h.}$$

(b) Hazel runs at a steady speed of 8 m/s.

(i) How far does she travel in 4·3 s? (ii) What is her speed in km/h?

(i)



$D = S \times T$

$= 8 \times 4{\cdot}3$

$= 34{\cdot}4$ m

(ii) Speed = 8 m/s = $8 \times 60 \times 60$ metres per hour

$= \frac{8 \times 60 \times 60}{1000}$ km/h

$= 28{\cdot}8$ km/h

M10.3

1 A plane flies 480 km at 320 km/h. How long does the journey take?

2 A hiker walks 28·5 miles at 3 mph. How long does the hiker walk for?

3 Find the speed in mph for each of the following:

Distance	Time	Speed (mph)
30 miles	30 minutes	
9 miles	15 minutes	
15 miles	20 minutes	
6 miles	5 minutes	
30 miles	45 minutes	

4 Terry cycles at 16 mph for 30 minutes then slows down to 12 mph for 15 minutes. How far does he travel in total?

5 John walks at 6 km/h for 1 hour 30 minutes then 4 km/h for 2 hours 15 minutes. How far does he walk in total?

6 Sima drives 50 miles from Leeds to Manchester at an average speed of 40 mph. If she left Leeds at 10:20, when did she arrive at Manchester?

7 The speed of light is 300 000 000 m/s. How long will it take light to travel 6 000 000 km?

8 Convert the following speeds into km/h:

(a) 3 m/s (b) 20 m/s (c) 35 km/min (d) 78 cm/s

9 Convert the following speeds into m/s:

(a) 18 km/h (b) 115·2 km/h (c) 61 200 m/h (d) 0·408 km/min

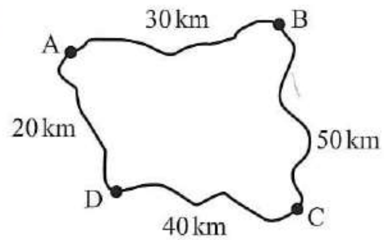
10 Two cyclists, Nerys and Ben, complete a race. Nerys has an average speed of 14·5 km/h and Ben has an average speed of 4 m/s. Who wins the race?

11 A car travels 80 km at an average speed of 50 km/h then travels 64 km at an average speed of 80 km/h. Find the average speed for the whole journey.

12 A plane travels 920 km at an average speed of 800 km/h. It then increases its speed by 50% and travels another 1020 km. Find the average speed for the whole journey.

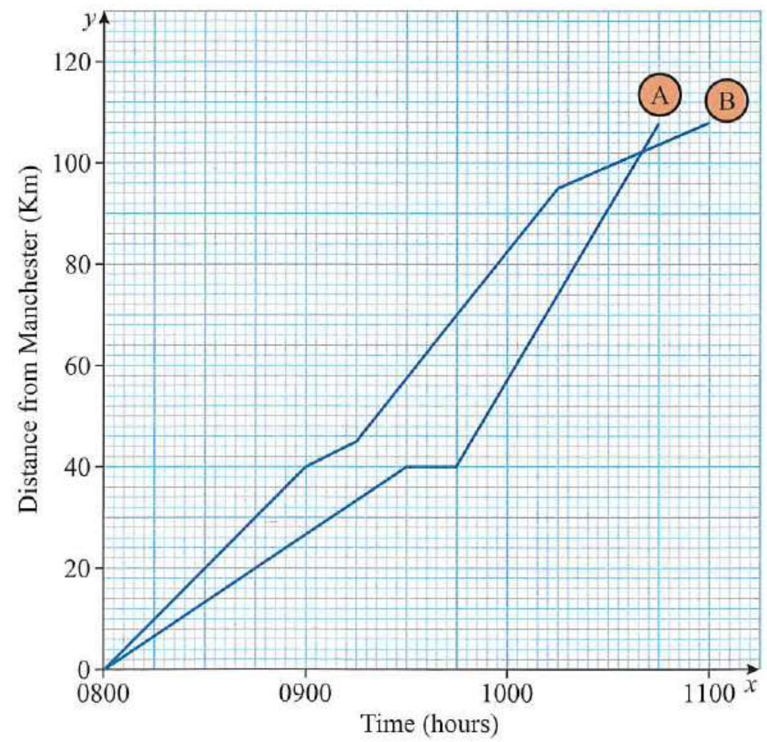
13 In a marathon race, Candice is 40 m behind Jess. Candice is running at 0·7 m/s but Jess is only running at 0.5 m/s. How long will it take Candice to catch up Jess?

14



Brooke travels from A to B at a steady speed of 45 km/h, from B to C at 60 km/h, from C to D at 36 km/h and from D to A at 60 km/h. Find Brooke's average speed for the whole journey.

15 The graph opposite shows the journeys of 2 cars from Manchester.



(a) How far from Manchester is car B at 10:30?

(b) At what time is car B 30 km from Manchester?

(c) Find the speed of car B in km/h between 09:00 and 09:15.

(d) Find the speed of car A in km/h between 09:45 and 10:45.

(e) Find the average speed of car B in m/s for the entire journey.

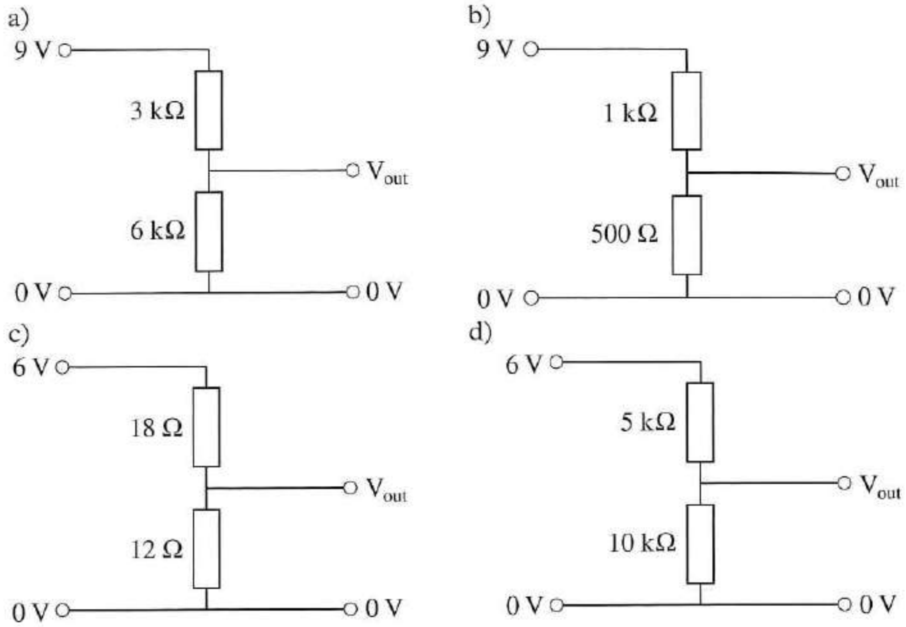
(f) Both cars travelled 108 km. Which car was the faster overall?

(g) Describe the journey of both cars.

KEY IDEA YOUTUBE VIDEO LINKS
Force and Motion Newtons laws: https://www.youtube.com/watch?v=NYVMlmL0BPQ **Newtons first law:** **https://www.youtube.com/watch?v=LEHR8YQNm_Q** **Newton's second law:** **https://www.youtube.com/watch?v=ZvPrn3aBQG8** **Newton's third law:** https://www.youtube.com/watch?v=MUgFT1hRTE4 Simulations of motion: http://phet.colorado.edu/sims/html/forces-and-motion-basics/latest/forces-and-motion-basics_en.html Distance-time graphs: https://www.youtube.com/watch?v=9LQdLDDEJ1g Calculating speed from d-t graphs: https://www.youtube.com/watch?v=JZD3WlqtRyo speed-time graphs and acceleration https://www.youtube.com/watch?v=3x22CpSB7zM worked examples of calculating acceleration https://www.youtube.com/watch?v=e6IOZb-03Zw
Momentum and energy Momentum **https://www.youtube.com/watch?v=wsVLU9A2TJM** **Gravitational potential energy and Kinetic Energy** **https://www.youtube.com/watch?v=MYwqb8m0jkM** Energy transfer diagrams/Sankey diagrams https://www.youtube.com/watch?v=NC8ItrcR2Ak Generating Electricity https://www.youtube.com/watch?v=hx_917HJiAI The National Grid https://www.youtube.com/watch?v=-1SLFzqLU5k Energy Resources **https://www.youtube.com/watch?v=SeXG8K5_UvU coal power 2min 12** https://www.youtube.com/watch?v=yGsPc3fptoY oil 4min18 **https://www.youtube.com/watch?v=f3S5UyBpymU gas 2min 03** **https://www.youtube.com/watch?v=_UwexvaCMWA nuclear 4min47** **https://www.youtube.com/watch?v=C1SDAgLn-tk biofuel 4min 56** https://www.youtube.com/watch?v=McByJeX2evM **electric mountain, wales,, 7min51 HEP** **https://www.youtube.com/watch?v=D67NnRjgJa0 solar power 6min32** **https://www.youtube.com/watch?v=D-OVU2RGNDo tidal power 6min03**
Waves https://www.youtube.com/watch?v=uWVS2aWy7KU
Light and em spectrum The E-M spectrum song https://www.youtube.com/watch?v=kOkv8ynpppk Tour of the E-M Spectrum https://www.youtube.com/watch?v=HPcAWNlVl-8 Refraction https://www.youtube.com/watch?v=7aU8sX8cFNs

Radioactivity https://www.youtube.com/watch?v=KYDil96NR5Q Uses of radioactivity https://www.youtube.com/watch?v=w0_VZuC4nfc Biological effects of radiation https://www.youtube.com/watch?v=EuKzI3g5ra4
Astronomy The Solar System https://www.youtube.com/watch?v=libKVRa01L8 Gravity and orbits https://www.youtube.com/watch?v=WUYWDjcFDro Life cycle of stars https://www.youtube.com/watch?v=NdmaprNK8yc Big Bang and red shift https://www.youtube.com/watch?v=KANnNIAIGAQ Evidence for the Big Bang https://www.youtube.com/watch?v=9f_i87aHKoo
Energy – forces doing work Work done and power https://www.youtube.com/watch?v=OyGG1Lm6L2Y Hooke's Law https://www.youtube.com/watch?v=zJs27xNdKOM
Electricity https://www.youtube.com/watch?v=3YSH-RRoNWI Series and parallel circuits https://www.youtube.com/watch?v=zx2yqSQ-sWs Electrical Power https://www.youtube.com/watch?v=3UY2SFelMaI Plugs and Fuses https://www.youtube.com/watch?v=0OKTejgaWTY
Static electricity https://www.youtube.com/watch?v=V1c61Q7qU-s Dangers https://www.youtube.com/watch?v=FzsTamPPnHc
Magnetism and the motor effect, electromagnetism https://www.youtube.com/watch?v=dSNB-PsC2Vw
Particle model, forces and matter Specific heat capacity https://www.youtube.com/watch?v=6PXTHsD24Tw Specific latent heat https://www.youtube.com/watch?v=SzNAoyIGUeA Pressure https://www.youtube.com/watch?v=zlLpKzPz84Q Gas pressure https://www.youtube.com/watch?v=zvh9uv2Hxx4 Density https://www.youtube.com/watch?v=SimFy9wOMXY

3 For each of the following circuits, calculate the value of V_{out}.



4 A capacitor is discharged through a resistor. How can the discharge time be increased?

5 Outline the reasons why some people feel that privacy is no longer possible.

6 What types of job can a robot do well and what types of job does a human worker do better?

7 Some people think that mobile phones are dangerous. Why is this?

Examination questions

1 A student did an experiment with two strips of polythene. She held the strips together at one end. She rubbed down one strip with a dry cloth. Then she rubbed down the other strip with the dry cloth. Still holding the top ends together, she held up the strips.

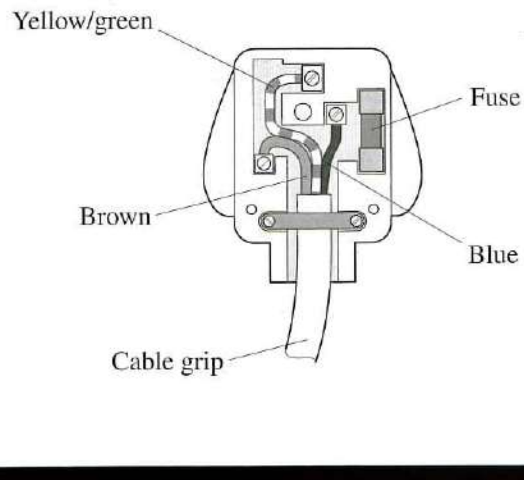
a) i) What movement would you expect to see? *(1 mark)*

ii) Why do the strips move in this way? *(2 marks)*

b) Copy and complete the **four** spaces in the passage.

Each strip has a negative charge. The cloth is left with a __________ charge. This is because particles called __________ have been transferred from the __________ to the __________ . *(4 marks)*

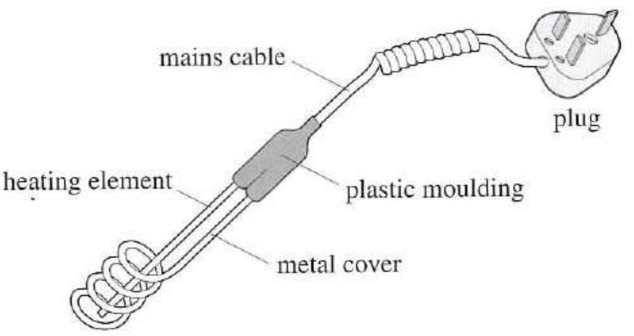
2 a) The diagram shows a 13 amp plug.



i) What is wrong with the way this plug has been wired? *(1 mark)*

ii) Why do plugs have a fuse? *(1 mark)*

b) The diagram shows an immersion heater which can be used to boil water in a mug.



i) Which part of the immersion heater should be connected to the earth pin of the plug? *(1 mark)*

ii) Complete the sentence by choosing the correct words from the box. Each word may be used once or not at all.

chemical	electrical	heat	light

When the immersion heater is switched on ________ energy is transferred to ________ energy. *(2 marks)*

3 a) Look at this table of results.

VOLTAGE (V)	0.0	3.0	5.0	7.0	9.0	11.0
CURRENT (A)	0.0	1.0	1.4	1.7	1.9	2.1

i) Plot a graph of current agains voltage. Place current, in amps, on the vertical axis and voltage, in volts, on the horizontal axis. *(3 marks)*

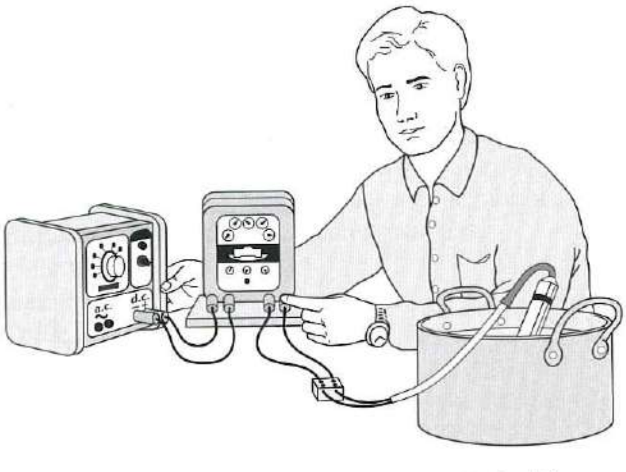
ii) Use your graph to find the current when the voltage is 10 V. *(1 mark)*

iii) Use your answer to (ii) to calculate the resistance of the lamp when the voltage is 10 V. *(2 marks)*

b) i) What happens to the resistance of the lamp as the current through it increases?

ii) Explain you answer. *(2 marks)*

4 The drawing shows an experiment using a low voltage supply, a joulemeter, a small immersion heater and a container filled with water.



The potential difference was set at 6 V d.c. The reading on the joulemeter at the start of the experiment was 78 882 and 5 minutes later it was 80 142.

a) Use the equation:

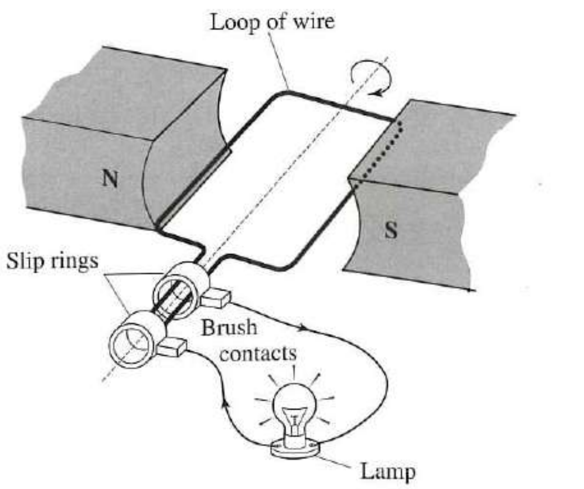
$$\text{potential difference} = \frac{\text{energy transferred}}{\text{charge}}$$

to work out the total charge which flowed through the immersion heater in five minutes. Clearly show how you get to your answer and give the unit. *(3 marks)*

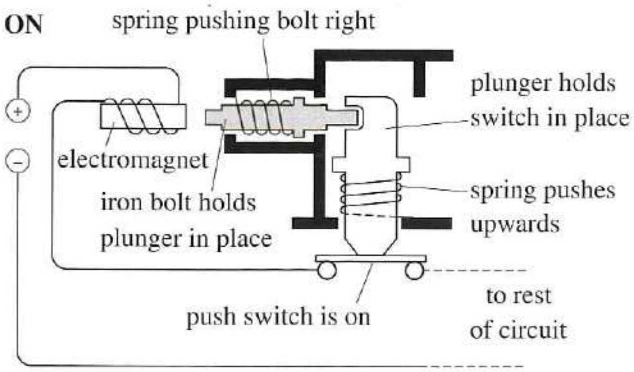
b) Calculate the current through the immersion heater during the 5 minutes. Write the equation you are going to use, show clearly how you get to your answer and give the unit. *(3 marks)*

5 The diagram shows a simple electricity generator. Rotating the loop of wire causes a current which lights the lamp.

State **three** ways to increase the current produced by the generator. *(3 marks)*

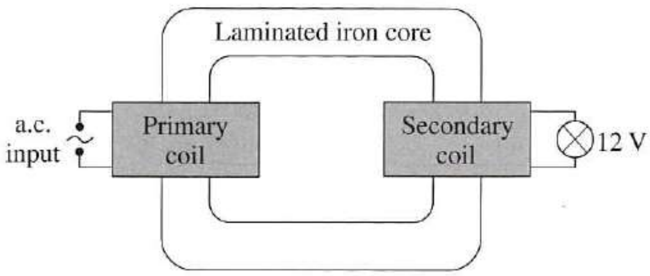


6 A fault in an electrical circuit can cause too great a current to flow. Some circuits are switched off by a circuit breaker.

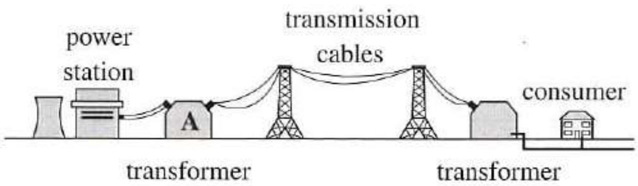


One type of circuit breaker is shown above. A normal current is flowing.
Explain, in full detail, what happens when a current which is bigger than normal flows.
(4 marks)

7 a) The diagram represents a simple transformer used to light a 12 V lamp. When the power supply is switched on the lamp is very dim.



Give **one** way to increase the voltage at the lamp with without changing the power supply.
(1 mark)



b) Electrical energy is distributed around the country by a network of high voltage cables.

i) For the system to work the power is generated and distributed using alternating current rather than direct current. Why? *(1 mark)*

ii) Transformers are an essential part of the distribution system. Explain why. *(2 marks)*

iii) The transmission cables are suspended high above the ground. Why? *(1 mark)*

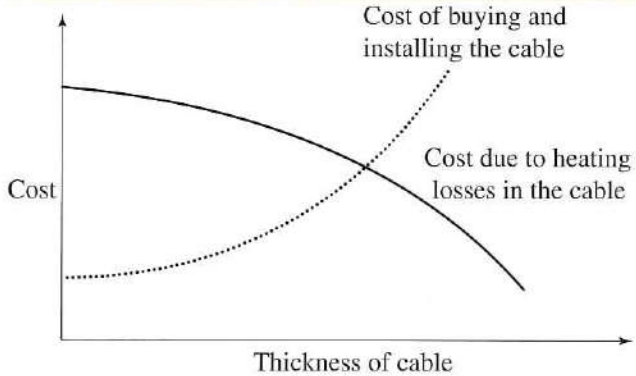
c) The power station generates 100 MW of power at a voltage of 25 kV. Transformer **A**, which links the power station to the transmission cables, has 44 000 turns in its 275 kV secondary coil.

i) Write down the equation which links the number of turns in each transformer coil to the voltage across each transformer coil. *(1 mark)*

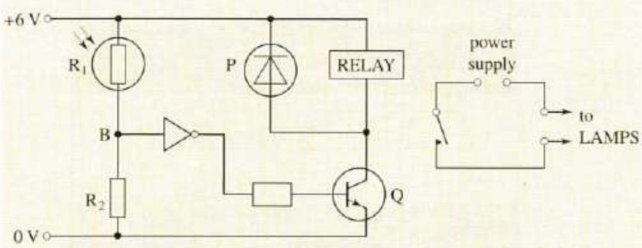
ii) Calculate the number of turns in the primary coil of transformer **A**. Show clearly how you work out your answer. *(2 marks)*

d) The diagram shows how the cost of transmitting the electricity along the cables depends upon the thickness of the cable.

Why does the cost due to the heating losses go down as the cable is made thicker? *(1 mark)*



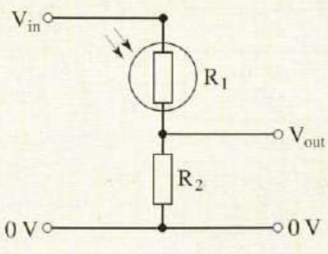
8 The diagram below shows a circuit which can be used as an automatic switch.



a) Name the following components: P, Q, R_1. *(3 marks)*

Use the following information for parts b) and c).

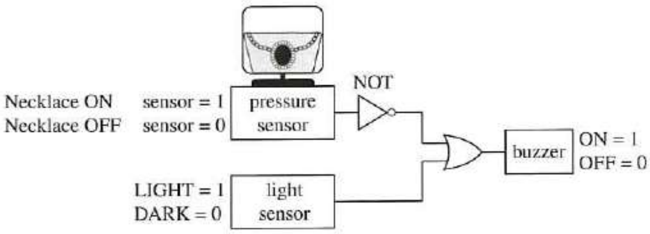
$$V_{out} = V_{in} \times \frac{(R_2)}{(R_1 + R_2)}$$



b) The resistance of $\mathbf{R_2}$ = 2000Ω.
V_{in} is 6V.
  i) In daylight the resistance of $\mathbf{R_1}$ = 500Ω. Calculate the voltage across $\mathbf{R_2}$.
  ii) In daylight the lamps will be OFF. Explain why. *(6 marks)*

c) In the dark the resistance of $\mathbf{R_1}$ is 198000Ω. Calculate the voltage across $\mathbf{R_2}$. *(2 marks)*

9 a) The diagram shows part of a simple alarm system used to protect a valuable necklace.



  i) Copy and complete the truth table for the NOT gate.

Input	Output
1	
0	

(1 mark)

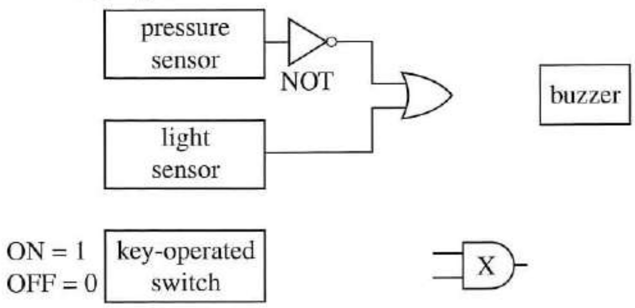
  ii) Copy and complete the truth table for the alarm system.

Pressure sensor	Light sensor	Buzzer
0	0	
0	1	
1	0	
1	1	

(2 marks)

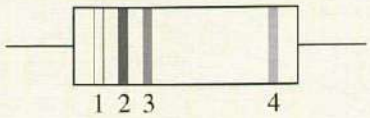
  iii) Explain how this alarm system would work. *(2 marks)*

b) The alarm needs to be able to be switched on and off. To do this a key-operated switch and a logic gate **X** are added to the circuit.



  i) What type of logic gate is **X**? *(1 mark)*
  ii) Copy and complete the circuit above to show how the key-operated switch and logic gate **X** should be connected into the alarm system. *(2 ma*

10 a) The diagram shows the arrangement of th colour coded bands on a typical resistor.

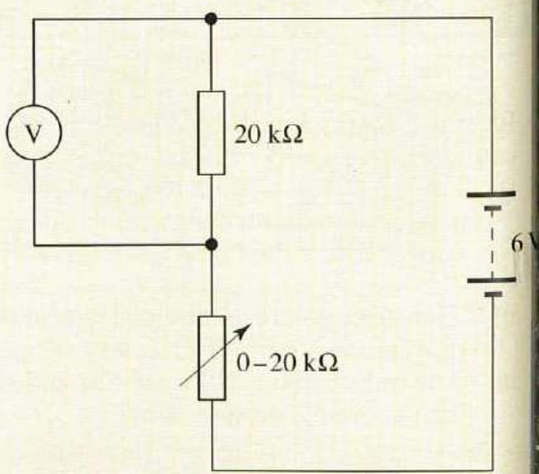


The colour code is given in the table belo

Figure	Colour
0	black
1	brown
2	red
3	orange
4	yellow
5	green
6	blue
7	violet
8	grey
9	white

  i) What are the colours of the first **thre** bands of a 20 kΩ resistor? *(2 ma*
  ii) What information is given by the **fourth** band? *(1 m*

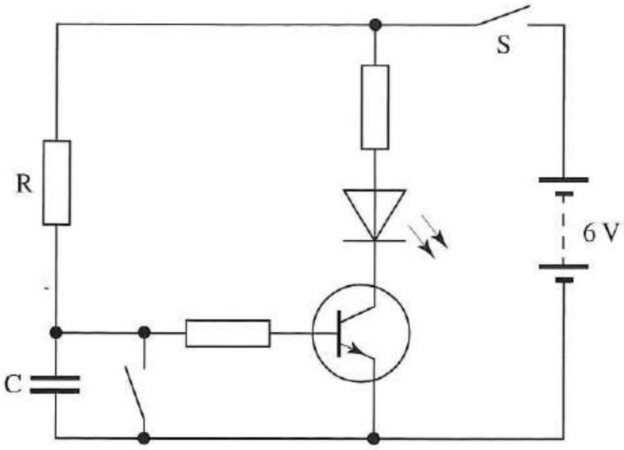
b) The diagram shows two resistors joined in series. The variable resistor can have any value between 0 and 20 kΩ.



  i) What is the smallest possible reading the voltmeter? *(1 m*

ii) What is the largest possible reading on the voltmeter? *(1 mark)*

c) The diagram shows one design for a time-delay circuit.



i) What is the function of a capacitor? *(1 mark)*

ii) When the switch **S** is closed, it is several minutes before the light emitting diode (LED) comes on. Explain why. The explanation has been started for you.

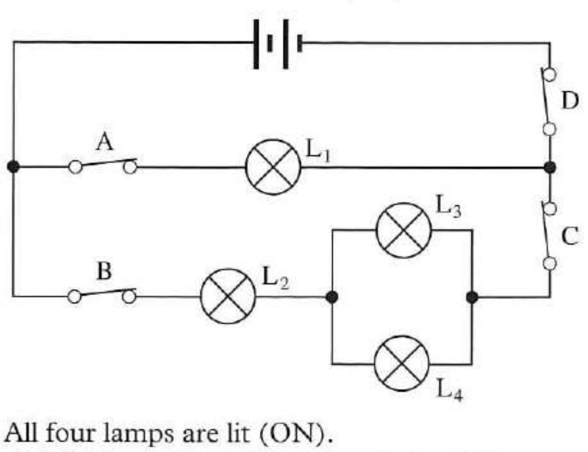
*When the switch **S** is closed, the voltage across the capacitor . . .* *(2 marks)*

iii) Give **one** practical use for this circuit. *(1 mark)*

iv) A pupil wires up the circuit. By mistake the positions of capactior **C** and the resistor **R** are swapped. Describe what will happen after the switch **S** is closed. *(2 marks)*

11 In the circuit shown below all four lamps are identical.

All four switches are closed (ON).



All four lamps are lit (ON).

a) Which **single** switch, **A** to **D**, should be opened in order to

i) turn OFF **all four** lamps?

ii) turn OFF one lamp only? *(2 marks)*

b) When **all four** switches are closed (ON), state which lamp L_1 to L_4 will be the brightest. Give a reason for your answer. *(2 marks)*

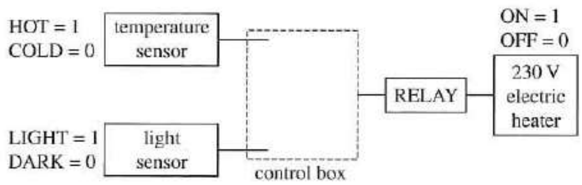
c) Lamps are sometimes used in electronic systems as output devices. Other devices are used as input sensors.

Below there is a list of output devices and input sensors.

Identify the **three** input sensors.

buzzer **heater** **LDR** **motor**
switch **thermistor** *(3 marks)*

12 a) The diagram shows part of a heating system. It is designed to switch on automatically when it is both cold and dark. The control box contains two logic gates which are not shown.



i) What is the name and circuit symbol for an input sensor which responds to light? *(2 marks)*

ii) What is the name and circuit symbol for an input sensor which responds to temperature? *(2 marks)*

iii) Copy and complete the truth table for the control system.

Light sensor	Temperature sensor	Heater
1	1	
1	0	
0	1	
0	0	

(2 marks)

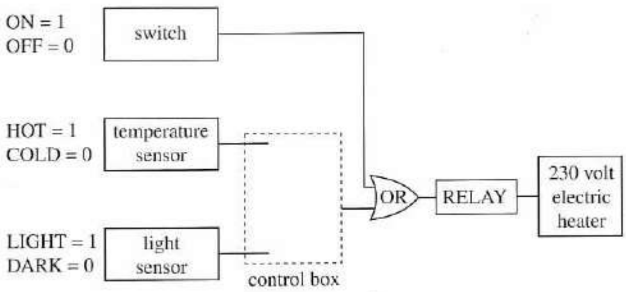
iv) Identify the names of the **two** logic gates that should be used inside the control box, from the list below.

AND NOT OR *(1 mark)*

v) Copy and complete the diagram in part a) to show how the two logic gates are used to connect the input sensors to the relay. Use the correct symbols for the logic gates. *(3 marks)*

vi) Why must a relay be used to operate the heater? *(1 mark)*

b) The diagram shows an additional logic gate and switch added to the system.



Explain how this change allows the heater to be switched on at any time. The explanation has been stared for you.

Closing the switch sends . . . *(2 marks)*

4 Under normal conditions the maximum speed that a lorry can go around a bend without skidding is 60 km/hr. Would the lorry be able to go around the bend faster, at the same speed, or would it need to slow down, if the road was muddy? Explain the reason for your answer.

Examination questions

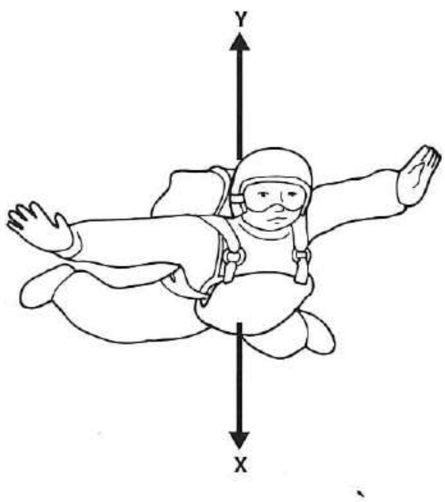
1 a) Two sky-divers jump from a plane. Each holds a different position in the air.

Copy and complete the following sentence.
Sky-diver __________ will fall faster because __________ *(2 marks)*

A

B

b) The diagram shows the direction of the forces acting on one of the sky-divers.



Copy the following sentences and complete them by choosing the correct endings from the boxes.

i) Force X is caused by [air resistance / friction / gravity]. *(1 mark)*

ii) Force Y is caused by [air resistance / gravity / weight]. *(1 mark)*

iii) When force X is bigger than force Y, the speed of the sky-diver will [go up / stay the same / go down]. *(1 mark)*

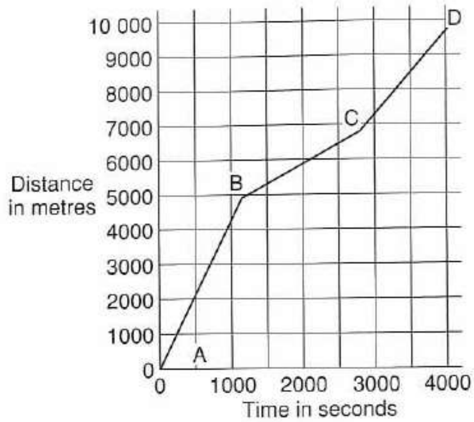
iv) After the parachute opens, force X [goes up / stays the same / go down]. *(1 mark)*

c) How does the area of an opened parachute affect the size of force Y? *(1 mark)*

2 Two students Anna and Graham took part in a sponsored run. The distance–time graph for Graham's run is shown. Four points have been labelled A, B, C and D.

a) Between which pair of points was Graham running the slowest? *(1 mark)*

b) Anna did not start the run until 10 minutes after Graham. She completed the whole run at a constant speed of 4 m/s.



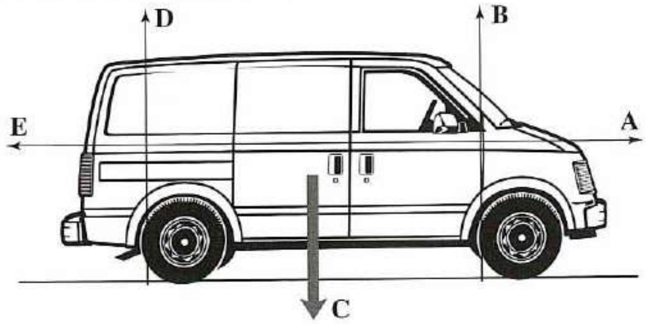
i) Write down the equation that links distance, speed and time. *(1 mark)*

ii) Calculate, in seconds, how long it took Anna to complete the run. Show clearly how you work out your answer. *(2 marks)*

iii) Copy the graph and draw a line to show Anna's run. *(2 marks)*

iv) How far had Graham run when he was overtaken by Anna? *(1 mark)*

3 Five forces, **A**, **B**, **C**, **D** and **E** act on the van.



a) Copy and complete the following sentences by choosing the correct forces from **A** to **E**.
Force __________ is the forward force from the engine.
Force __________ is the force resisting the van's motion. *(1 mark)*

b) The size of forces **A** and **E** can change.
Copy and complete the table to show how big force **A** is compared to force **E** for each motion of the van.
Do this by placing a tick in the correct box.
The first one has been done for you.

Motion of van	Force A smaller than force E	Force A equal to force E	Force A bigger than force E
Not moving		✓	
Speeding up			
Constant speed			
Slowing down			

(3 marks)

c) When is force **E** zero? *(1 mark)*

d) The van has a fault and leaks one drop of oil every second.
The diagram below shows the oil drops left on the road as the van moves from **W** to **Z**.

Describe the motion of the van as it moves from **W** to **X**, **X** to **Y** and **Y** to **Z.** *(3 marks)*

e) The driver and passengers wear seatbelts. Seatbelts reduce the risk of injury if the van stops suddenly.

backwards downwards force forwards mass weight

Copy and complete the following sentences, using words from the list above, to explain why the risk of injury is reduced if the van stops suddenly.

A large __________ is needed to stop the van suddenly.
The driver and passengers would continue to move __________ .
The seatbelts supply a __________ force to keep the driver and passengers in their seats. *(3 marks)*

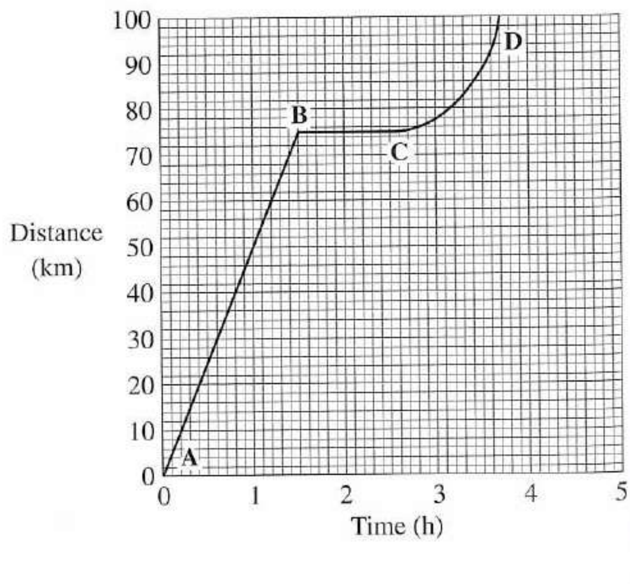
f) The van was travelling at 30 m/s. It slowed to a stop in 12 seconds. Calculate the van's acceleration. *(3 marks)*

4 The graph shows three stages of a van's journey.

a) During which stage of the journey **A–B**, **B–C** or **C–D**:

i) is the van stationary? *(1 mark)*

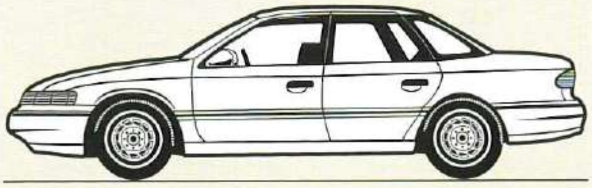
ii) is the van moving at a constant speed? *(1 mark)*



b) Calculate the gradient of the graph from **A** to **B**. *(2 marks)*

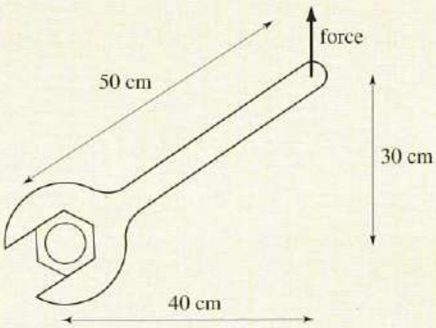
c) What does this gradient measure? *(1 mark)*

5 The diagram shows a parked car.



When the car is driven away, its engine gives a constant forward force.
The speed increases quickly at first, then more slowly. After a time the car reaches a constant speed.
Explain why the motion of the car changes in this way. *(3 marks)*

6 The diagram shows a spanner being used to undo a tight nut.



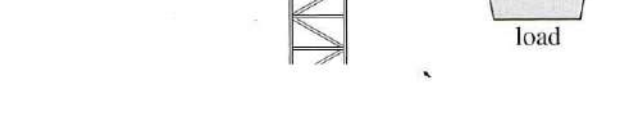
The nut was tightened using a moment of 120 newton metres.

Use the following equation to calculate the force needed to undo the nut. Show clearly how you work out your answer.

moment = force × perpendicular distance from pivot

(2 marks)

7 The diagram shows a tower crane.



a) Explain why the crane would be unstable without the counterbalance. *(2 marks)*

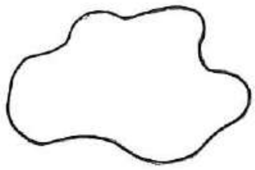
b) The counterbalance can be moved to the left or right, as shown by the arrows on the diagram. Explain the advantage of having a movable counterbalance. *(2 marks)*

c) The load shown in the diagram is 75 000N. The load is 6 m from the tower. Calculate the turning effect (moment) of the load in newton metres. *(2 marks)*

d) The crane is balanced and horizontal. What is the turning effect (moment) of the counterbalance in newton metres? Explain your answer.

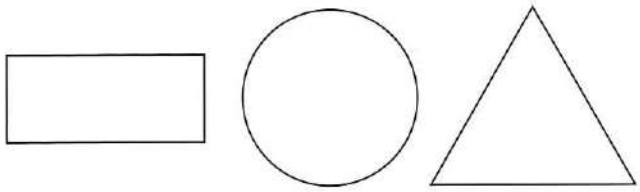
(3 marks)

8 a) A thin sheet of cardboard is cut to the shape below. Describe, with a diagram, an experiment to find its centre of mass.



(5 marks)

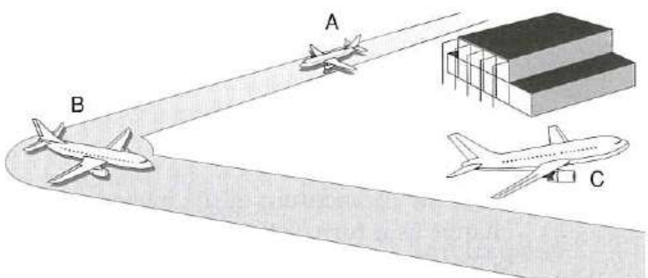
b) Copy and label with an **X** the centre of mass of each of the three objects below.



(3 marks)

c) Explain why a mechanic would choose a long spanner to undo a tight nut. *(2 marks)*

9 a) The diagram shows three aeroplanes at an airport.

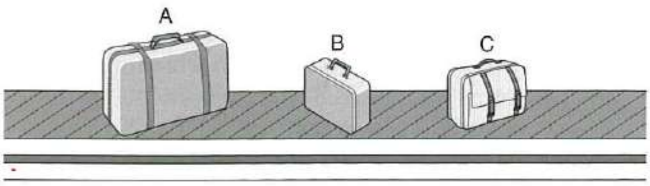


Aeroplane **A** is moving at constant velocity towards the main runway.
Aeroplane **B** is stationary, waiting to take off.
Aeroplane **C** has just taken off and is accelerating.

i) Which, if any, of the aeroplanes has zero momentum? *(1 mark)*

ii) The momentum of **one** of the aeroplanes is changing. Which one? Give a reason for your answer. *(2 marks)*

10 The picture shows luggage which has been loaded onto a converyor belt.



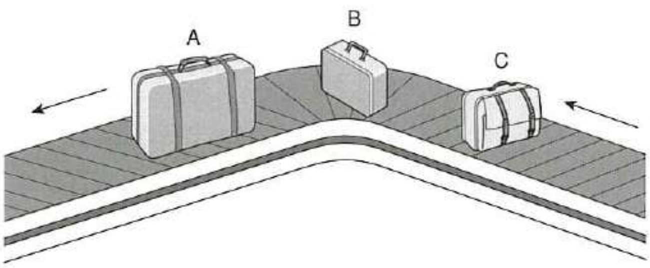
Each piece of luggage has a different mass.

mass of **A** = 22 kg
mass of **B** = 12 kg
mass of **C** = 15 kg

a) i) What is the momentum of the luggage before the conveyor belt starts to move? Give a reason for your answer. *(2 marks)*

ii) When the conveyor belt is switched on the luggage moves with a constant speed. Which piece of luggage **A**, **B** or **C** has the most momentum? Give a reason for your answer. *(2 marks)*

iii) At one point the conveyor belt turns left. The luggage on the belt continues to move at a constant speed.

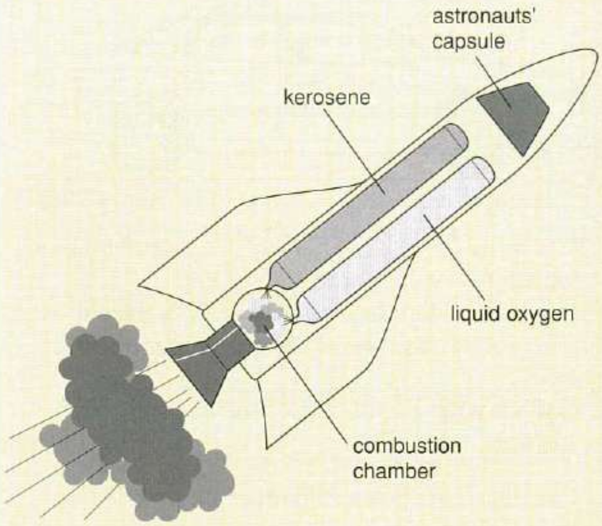


Does the momentum of the luggage change as it turns left with the conveyor belt? Give a reason for your answer. *(2 marks)*

b) Which of the following units can be used to measure momentum?

J/s   kg m/s   Nm   *(1 mark)*

11 a) The diagram shows a simple design for a space rocket.

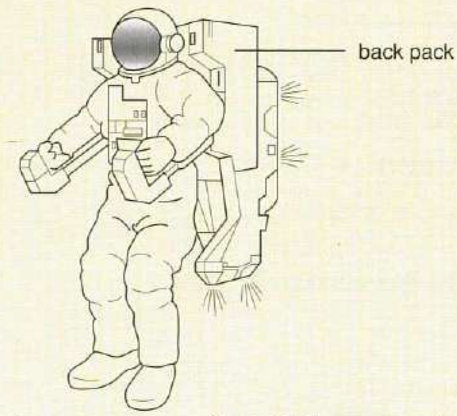


i) Explain, using the idea of momentum, how the initial propulsion of a rocket is produced. *(3 marks)*

ii) State and explain **one** way the acceleration of a rocket can be increased. *(2 marks)*

iii) In what unit is momentum measured? *(1 mark)*

b) The diagram shows an astronaut working in space. Releasing compressed gas from the back pack allows the astronaut to move around.

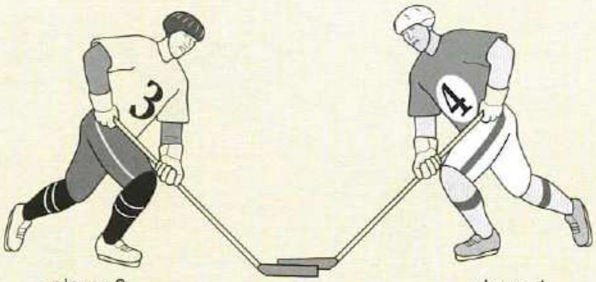


During one spacewalk, 0.5 kilograms of gas was released in 2 seconds. The gas had a speed of 60 metres per second. Use the following equation to calculate the force, in newtons, exerted on the astronaut by the gas. (Ignore the change in mass of the back pack).

$$\text{force} = \frac{\text{change in momentum}}{\text{time}}$$

(2 marks)

12 a) The picture shows two ice hockey players skating towards the puck. The players, travelling in opposite directions, collide, fall over and stop.



i) Use the following equation and the data given in the box to calculate the momentum of player number **3** before the collision. Show clearly how you work out your answer and give the unit.

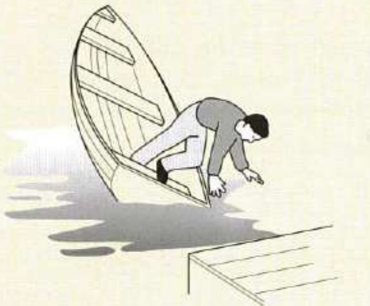
momentum = mass × velocity

(3 marks)

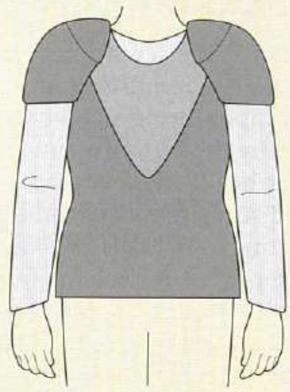
ii) What is the momentum of player **4** just before the collision? *(1 mark)*

iii) The collision between the two players is **not** *elastic*. What is meant by an *elastic* collision? *(1 mark)*

b) The pictures show what happened when someone tried to jump from a stationary rowing boat to a jetty. Use the idea of momentum to explain why this happened. *(2 marks)*

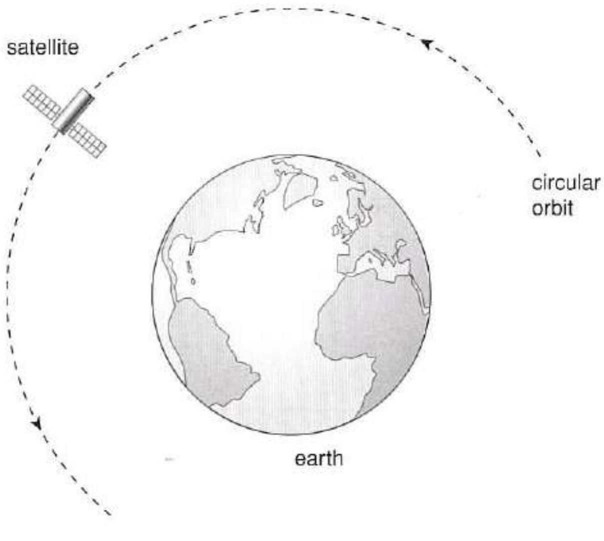


c) The diagram shows one type of padded body protector which may be worn by a horse rider.



If the rider falls off the horse, the body protector reduces the chance of the rider being injured. Use the idea of momentum to explain why. *(3 marks)*

13 The diagram shows a satellite in orbit around the Earth.



a) Copy and complete the diagram, drawing an arrow on the diagram to show the direction of the centripetal force which acts on the satellite. *(1 mark)*

b) Use words from the following list to complete the sentences.

greater **less** **unchanged**

i) If the mass of the satellite decreases then the centripetal force needed is ——— .

ii) If the speed of the satellite increases then the centripetal force needed is ——— .

iii) If the radius of the orbit increases then the centripetal force needed is ——— .

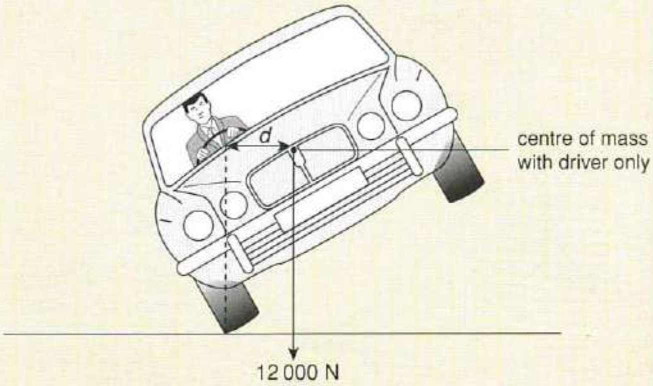
(3 marks)

14 The following paragraphs appeared in a newspaper.

> **JEEP FAILS GOVERNMENT TEST**
>
> A car manufacturer confirmed yesterday that one of its four-wheel drive mini-jeeps rolled over at 38 mph during stability tests conducted by the Government.
> Testing has been halted until safety cages can be fitted to seven makes of mini-jeeps which the Department of Transport has agreed to test after repeated complaints from the Consumer's Association. The Association claims its own tests show that the narrow track, short-wheelbase vehicles are prone to rolling over. The Government's tests highlight how passengers raise the centre of mass. All seven vehicles passed the test unladen, although one raised two wheels.

a) Write down **two** factors mentioned in the newspaper article which affect the stability of vehicles. *(2 marks)*

b) The diagram shows a tilted vehicle.



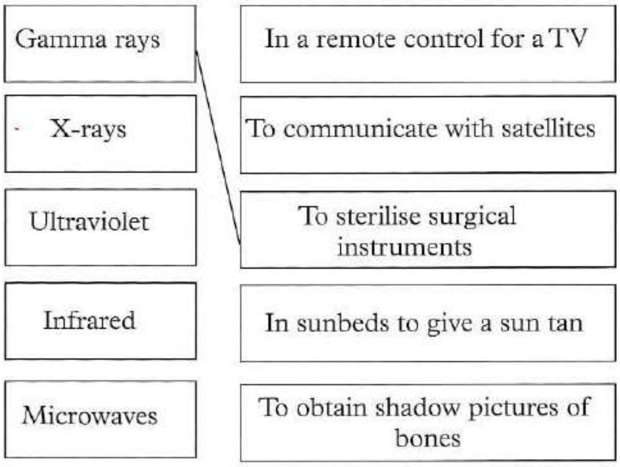
The distance ***d*** shown in the diagram is 50 cm. Calculate the moment of the force about the point of contact with the road. *(3 marks)*

c) Explain how passengers make the vehicles more likely to roll over (less stable). You may use diagrams if you wish. *(4 marks)*

Examination questions

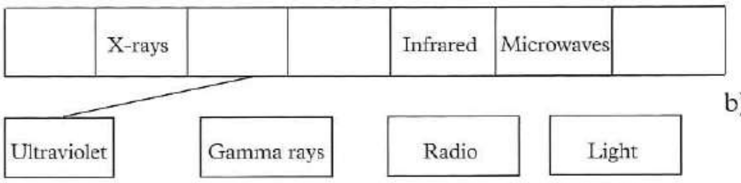
1 The boxes on the left show some types of electomagnetic radiation. The boxes on the right show some uses of electromagnetic radiation.

Copy the diagram and draw a straight line from each type of radiation to its use. The first one has been done for you.



(3 marks)

2 a) The diagram represents the electromagnetic spectrum. Four of the waves have not been named. Copy the diagram and draw lines to join each of the waves to its correct position in the electromagnetic spectrum. One has been done for you.



(2 marks)

b) Copy and complete the following sentence by choosing the correct answer from the three lines in the box.

The speed of radio waves through a vacuum is

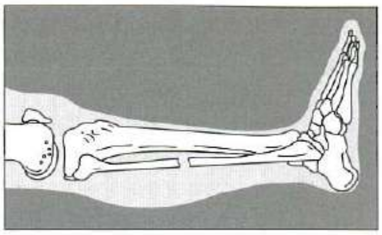
faster than the same as slower than	the speed of light through a vacuum.

(1 mark)

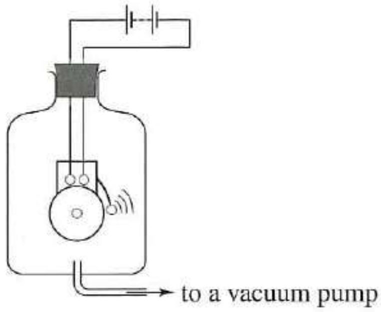
c) i) Before sunbathing it's a good idea to apply a sun cream to your exposed skin. Why? *(1 mark)*

ii) From which type of electromagnetic wave is sun cream designed to protect the skin? *(1 mark)*

d) The diagram shows an X-ray photograph of a broken leg.
Bones show up white on the photographic film.
Explain why. *(2 marks)*



3 a) The diagram shows an electric bell inside a glass jar. The bell can be heard ringing.



Copy and complete the following sentences, by choosing the correct line in each box.

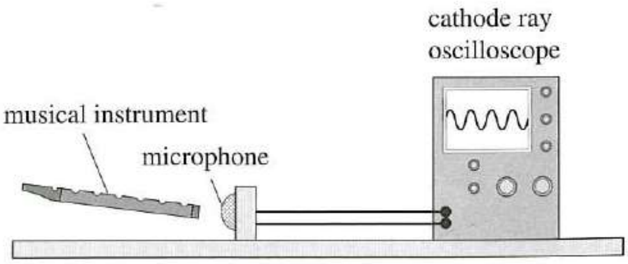
When all the air has been taken out of the glass jar,

the ringing sound will	stop. get louder. get quieter.

This is because sound	travels faster travels slower cannot travel	through

a vacuum. *(2 marks)*

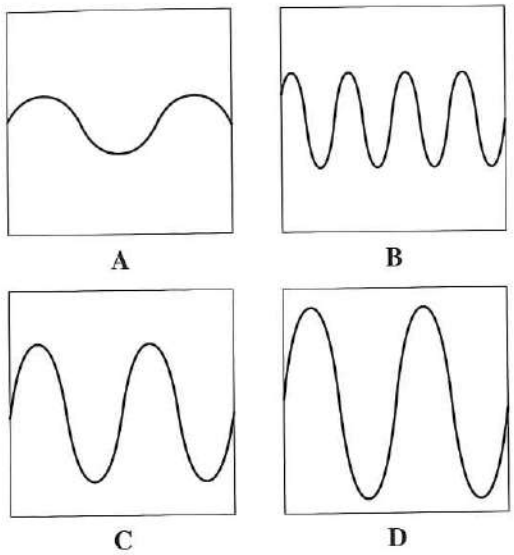
b) The microphone and cathode ray oscilloscope are used to show the sound wave pattern of a musical instrument.



One of the following statements describes what a microphone does. Identify the correct statement. *(1 mark)*

- A microphone transfers sound energy to light energy.
- A microphone transfers sound energy to electrical energy.
- A microphone transfers electrical energy to sound energy.

c) Four different sound wave patterns are shown. They are all drawn to the same scale.



i) Which sound wave pattern has the highest pitch? Give a reason for your answer. *(2 marks)*

ii) Which sound wave pattern is the loudest? Give a reason for your answer. *(2 marks)*

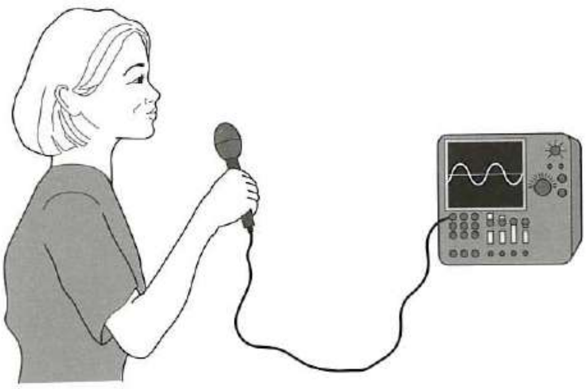
d) i) The frequency of some sounds is too high for humans to hear. Which of the following words describes this sound.

microwave **ultrasound** **ultraviolet**

(1 mark)

ii) Give **one** use for this type of sound wave. *(1 mark)*

4 a) The student is using a microphone connected to a cathode ray oscilloscope (CRO).



The CRO displays the sound waves as waves on its screen. What does the microphone do? *(2 marks)*

b) The amplitude, the frequency and the wavelength of a sound wave can each be either increased or decreased.

i) What change, or changes, would make the sound quieter? *(1 mark)*

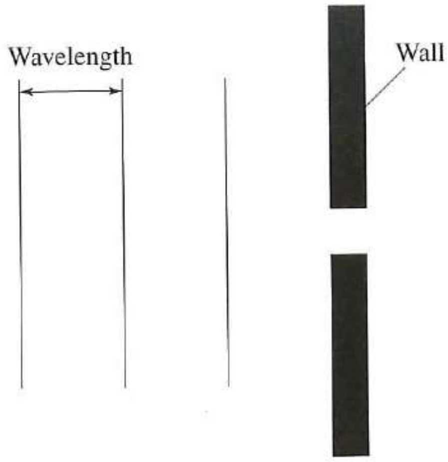
ii) What change, or changes, would make the sound higher in pitch? *(1 mark)*

c) People can generally hear sounds in the frequency range 20 Hz to 20 000 Hz.

i) What are very high frequency, and inaudible, sounds with frequencies **greater** than 20 000 Hz called? *(1 mark)*

ii) Give **two** uses for very high frequency sounds. *(2 marks)*

d) The diagram shows sound waves approaching a gap in a wall.

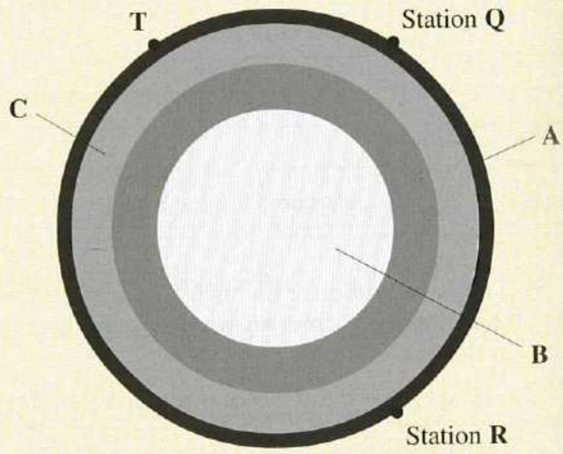


i) Copy and complete the diagram to show what will happen to the sound waves on the other side of the wall. *(2 marks)*

ii) What is the name of this effect? *(1 mark)*

iii) What would the width of the gap need to be for this effect to be most pronounced? *(1 mark)*

5 The diagram represents the structure of the Earth.



a) On the diagram, name the parts **A**, **B** and **C**.

b) An earthquake occurs at the point **T** on the Earth's surface. Two types of shock wave are produced by the earthquake, P waves and S waves.

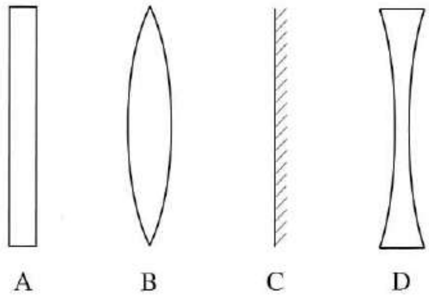
Describe **two** similarities and **two** differences between P waves and S waves as they travel through the Earth. *(4 marks)*

c) State whether P waves or S waves or both will reach:

i) Station Q *(1 mark)*

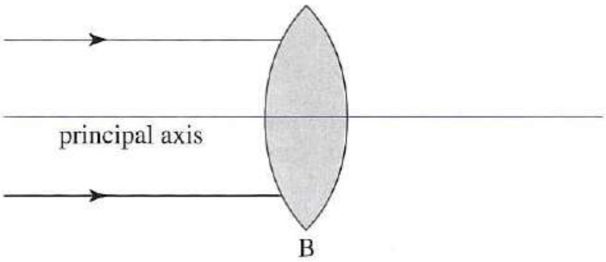
ii) Station R. *(1 mark)*

6 The diagrams below show some pieces of glass.



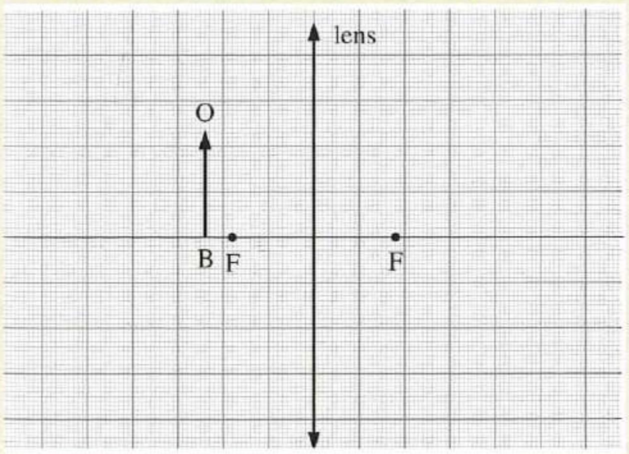
a) Which of **A**, **B**, **C** and **D** is
   i) a converging lens?
   ii) a diverging lens? *(2 marks)*

b) Copy and complete the diagram below to show what happens to the rays of light when they pass through **B**.



(4 marks)

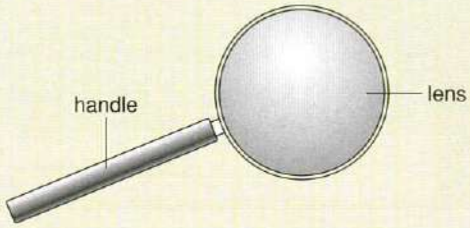
7 a) An object OB is placed 12 cm in front of a converging lens of focal length 9 cm. The diagram below is drawn to scale.



   i) Draw the ray diagram on graph paper to show the position and size of the image. Draw and label the image.
   ii) Write down two ways in which the image is different from the object.

(6 marks)

b) Cameras use converging lenses to produce an image of an object. Give two ways in which the image produced on the film is different from the object. *(2 marks)*



8

When some people are reading a book with very small print, they may use a lens like the one shown in the diagram.

a) State the type of lens used.

b) Explain, in as much detail as you can, how the lens makes it easier to read the print.

(4 marks)

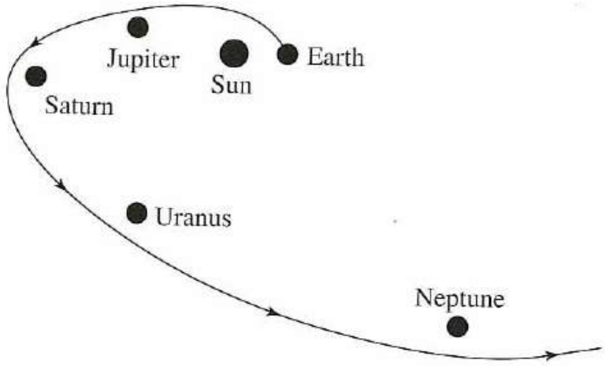
Examination questions

1 a) Copy and complete each sentence by choosing the correct word or phrase from the box. Each word or phrase should be used once or not at all.

milky way	moon	planet	solar system	star	universe

The Sun is the nearest ________ to the Earth.
The Sun is in the galaxy called the ________.
Within the ________ there are millions of galaxies.
Pluto is orbited by one ________. *(4 marks)*

b) The diagram shows the path taken by the Voyager 2 spacecraft.



Choosing from the forces in the box, which force caused the spacecraft to change direction each time it got close to a planet?

air resistance	friction	gravity

(1 mark)

2 a) The table gives some information about four planets.

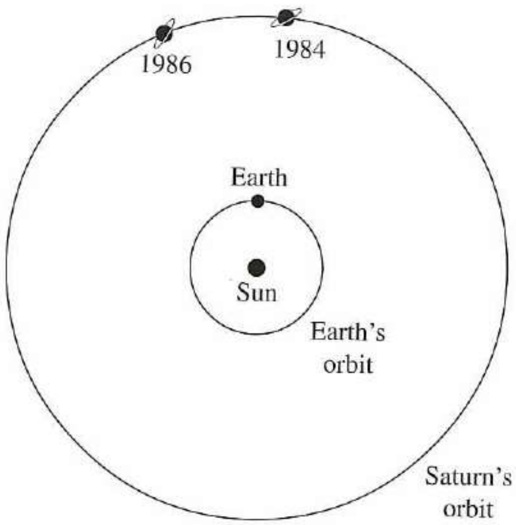
Planet	Average distance from the Sun in million km	Average time to complete one orbit in Earth years	Average orbital speed in km/sec
Jupiter	800	12	13.0
Saturn	1400	30	9.6
Neptune	4500	165	5.2
Pluto	5900	248	4.7

i) Draw a graph of each planet's average orbital speed against the distance the planet is from the Sun. Plot distance from the Sun on the horizontal axis and orbital speed on the vertical axis. *(3 marks)*

ii) How does the average orbital speed of a planet vary with its average distance from the Sun? *(1 mark)*

iii) The average distance between Uranus and the Sun is 2900 million kilometres. Use the graph to predict the average orbital speed of Uranus. *(1 mark)*

b) The diagram shows the position of Saturn in July 1984 and July 1986.



i) Saturn takes 30 Earth years to complete one orbit of the Sun. Copy the diagram and mark the position of Saturn in the year 2000. *(1 mark)*

ii) Suggest why it was difficult to see Saturn in July 2000. *(1 mark)*

3 A satellite in a stable Earth orbit moves at constant speed in a circle, because a single force acts on it.

a) i) Name the force acting on the satellite. *(1 mark)*

ii) State the direction of this force. *(1 mark)*

b) Communications satellites and satellites used to observe the Earth are placed in different orbits.

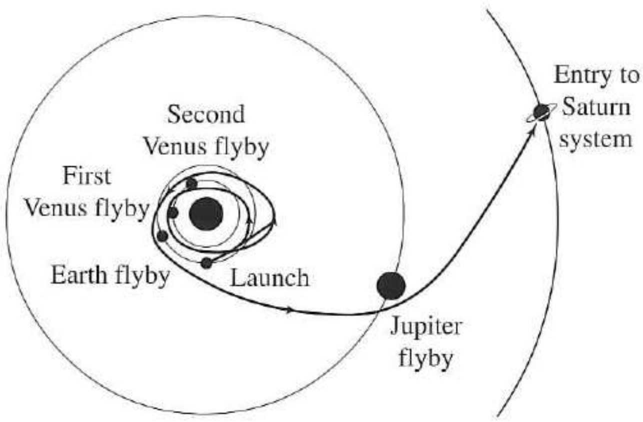
i) Describe the orbit of a communications satellite. *(3 marks)*

ii) Describe the orbit of a satellite used to observe the Earth. *(2 marks)*

iii) Explain why the satellites are placed in different types of orbit. *(3 marks)*

c) Explain, in terms of its orbit, why a comet is rarely seen from Earth. *(2 marks)*

4 a) The Cassini spacecraft launched in 1997 will take seven years to reach Saturn. The journey will take the spacecraft close to several other planets.



Each time the spacecraft approaches a planet it changes direction and gains kinetic energy. Explain why. *(2 marks)*

b) The Big Bang theory attempts to explain the origin of the Universe.
   i) What is the Big Bang theory? *(1 mark)*
   ii) What can be predicted from the Big Bang theory about the size of the Universe? *(1 mark)*

c) i) Explain how stars like the Sun were formed. *(2 marks)*
   ii) The sun is made mostly of hydrogen. Eventually the hydrogen will be used up and the Sun will 'die'.
   Describe what will happen to the Sun from the time the hydrogen is used up until the Sun 'dies'. *(3 marks)*

Examination questions

1 a) Using words or phrases from the list copy and complete the sentences.

elastic **frictional** **gravitational**
less than **more than** **the same as**

When a child goes down a slide the __________ force makes him go faster.
On a damp day the child takes longer to go down the slide. This is because on a damp day the force of friction is __________ on a dry day. *(2 marks)*

b) Using words or phrases from the list copy and complete the sentence.

elastic **gravitational potential** **sound**
kinetic (movement) **light** **thermal (heat)**

When the child goes down the slide, the energy transfers are from __________ energy to __________ energy and __________ energy. *(3 marks)*

2 a) The list gives energy resources which can be used to produce electricity.

coal **gas** **nuclear fuel** **oil**
sunlight **tides** **waves** **wind** **wood**

Write down the **four non-renewable** energy resources.

(4 marks)

b) Using words from the list copy and complete the sentences about generating electricity.

energy **gas** **generator** **smoke**
steam **transformer** **turbine** **water**

In a coal-fired power station, coal is burnt to release __________. This is used to change __________ into __________ which drives a

b) To lift the weight, the weightlifter does 4500 joules of work in 3.0 seconds.
Use the following equation to calculate the power developed by the weightlifter. Show clearly how you work out your answer.

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

(2 marks)

4 The diagram shows a high jumper.

In order to jump over the bar, the high jumper must raise his mass by 1.25m.
The high jumper has a mass of 65kg. The gravitational field strength is 10 N/kg.

a) The high jumper just clears the bar.
Calculate his gravitational potential energy. *(4 marks)*

b) Calculate the minimum speed the high jumper must reach for take-off in order to jump over the bar. *(3 marks)*

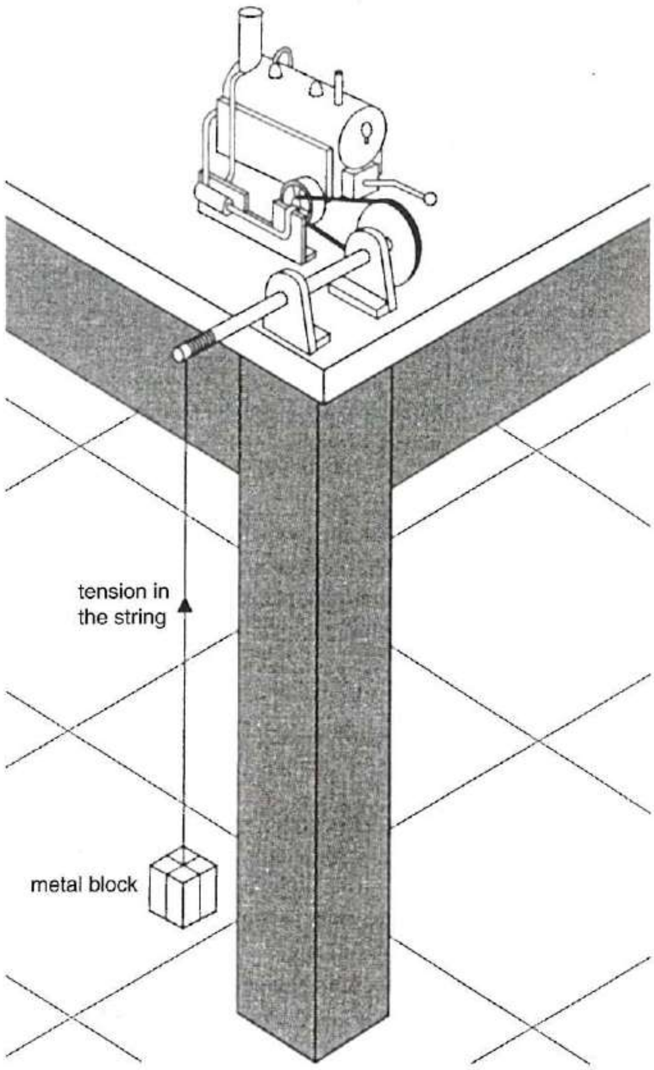
5 The drawing shows an investigation using a model steam engine to lift a load.
In part of the investigation, a metal block with a weight of 4.5 N was lifted from the floor to a height of 90 cm.

a) i) Calculate the work done in lifting this load. Write the equation you are going to use, show clearly how you get to your answer and give the unit. *(3 marks)*

ii) How much useful energy is transferred to do the work in part a) i)? *(1 mark)*

b) In another part of the investigation, 250 J of work is done in one minute.

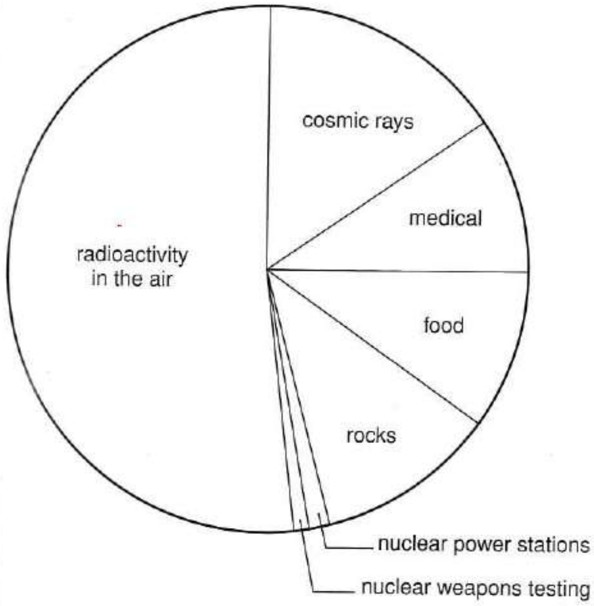
Calculate the useful power output. Give the unit. *(2 marks)*



6 State and explain the advantages and disadvantages of using nuclear power stations to produce electricity compared with coal-fired power stations. *(4 marks)*

Examination questions

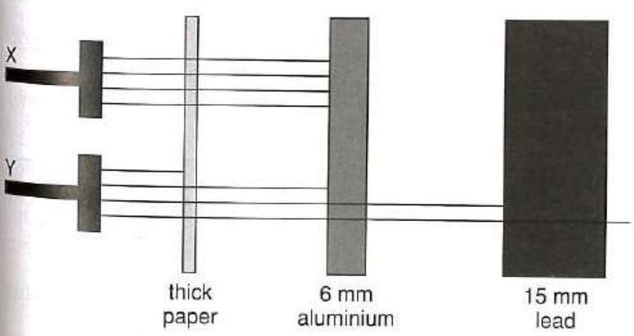
1 a) The different sources of radiation to which we are exposed are shown in the pie chart. Some sources are natural and some artificial.



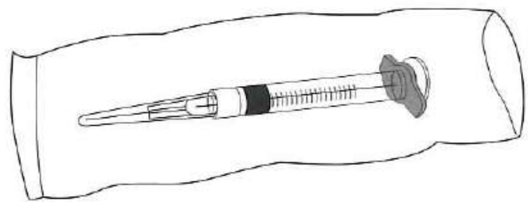
i) Name *one* natural source of radiation shown in the pie chart. *(1 mark)*

ii) Name *one* artificial source of radiation shown in the pie chart. *(1 mark)*

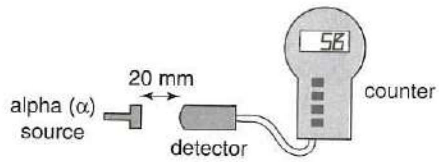
b) A radioactive source can give out three types of emission: alpha particles, beta particles, gamma radiation.
The diagram shows the paths taken by the radiation emitted by two sources, X and Y.
What types of radiation are emitted by each of the sources? *(2 marks)*



c) The diagram shows a disposable syringe sealed inside a plastic bag. After the bag has been sealed the syringe is sterilised using radiation. Explain why radiation can be used to sterilise the syringe. *(3 marks)*



2 a) The diagram shows the apparatus used by a teacher to investigate an alpha (α) source.



i) Which piece of apparatus could be used as a radiation dectector?

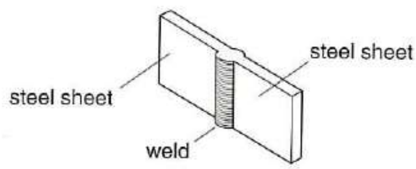
Geiger-Müller Tube	Oscilloscope	Voltmeter

(1 mark)

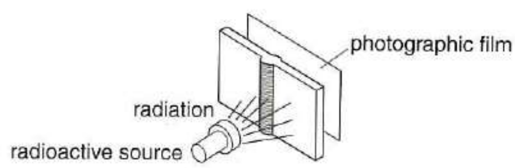
ii) Copy and complete the following sentence.
When a piece of paper is placed between the detector and the alpha source the count rate will go ________ .

(1 mark)

b) Two sheets of steel were joined together by welding.



Radiation was used to check how well the welding had been done.

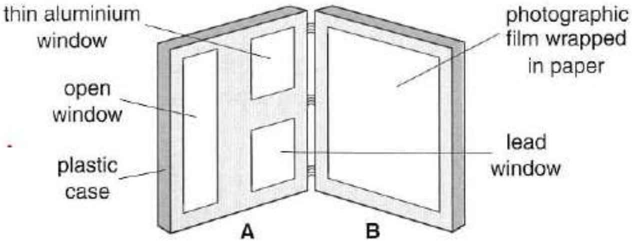


i) Which type of radiation should be used? Give a reason for your answer. *(2 marks)*

ii) The diagram shows the exposed photographic film.

Does the photographic film show that the weld was good or bad? Give a reason for your answer. *(2 marks)*

3 The diagram shows a film badge worn by people who work with radioactive materials. The badge has been opened. The badge is used to measure the amount of radiation to which the workers have been exposed.



The detector is a piece of photographic film wrapped in paper inside part **B** of the badge. Part **A** has "windows" as shown.

a) Use words from the list to complete the sentences.

alpha **beta** **gamma**

When the badge is closed

i) __________ radiation and __________ radiation can pass through the open window and affect the film. *(1 mark)*

ii) __________ radiation and __________ radiation will pass through the thin aluminium window and affect the film. *(1 mark)*

iii) Most of the __________ radiation will pass through the lead window and affect the film. *(1 mark)*

b) Other detectors of radiation use a gas which is ionised by the radiation.

i) Explain what is meant by *ionised.* *(1 mark)*

ii) Explain why ionising radiation is dangerous to people who work with radioactive materials. *(2 marks)*

4 a) The table gives information about five radioactive isotopes.

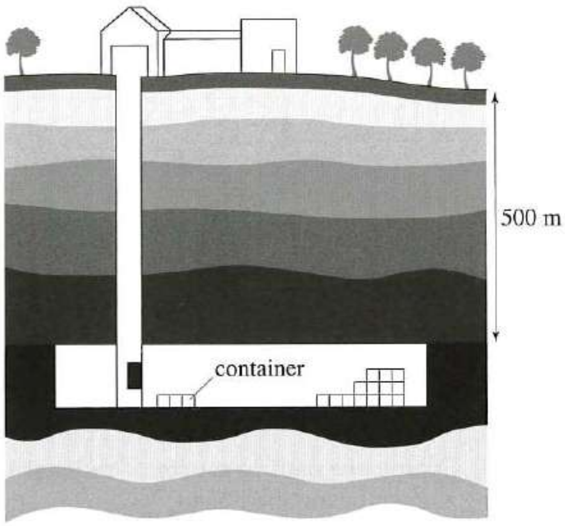
Isotope	Type of radiation emitted	Half-life
Californium-241	alpha (α)	4 minutes
Cobalt-60	gamma (γ)	5 years
Hydrogen-e	beta (β)	12 years
Strontium-90	beta (β)	28 years
Technetium-99	gamma (γ)	6 hours

i) What is an alpha (α) particle? *(1 mark)*

ii) What is meant by the term half-life? *(1 mark)*

iii) Which **one** of the isotopes could be used as a tracer in medicine? Explain the reason for your choice. *(3 marks)*

b) The increased use of radioactive isotopes is leading to an increase in the amount of radioactive waste. One method for storing the waste is to seal it in containers which are then placed deep underground.



Some people may be worried about having such a storage site close to the area in which they live. Explain why. *(3 marks)*

5 a) The graph shows how a sample of barium-143, a radioactive *isotope* with a short *half-life,* decays with time.

i) What is meant by the term *isotope*? *(1 mark)*

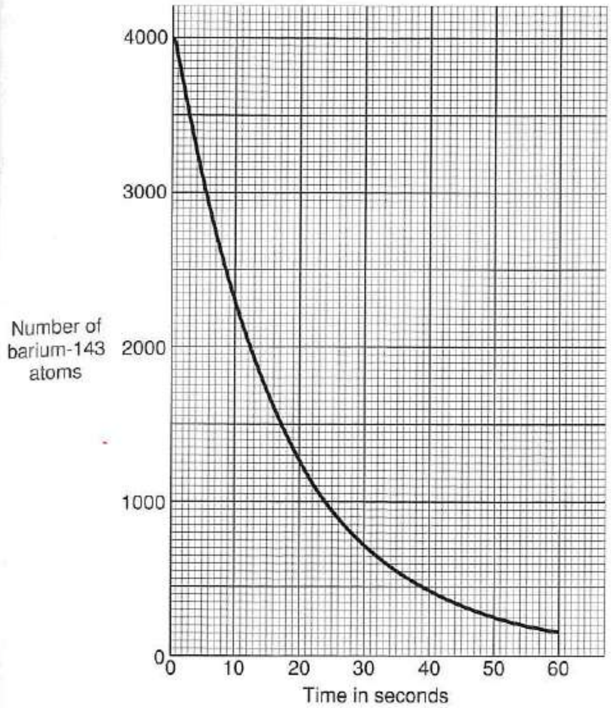
ii) What is meant by the term *half-life*? *(1 mark)*

iii) Use the graph to find the half-life of barium-143. *(1 mark)*

b) Humans take in the radioactive isotope carbon-14 from their food. After their death, the proportion of carbon-14 in their bones can be used to tell how long it is since they died. Carbon-14 has a half-life of 5700 years.

i) A bone in a living human contains 80 units of carbon-14. An identical bone taken from a skeleton found in an ancient burial ground contains 5 units of carbon-14. Calculate the age of the skeleton. Show clearly how you work out your answer. *(2 marks)*

ii) Why is carbon-14 unsuitable for dating a skeleton believed to be about 150 years old? *(1 mark)*



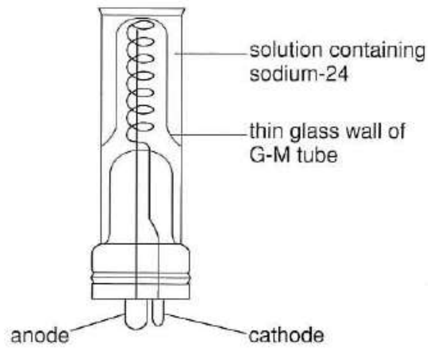
c) The increased industrial use of radioactive materials is leading to increased amounts of radioactive waste. Some people suggest that radioactive liquid waste can be mixed with water and then safely dumped at sea. Do you agree with this suggestion? Explain the reason for your answer. *(3 marks)*

6 The isotope of sodium with a mass number of 24 is radioactive. The following data were obtained in an experiment to find the half-life of sodium-24.

Time in hours	Count rate in counts per minute
0	1600
10	1000
20	600
30	400
40	300
50	150
60	100

a) Draw a graph of the results and find the half-life for the isotope. On the graph show how you obtain the half-life. *(4 marks)*

b) Sodium-24 decays by beta emission. The G-M tube used in the experiment is shown in the diagram. Each beta particle which gets through the glass causes a tiny electric current to pass in the circuit connected to the counter.



i) Why must the glass wall of the G-M tube be very thin?
ii) Why is this type of arrangement of no use if the radioactive decay is by alpha emission? *(1 mark)*

c) Sodium chloride solution is known as saline. It is the liquid used in 'drips' for seriously-ill patients. Radioactive sodium chloride, containing the isotope sodium-24, can be used as a tracer to follow the movement of sodium ions through living organisms. Give one advantage of using a sodium isotope with a half-life of a few hours compared to using an isotope with a half-life of:
i) five years *(1 mark)*
ii) five seconds. *(1 mark)*

Graphene

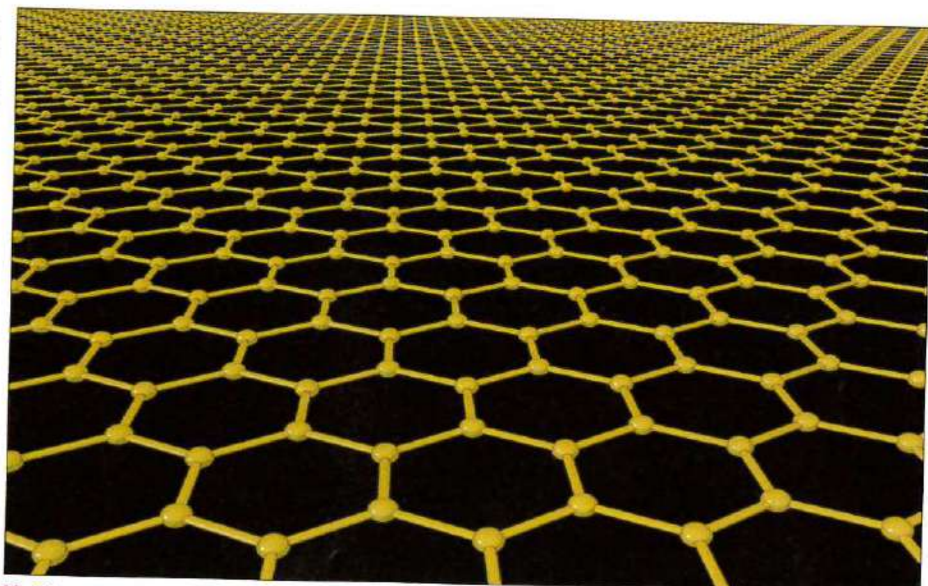
Commercializing the 'wonder material'

Extracting graphene from the laboratory has proved challenging, but as **Ray Gibbs** explains, improvements in material processing are beginning to pay off, particularly in the aerospace sector

Graphene has many amazing properties, including high strength and stiffness, high conductivity and impermeability to gases, to name but a few. These headline-grabbing properties have generated a considerable amount of hype, with potential new applications announced almost every day. However, as the graphene story has progressed, the task of translating properties measured in the laboratory into commercial applications has proved a greater challenge than many had anticipated. In particular, producing consistent single layers of graphene – the starting point for many potential electronics applications – is a technically difficult task, and doing so on a commercial scale is expensive.

Fortunately, other types of graphene are beginning to prove their worth in other industry sectors. At my firm, Haydale, our focus is on stacks of graphene with 5–100 layers. Materials at the lower end of this range are generally known as few-layered graphene (FLGs), while those at the higher end are termed graphene nanoplatelets (GNPs). When these materials are added to a resin or other thermoplastic material, the resulting mixture can become stronger, and may also become thermally conductive, electrically conductive or both. These enhancements could have applications in many areas, but they appeal particularly to the aerospace industry. Many key aircraft parts are made from carbon fibres bonded together with a thermoset resin. If this resin had better mechanical properties, it might be possible to reduce the number of carbon-fibre layers required – saving weight and thus cost.

Our experiments indicate that substantial improvements are possible: in one recent test, a carbon-fibre composite with FLGs added to the resin showed a 20% improvement in almost all mechanical properties. However, getting there involved much more than simply adding graphene to resin. The key to realizing the well-documented properties of graphene lies in starting with the right material and knowing how to process it for particular applications.



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Limitless possibilities Some forms of graphene are finding applications in hi-tech manufacturing.

Producing graphene

Graphene can be produced in a number of ways, and individual manufacturers use slightly different processes. One common approach is the "top down" method, where mined organic graphite is exfoliated to produce flakes of fewer layers. Getting down to the desired number of layers may require multiple production stages, since the thickness of most organic materials varies. However, in bulk systems such as the composite inks, pastes and resins we work with, this is not a huge issue.

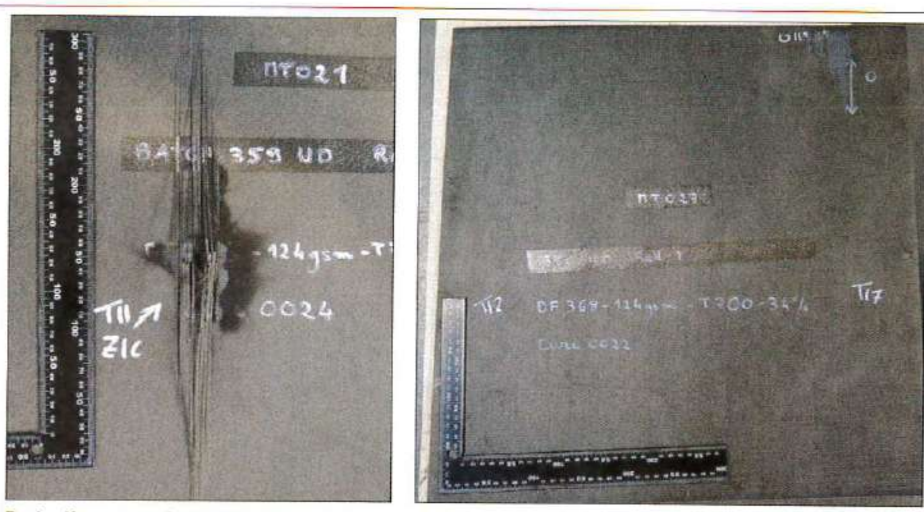
Alternatively, graphene can be produced layer-by-layer in a "bottom up" method such as chemical vapour deposition using methane gas or another carbon source. This process typically requires operating a reactor at energy-intensive temperatures (900 °C or more), and the reactors must also be cleaned after each batch is produced. Additionally, in many cases the graphene sheets produced by this method are not single layers but FLGs two or three layers thick. Expensive "release tapes" must then be used to peel off individual layers.

Clearly, graphene produced via the top-down method is very different from the bottom-up variety, both in its properties and in its manufacturing cost. However, due to a lack of industry standards, many different carbon nanomaterials can be described as "graphene". As a result, the prices of similarly labelled products can range from \$50 to more than \$2000 per kilogram. The temptation is to plump for the cheapest one available, but often this is not the best option. This is because every material produced at the nanoscale is different – in flake size, thickness and, crucially, the types and amounts of chemicals bonded to its surface and ends. These chemical groups are often involved in binding the graphene to other materials, and can thus affect the properties of the mixture. For example, a material with a lot of oxygen groups will act as an insulator, not a conductor. The size and shape of the flakes can also affect thermal conductivity, electrical conductivity and/or mechanical uplift.

> Mixing and dispersion know-how is crucial to functionalizing graphene

In our experience, whatever the desired application, mixing and dispersion know-how is crucial to "functionalizing" graphene (that is, getting other chemical groups to bond with it). Carbon as an additive is inert and does not mix well with other materials, so to get it to disperse in a homogeneous fashion, one needs both a good understanding of functionalization and a detailed knowledge of the particles' size and shape – which, when the particles are 2–5 μm across, requires special skills and equipment. It is also worth pointing out that adding nanomaterials to other substances does have some potential drawbacks; for example, it could change the viscosity of a resin, which can affect later steps in the production process. Often, there will be a trade-off between the desired performance of the final product and other properties that existed before the nanomaterials were added.

To develop our understanding of these issues, Haydale has conducted an 18-month programme of research in collaboration with Huntsman Advanced Materials using its high-end epoxy resin, Araldite. This work has given us considerable expertise of the mixing and processing techniques required to properly disperse graphene and other nanomaterials into a thermoset or thermoplastic resin. It has also become abundantly clear that adding a second nanomaterial (such as carbon nanotubes or silicon carbide) alongside graphene can have significant effects on performance, over and above the effects of purely adding graphene alone. We believe that this process, which we term "material hybridization", holds great promise for the future commercialization of composites and, indeed, other materials such as inks.



Protective properties The image at left shows the back face of a conventional, unprotected (that is, without copper mesh) composite panel after it was subjected to a severe lightning-strike event. "Punch-through" damage is clearly evident. The image at right shows a similar post-strike image, but in this case the panel had been modified with functionalized nanoparticles, and shows no visible back-face damage.

Haydale Composite Solutions

Taking flight

Since 2014 scientists at Haydale have been using functionalized graphene to improve the performance of carbon-fibre composites in the aerospace industry. This project was based on requirements specified by the Centro Italiano Ricerche Aerospaziali (CIRA), and was managed by an integrated team from CIRA, Haydale and the school of engineering at Cardiff University in the UK, with financial support from the Europe-wide Clean Sky Joint Technology Initiative.

Compared to resins, carbon fibres are immensely stiff and strong, so the structural properties of a component made from a fibre-reinforced composite is dominated by the properties of the fibre, not the resin. Hence, even though adding functionalized graphene to neat resin has been shown to double the resin's stiffness, one would expect the effect on the macrocomposite to be smaller. Our research investigated the effects of adding both GNPs and carbon nanotubes to resins, and we observed a 13% increase in compression strength and a 50% increase in compression after impact performance. These are both significant results, since damage resistance and compression properties are of paramount importance in high-performance structures such as composite aircraft wings.

In addition to stiffer, stronger materials, the aerospace industry is also looking for better ways of preventing lightning-induced damage to aircraft. Currently, copper mesh is built into the body of aircraft to dissipate electric charge from lightning strikes, but this mesh adds considerably to the aircraft's weight. If the actual fabric of the aircraft could be made to conduct electricity, this would render the mesh unnecessary and save substantial amounts of fuel.

Scientists in Haydale's composite division (Haydale Composite Solutions) are currently working with industrial partners such as Cobham Technical Services, Airbus and BAE Systems on two research projects that use functionalized nanoparticles to make aircraft components electrically conductive. The first project, Graphene Composites Evaluated in Lightning Strike (GraCELS), is investigating how functionalized nanoparticles affect the conductivity of carbon fibre-reinforced epoxy panels. The GraCELS experiments have shown that adding nanoparticles to the epoxy substantially improved the panels' electrical conductivity, and greatly enhanced their tolerance of lightning-strike damage. In particular, the modified panels showed no sign of "punch through" damage when subjected to a severe lightning-strike event (see images above).

The second project, known as Graphene-Enhanced Adhesive Technology through Functionalization (GrEAT Fun), is focused on the bonds between carbon-fibre panels in aircraft, rather than the panels themselves. Adhesive bonds made using conventional techniques are generally electrical insulators, which is a problem if we want the structure of the aircraft to conduct electricity. Previous studies have attempted to improve the electrical conductivity of structural adhesive bonds by adding metallic particulates or carbon nanotubes to the bonding material, but these efforts have had limited success in producing bonds that are strong and reliable as well as electrically conductive.

The GrEAT Fun project, in contrast, will use a patented technology for functionalizing GNPs to significantly improve the electrical conductivity of adhesive bonds as well as enhancing the strength of the bonded layer. This functionalized graphene can be incorporated into a thermosetting matrix resin. Inevitably, there will be a trade-off between mechanical and electrical performance and the ease of processing the modified resin; one of the project's goals is to establish the level of graphene loading that leads to the best overall performance of the adhesive.

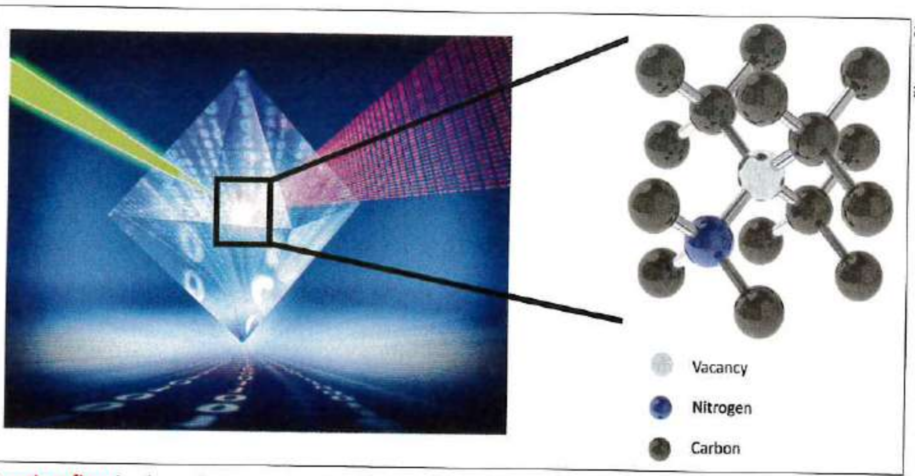
Future applications

The aerospace industry is likely to be an early adopter of the adhesives developed during the GrEAT Fun project, but other fields may also benefit. For example, improvements in the electrical conductivity of structural adhesive resin systems could enhance the performance of large off-shore wind turbines, while in the oil and gas industry, conducting resins could make it easier to dissipate static electricity and prevent it from causing damage to pipelines. As for the structural properties of functionalized graphene, there are myriad potential applications, from damage-resistant shower trays to tougher sporting equipment. The transport industry is likely to benefit, too, although here the time frame will be longer due to the regulated nature of the industry. Commercial applications for graphene may have taken longer to emerge than the hype suggested, but these recent developments could finally harness the wonder material's amazing properties.

Ray Gibbs is the chief executive officer of Haydale Graphene Industries, e-mail ray.gibbs@haydale.com

The cutting edge of quantum physics

Daniel Twitchen and **Matthew Markham** explain why carbon's most alluring allotrope could be a quantum physicist's best friend, and how the market for diamond-based quantum technologies is starting to take shape.



Quantum flaw A schematic representation of the nitrogen vacancy (NV) defect in diamond. The surrounding lattice of carbon atoms helps to shield the electronic spin state of the NV centre from noise.

In the 20th century many aspects of quantum physics were harnessed into world-changing technologies, including semiconductors, lasers and other now-ubiquitous devices. Throughout this first quantum revolution, however, one key aspect of quantum physics – superposition – has largely remained in the laboratory, a fundamental curiosity rather than a promising feature to be exploited.

However, this is about to change, thanks to several significant initiatives that aim to bring about a second quantum revolution. The key to this revolution's success will be the ability to "easily" engineer and control quantum bits. We use the word "easily" with caution, because initializing a quantum state and keeping it in a superposition for significant lengths of time is a difficult undertaking. Scientists are exploring many different approaches, using materials as varied as superconductors, synthetic diamonds, cold atoms and quantum dots, and the race is currently wide open. But diamond does have some intriguing advantages, both for quantum computation and for other applications such as magnetic-field sensing. The challenge for our organization, the industrial diamond firm Element Six, has been to support research in this area while also staying true to our core business interests in materials applications.

A useful flaw

The type of diamond that attracts would-be quantum revolutionaries has a defect in its otherwise uniform lattice of carbon atoms. This defect consists of a single nitrogen atom adjacent to a missing carbon atom, or vacancy. The nitrogen-vacancy (NV) centre has unique optical absorption and emission properties – among other effects, it gives diamond a red-to-pink colouration – and these properties have long been the focus of fundamental research on crystal structures.

In addition to its unusual optical properties, the negative charge state of the NV centre also has an electronic spin, $S = 1$, in its ground state. Remarkably, the state of this electronic spin can be controlled and read out at room temperature. The reason is that unlike most materials, the crystal lattice in diamond forms a low-noise environment, so fragile quantum properties are not lost and information can be stored and probed for longer time periods. The spin state can be read out by measuring the intensity of light given off by an NV centre as the system is excited by microwave radiation. At the NV centre's resonance frequency of 2.88 GHz, the spin state will flip from 0 to a +1 or −1, causing a dip in the intensity of red light emitted.

The robustness of this spin state, and the ease of reading it out, make NV diamond a very promising platform for a wide range of quantum technologies, with potential applications in secure communications, computing, imaging and sensing. A recent focus area for the diamond community is the use of NV defects to measure magnetic fields. Thanks to the Zeeman interaction, the gap between the frequencies of the $0 \rightarrow 1$ and $-1 \rightarrow 0$ microwave transitions in NV diamond increases as a function of magnetic field. Hence, in the simplest case, one can estimate the magnitude of the magnetic field by exposing the NV centre to a range of microwave frequencies and measuring the separation between the two dips in intensity. Remarkably, a basic measurement of this type can be performed using a single NV centre at room temperature. With multiple NV centres, the geometry of the diamond lattice means that one can make extremely sensitive measurements of the field's direction as well as its magnitude.

> Advances in our diamond-making capabilities have opened up a wide range of potential applications

Raw materials

Of course, numerous technologies for estimating magnetic field already exist. These include superconducting quantum interference devices (SQUIDs), vapour cells, flux-gate sensors and the Hall-effect sensors that constitute the compass in modern smartphones. However, SQUID-based magnetometers must be cryogenically cooled, making them relatively bulky and costly to run, while other sensor technologies require frequent recalibration and offer limited frequency bandwidth for measuring changing magnetic fields. In contrast, NV diamond-based sensors do not need to be recalibrated, have a broad bandwidth and could be incorporated into a lightweight, low-powered device. Critically, NV centres can also be used to construct maps of magnetic field across a surface, thanks to the high spatial resolution provided by a microscopic probe. For these reasons, diamond-based magnetometers have strong potential both as replacements for existing technologies and as the enablers

Diamond quantum technologies

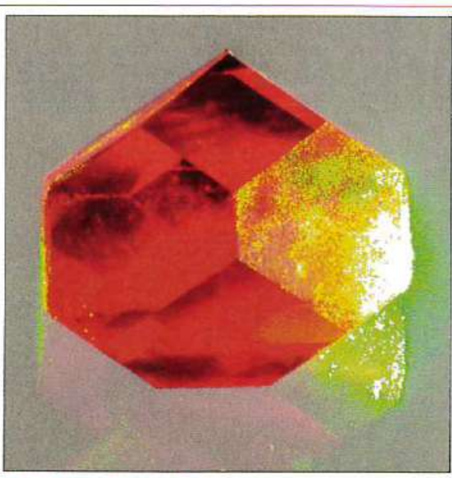
of applications where competing technologies do not yet exist.

For these applications to become a reality, though, we need a ready supply of high-quality NV diamonds. NV centres are rare in natural diamonds, and it is difficult to do much research if you are limited to working with a single sample. At Element Six we have developed methods for growing NV diamond synthetically using chemical vapour deposition (CVD). This process involves filling a microwave chamber with a mixture of hydrogen, methane and nitrogen gas, and heating it to 2500–3000 K to create a plasma. Diamond "seeds" placed in the chamber become the nuclei for new diamonds as carbon atoms from the plasma deposit onto their surfaces layer by layer. The hydrogen stabilizes the surface against forming graphite instead of diamond, while the nitrogen acts as a dopant, making it possible for NV centres to form.

This process is the result of more than 15 years of intensive R&D and it enables us to grow diamond in a controlled and scalable fashion, with a purity far exceeding that of natural diamonds. It also makes it possible to control the number of NV centres. Under high-purity conditions, small numbers of NV centres are produced via the chemistry of the growth process. These isolated vacancies can be probed individually in an experiment, so this type of NV diamond is well-suited for quantum-computation applications. Magnetic-sensing applications require higher numbers of NV centres, and we achieve this by increasing the nitrogen concentration during synthesis and then bombarding the crystal with high-energy electrons to create additional vacancies. Heating the diamond to 800 °C causes these vacancies to migrate through the crystal lattice until they encounter nitrogen atoms; at that point, the structure stabilizes, since the NV centre has a lower potential energy than a separate nitrogen and vacancy.

The value chain

Over the past decade advances in our diamond-making capabilities, coupled with a deepening understanding of the physics of quantum spins in NV diamond, have opened up a wide range of potential applications. Element Six has supported this nascent field by supplying state-of-the-art diamond samples and diamond engineering expertise to external partners, while focusing internally on making further improvements to the material. In recent years, however, we have also become more active in supporting commercial start-ups to allow them to incubate the technology and in helping larger companies assess the applicability of our diamonds to various market opportunities.



Element Six

Precision engineering Advances in synthetic diamond manufacturing have made it possible to create diamonds with the right number of NV centres for particular applications.

The breadth and depth of knowledge needed to appreciate these opportunities is significant. It requires one to consider an entire value chain: a material; a device made from that material; the package surrounding that device; the subsystems and systems the device fits into; and finally the user. As is often the case, the commercial value of this chain is concentrated at the subsystem and system level. But Element Six is a materials company, and we have grown by developing novel materials that address problems across multiple markets and industries. Making devices, let alone complete systems for end users, is not really our speciality. So how can we access the value at the other end of the chain?

Rather than changing our strategic focus, we have instead sought to exploit diamond quantum devices by communicating their "value proposition" to end users. A basic demonstration of the NV centre's ability to measure magnetic field is not difficult, and a prototype device can be made using remarkably simple components such as off-the shelf diode lasers and photodiodes, and coils of wire to deliver the microwaves to the sample. Packaging all of this together into a robust unit is less trivial, of course; ultimately, the performance of a diamond sensor will depend not only on the material itself and Element Six's expertise, but also on the stability of surrounding components and the data-processing algorithms used to transform raw measurements of light intensity into an accurate and highly sensitive map of the vector magnetic field. Nevertheless, it is always much easier to convince people of a device's potential with a demo than with PowerPoint slides.

Another component of our strategy has been to partner with university researchers who are developing diamond-quantum-device technology. This has enabled us to secure some intellectual property (IP) on the physics needed to make working devices – although, crucially, we actively avoided filing patents for the actual applications because we wanted to leave third parties free to develop their own. Our university partners have also been an important bridge between us and potential end users. Making a diamond-based quantum device (or indeed any quantum device) requires knowledge of quantum physics, and since this is an emerging industry most organizations do not yet have that expertise. Combining our IP and materials know-how with their quantum-physics expertise enabled us to start talking to organizations that were actually in a position to develop this technology. In addition, many of the academic groups we work with have produced spin-out companies. We have supported these companies with materials sales and knowledge-sharing, and we anticipate that the applications they develop will be a growth area for Element Six over the coming years.

Potential gems

Diamond quantum technologies are extremely promising, with many applications already at the proof-of-concept stage. These include applications in materials characterization such as nanoscale imaging of the write heads for next-generation magnetic hard drives, and biological imaging. New sensing methods for pressure and temperature, plus the alluring possibility of diamond-based quantum computing, makes this an exciting and productive area.

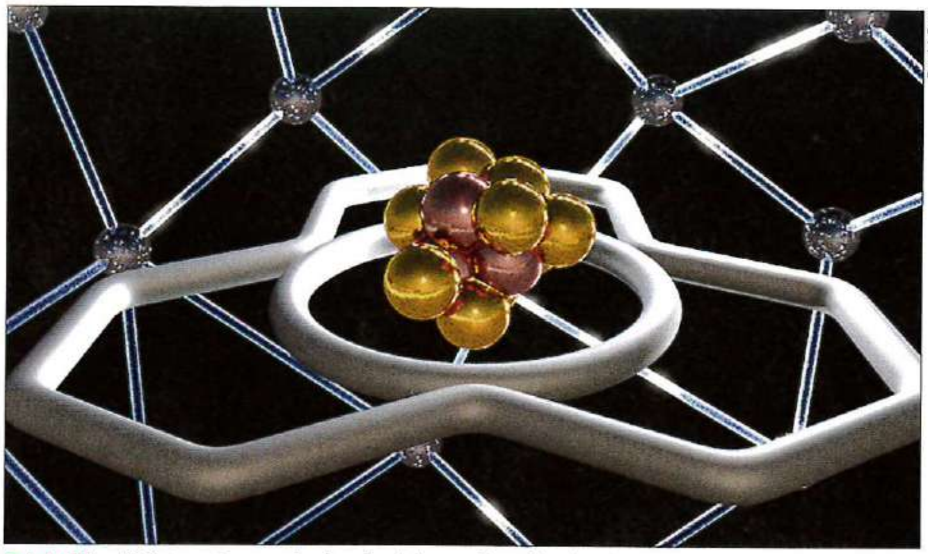
We foresee that diamond will continue to be used as a tool to aid our understanding of the quantum world. However, the real excitement concerns the possible technologies that this understanding will enable. In late 2016 a group of researchers led by Ron Walsworth at Harvard University, US used NV centres in diamond to study neuron activity in marine worms, measuring the tiny magnetic pulses from single neurons with high spatial resolution. No other existing technology can perform measurements at such high sensitivity and resolution; the maximum spatial resolution of standard MRI scans is about 1 mm^3, whereas diamond-based magnetic field sensing could, in theory, give us cellular-level images of chemical processes. We expect that this proof-of-principle experiment will be followed by breakthroughs in our understanding of how the brain works, as well as new diagnostic methods and treatments.

Daniel Twitchen is head of CVD business development (www.e6cvd.com) and **Matthew Markham** is a principal research scientist at Element Six, e-mail daniel.twitchen@e6.com

Computational nanoscience

Bridging the gap

Theoretical physicist and nanotech start-up founder **David Gao** tackles the challenges of working at the interface between academia and industry



David Gao

Catalyst Combining quantum-mechanics simulations and machine-learning techniques to design the next generation of catalyst nanoparticles.

When academics go looking for funding it is common (some might even say required) for their grant proposals to play up the industrial applications of their research. However, bridging the gap between fundamental research and industry is often difficult. One of the most important aspects of scientific research is the way it explores the unknown. This comes with a significant level of risk. The most interesting problems are often the most challenging ones and even seemingly straightforward questions are never as simple as they initially appear.

Academic research embraces the uncertainty that comes with this risk, celebrating the discovery of new questions and, in some cases, finding answers that are unrelated to the original line of enquiry. In contrast, within a commercial enterprise the most critical aspects of research projects are specific "deliverables" and the particular business needs they serve. Most companies do not have the resources to exploit new discoveries in unrelated fields or sectors, and instead focus on generating tangible returns within their own space.

Best of both worlds

In an attempt to get the best out of both of these worlds, companies and academics sometimes form customer–provider relationships in which the industrial partner essentially pays for a research service. In this way, the company can retain all the desired intellectual property rights and clearly define the work plan and goals. The academic partner in turn receives much needed funding, as well as a valuable route towards applying their results. However, this relationship can become strained if the academic partner aims to develop new methods and build fundamental knowledge while their industry counterpart is expecting a specific deliverable or product. Unfortunately, in many sectors fundamental research is seen as an extremely long-term investment, making it one of the first budgets to be cut during a downturn. This can be problematic for academic partners.

I have personally experienced situations like this from both sides, having been both the industrial partner and the academic at various points of my research career. While working as an industrial materials scientist at Chevron I could see my research turn into tangible advances in technology. However, I was often frustrated by the fact that studying fundamental mechanisms and method development was given low priority. I then decided to return to academia, joining Alexander Shluger's group at University College London (UCL) to focus on the theoretical modelling of material properties. While I was now able to throw myself into studying fundamental mechanisms, it became difficult to see how my work developed into real-world products. So, when I took a step back and looked closely at what research and development means to me, and where I wanted to position myself, I decided to use my experience in both academia and industry to try to reconcile these two goals. My interest lies in pushing the boundaries of our knowledge, so becoming a contracted problem-solver was not an ideal arrangement. Instead, I decided to embark on an exciting journey: I started a new company, Nanolayers Research Computing, with a few like-minded colleagues.

People are often curious about this approach, and I am sometimes asked how I balance an academic life and an industrial one. In fact, this is quite demanding. I have given up a lot of nights, weekends and holidays, and even so, it would have been extremely difficult to stay motivated without the encouragement and support of close friends, family and my group at UCL. Another common question is "What does your company actually do?" The short answer is that we apply computational chemistry, physics and machine-learning techniques to design and develop new materials for a variety of industrial applications. However, what that statement actually means in practice is not transparent. How does a materials design firm – and a heavily theory-based one at that – fit in to a landscape of chemical, pharmaceutical and electronics companies?

Novel nanoparticles

For Nanolayers, part of the answer lies in the European Union's Horizon 2020 framework. This framework incorporates a role for companies that are designated as "translators" because they help research groups connect with people in industry who might want to use the group's software or methods. Our years spent in the theoretical physics community gave us an excellent network of potential university collaborators, while my past life as an industrial materials scientist provided several useful industry connections. Before long, we were invited to join a Horizon 2020 project that aims to replace certain critical, industrially-relevant catalyst materials with novel transition-metal nanoparticles.

Computational nanoscience

Within this project, known as CritCat (www.CritCat.eu), our role is to apply machine-learning techniques to results and data collected by our academic partners. We then use our findings to develop catalyst materials that do not incorporate elements such as platinum-group metals, which are of critical importance in Europe due to their cost and scarcity. Our strategy for catalyst design is to figure out what features are relevant in describing these materials and then train neural networks to learn how these features correlate to catalytic activity. This allows us to learn the mechanisms behind what makes a good material, and thus design and control the properties of our materials. We then design new nanoparticles that are subsequently produced by our manufacturing partners and then validated in real-world trials.

As a small-to-medium-sized enterprise (SME) capable of interfacing not only between academia and industry but also between theory and experiment, we hold a unique position within the CritCat project. We have taken a leading role in the dissemination and the exploitation of our technology, and have also leveraged our expertise in computational chemistry and theory techniques to provide additional support services and method development for our theory partners, who are based at Finland's Aalto and Tampere universities.

Beyond materials science

When I got the opportunity to network with other materials design-focused companies and projects such as NoMAD (novel materials discovery), one of my take away messages was the importance of developing a marketable product along with a diverse skillset. Since Nanolayers' core values involve performing exploratory research rather than commercializing something that has already been tested, we decided to take on more of a consulting or partner role and looked for an opportunity to apply our expertise and experience in other sectors. Our goal was to use our simulations and machine-learning techniques in an equal partnership with someone capable of producing marketable devices or software.

To this end we recently formed a partnership with two firms (GV Concepts in the US and eQuumSoft in Asia) to develop new technologies for monitoring vital signs and conducting medical pre-screenings remotely. By continuously monitoring patients' vital signs, clinicians may be able to spot qualitative early-warning signals for a variety of potential illnesses, and intervene if the risk is deemed high enough. The challenge is to do this outside a clinical setting, so that patients – particularly those who are elderly, high-risk or suffering from chronic diseases – can record these vital signs in the comfort of their own homes. This is a complex task, one that involves digital devices, diagnostic tools and complementary vital-signs data-collection software. Our solution will make it possible for patients to monitor their own health in a personalized way using a set of patented diagnostic tools including a digital stethoscope, otoscope, blood-pressure monitor, oximeter, thermometer, ophthalmoscope and camera. These devices are all integrated with a smartphone-based software suite that not only enables patients to connect remotely with healthcare professionals, but also allows doctors to remotely control diagnostic tools during the "virtual visit".

David Gao

Personalized care A suite of patented electronic medical attachments enhances a doctor's "virtual visit", reducing wait time, eliminating distance barriers and cutting overhead costs.

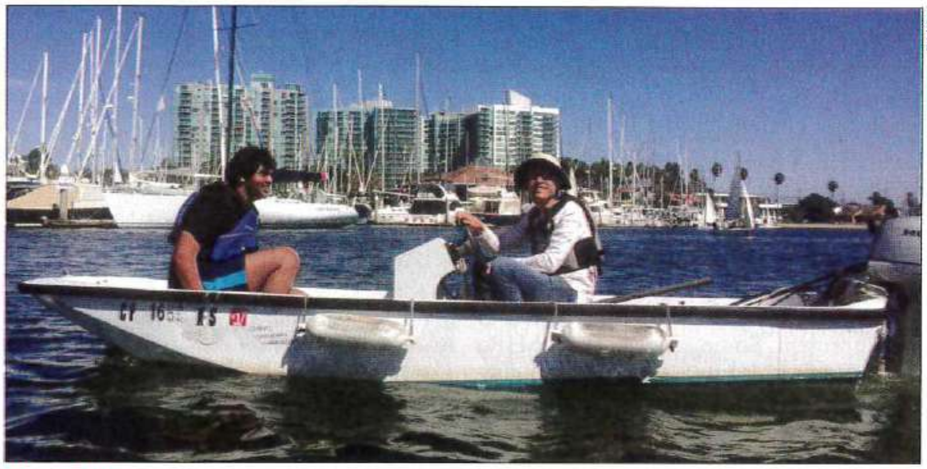
We use the data collected in this project in a way that is similar to the method we employ for designing novel materials. In this case, we are seeking symptom–disease relationships rather than structure–performance ones, but the strategy of using machine-learning techniques to identify relevant relationships is the same. For example, we use neural networks for image recognition and signal processing to help healthcare professionals interpret the collected data.

As we pick up more projects and partnerships that are structured in a similar way, Nanolayers continues to expand while focusing on the theme of bridging the gap between fundamental scientific knowledge and techniques and industrial applications. In this way, we can enjoy the best of both worlds by exploring new materials and techniques while making sure that the discoveries and advancements we make are applied in a meaningful way. Time spent on improving our own methods and gaining experience is not wasted. After all, one of our most important products will always be the research team itself.

David Gao is a research fellow at University College London, UK, and the founder and director of Nanolayers Research Computing, e-mail david@nanolayers.com

Smooth sailing

Superhydrophobic surfaces with the right microscopic structures could be the key to reducing friction on marine vessels, as **CJ Kim** explains



CJ Kim

The call of the sea University of California, Los Angeles graduate students Shashank Gowda and Muchen Xu are testing superhydrophobic surfaces attached to the bottom of a motorboat in Marina del Rey, California.

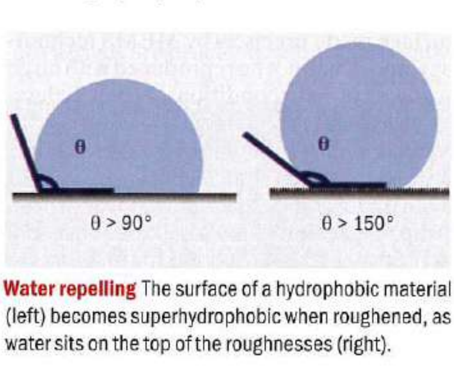
Ocean-going ships face a constant struggle. In order to maintain their motion, they must continuously overcome the drag of the water that surrounds them. When one considers that marine shipping accounts for 4% of all fossil-fuel use, a similar percentage of climate-change-causing emissions and more particulate pollution than all of the world's cars combined, it is clear that reducing this drag by even a small fraction would bring considerable benefits. Since the drag consists mostly of friction between the skin of the moving hull and the stationary water around it, lubricating this surface to reduce frictional motion would be a big help in reducing total drag.

We usually think of lubricants as being liquids, such as oil, but when friction occurs between a solid and a liquid, gas is the only real option for lubrication. For example, a torpedo can "fly" underwater, reaching otherwise unimaginable speeds, if a large pocket of water vapour engulfs its entire body via a method known as supercavitation. Also, blowing air bubbles onto the bottom side of a ship's hull would allow the ship to move faster at a given propelling power.

You might ask, then, why we do not have gas-lubricated boats around us already. The problem is that unlike a liquid lubricant on a solid surface, a gas lubricant on a solid surface in a liquid (such as air in water) will leave the surface rather than staying on it. And unfortunately, providing a continuous supply of a rapidly disappearing gas consumes a lot of energy, which tends to cancel out the energy saved through lubrication, limiting the overall benefit.

Superhydrophobicity to the rescue

This frustration helps to explain why superhydrophobic (SHPo) surfaces were so exciting when, in the early 2000s, researchers began considering their applications for drag reduction. A SHPo surface is one that repels water much more strongly than usual. For example, a Teflon surface will repel water, forming a contact angle of around 110° between the water droplet and the surface. However, when such a naturally water-repellent material is roughened, water will sit on top of the roughness as if levitated by the air in the rough surface, with contact angles increasing to more than 150°. As a result, the water will bead up and roll straight off when the surface is tilted. This "lotus leaf" effect has been a very popular topic in science and engineering for the past two decades, and thousands of images and online videos vividly demonstrate its intriguing properties.



Water repelling The surface of a hydrophobic material (left) becomes superhydrophobic when roughened, as water sits on the top of the roughnesses (right).

If SHPo surfaces repel water so well, they must reduce the drag of water – or so the thinking went. Returning to the gas lubrication process discussed above, it was speculated that gases would persist on the SHPo surface, and thus finally make it possible to lubricate water friction with a gas layer that would not dissipate. Yet despite this logical expectation, and a torrent of research activities worldwide over the last 15 years, so far no publication has reported a successful demonstration of superhydrophobicity reducing the drag on a boat in ocean water. This article discusses why this is so, and whether there is a light at the end of this tunnel.

To identify the problems, let's break down the issues. First, is it at least theoretically possible to obtain an appreciable drag reduction using a SHPo surface for real applications, such as a boat? Second, would a SHPo surface really save us from having to constantly supply a gaseous lubricant? Third, would it be economical to produce and implement such a SHPo surface for practical applications?

Enough drag reduction?

Scientifically, there is no doubt that a layer of air (known as a "plastron") on the SHPo surface in water will lubricate the motion and help reduce the drag. The real question is, just how much reduction are we talking about? If it is more than 10%, for example, that would be meaningful in practice. If the reduction is below 1%, however, we would not expect much impact in the real world even though it would still be scientifically interesting.

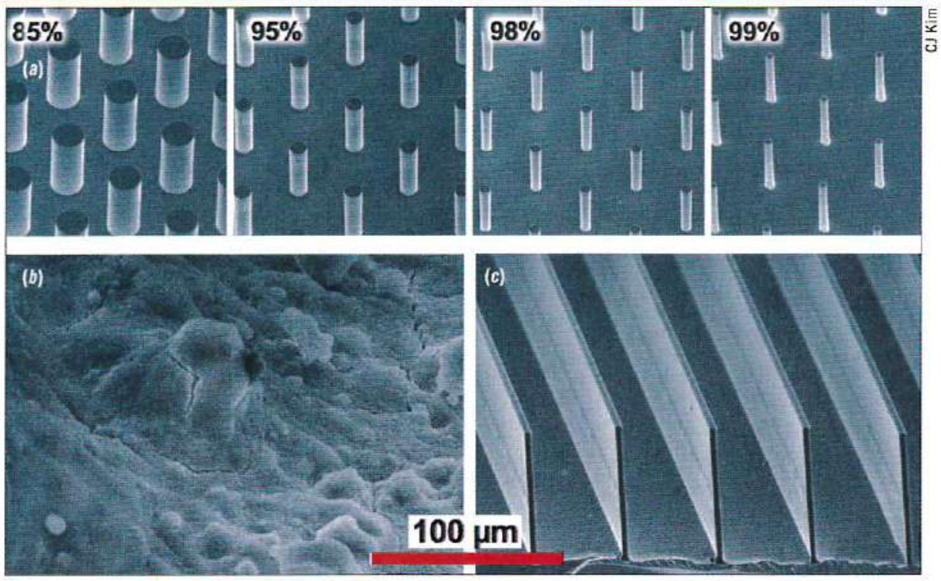
Despite the excitement generated by the idea's scientific merit, and some encouraging early experimental results, it took nearly 10 years for the research community to understand how much drag reduction would be possible, and on what kinds of SHPo surfaces. Looking back, a few reasons for the slow advance are apparent. First, hydrophobicity is related to drag reduction, but not directly. Since the dynamics of bulk water and droplets are fundamentally different, a surface more favourable for droplet rolling is not necessarily more slippery to water continuously flowing by. The underlying, and still widespread, notion – that if a SHPo is very repellent to water, then it must be also very slippery to water flowing on it – is now known to be flawed.

Second, there was some confusion between drag reduction and the "slipperiness" of the surface. These concepts are linked, but they are not the same thing: the amount of drag reduction is determined not only by how slippery a surface is, but also by the flow system where the surface is employed. This means that for a given drag-reducing surface, one may obtain 50% drag reduction in a microscopic channel but not even 0.1% reduction on a boat. This mix-up made it difficult to objectively compare one SHPo surface with another in terms of their ability to reduce drag.

A third source of delays was related to measurement. Some early work on SHPo surfaces reported fantastically large reductions in drag – reductions that we now know were impossible. In many cases, the errors seem to have come from the challenge of measuring drag reduction accurately. While the observed trends may have been correct, the actual amount of reduction was simply wrong. These early, incorrect experimental data probably slowed down the establishment of the knowledge base for SHPo drag reduction.

Today, the research community uses an objective measure called "slip length" to describe the slipperiness of a given SHPo surface. We also understand how much drag reduction a particular slip length will entail under a certain flow condition, at least for turbulence-free (laminar) flows. Micro and nanofabrication technologies using microelectromechanical systems (MEMS) have played a key role in advancing the field. By enabling researchers to construct SHPo surfaces with exact and deterministic micro and nano structures (figure *a*), rather than random roughness (figure *b*), these technologies have made it possible to confirm theoretical predictions about SHPo behaviour. In a nutshell, we have learned that a SHPo surface will be more slippery (that is, its slip length will be large) if its microstructures are slender (more void spaces and fewer solid portions) and dispersed (greater distances between the structure peaks).

These theoretical predictions have been confirmed for laminar flows. If we extrapolate this to turbulent flows, it seems that a highly slippery surface, consisting of slender microstructures dispersed far apart, is required before a boat will enjoy an appreciable reduction in drag. By fabricating SHPo surfaces full of parallel trenches tens of microns apart with a large void space between them (figure *c*), our lab has reported drag reductions as large as 75% in turbulent flows. This level of reduction was obtained under well-controlled flow experiments using a small (2×2 cm) SHPo surface made precisely by MEMS technology, and it may not be reproduced with large surfaces in field conditions. Nevertheless, it demonstrates the potential of SHPo drag reduction. The theory also suggests that more typical SHPo surfaces (figure *b*) with microscopic random roughness would have a slip length that is too small to induce any appreciable drag reduction for macroscale applications such as a boat in open water.



Exact construction vs random roughness These periodic microstructures were made precisely by microelectromechanical systems and helped to establish a quantitative relationship between a superhydrophobic surface's geometry and how slippery it is. The percentages are the proportion of void spaces on the superhydrophobic surface; surfaces with higher void percentages proved more slippery in tests.

Maintaining an air layer

Recall that in the early days, SHPo surfaces were considered promising for drag reduction because of their presumed ability to retain an air layer (plastron) between the surface and water even when fully submerged. The assumption was that this plastron would persist for as long as it was needed to keep the surface lubricated. The reality is not so simple, and plastron behaviour is currently the most critical issue in the field of SHPo drag reduction.

Unlike in air, where microscopic voids between microstructures or roughness in a SHPo surface stay dry even after temporary wetting by water droplets, a SHPo surface fully submerged in water will become wetted once it loses the plastron. And unfortunately, the plastron is easily lost underwater because hydrostatic pressure forces the surrounding water into the spaces between microstructures. The smaller the spaces, the more persistent the plastron – but as we have already noted, narrowly packed SHPo structures are not very slippery. We cannot have it both ways: careful quantitative studies have shown that SHPo surfaces capable of providing an appreciable (>10%) drag reduction for a boat simply cannot retain the plastron if the SHPo surface is submerged to a depth of more than a few centimetres. For most applications the surface would need to be much deeper than that.

> We now understand the potential as well as the limitations of SHPo drag reduction much better than we once did.

If the plastron will be lost for most applications, the principle of SHPo drag reduction won't apply to them either. Faced with this fundamental limitation, the only reasonable approach that is valid for all flow applications is to replenish the lost gas – ideally, using a method that is simple to implement and consumes a minimal amount of energy. Our lab is pioneering such an approach, but much still needs to be learned before it becomes practical for real-world applications.

Most drag-reduction research has been performed using SHPo surfaces with microstructures that are randomly rough. This is mainly because such surfaces are easy to fabricate: all you need to do is apply a commercially available SHPo spray coating. In contrast, SHPo surfaces with well-defined periodic microstructures, and thus a superior slip, would be simply too expensive to manufacture and implement on a ship hull – or so the discussion goes. The inconvenient fact, however, is that a surface with micro-

Advanced nanomaterials

scopic random roughness is simply not capable of providing enough slip to achieve meaningful drag reduction in most practical applications. A better approach would be to address this economic challenge by developing techniques to mass-manufacture SHPo surfaces that do have a chance of providing appreciable drag reduction.

Another reason why random SHPo surfaces continue to be studied, despite the well-established theory that indicates their severe limitations, is that one can also find many successful results reported in the literature. The reason for this apparent contradiction lies in the way drag reductions are tested. Most drag-reduction experiments have been performed in a water tunnel, in keeping with traditional flow experiments. But as my group confirmed recently, in flow tests using a water tunnel, the water quickly becomes supersaturated with air. In this supersaturated condition a very thick plastron forms on random SHPo surfaces, assisted by the few tallest rough protrusions. Naturally, one obtains a large drag reduction in these conditions: basically, you get the plastron you would expect from a SHPo surface with slender microstructures spaced far apart.

The problem, unfortunately, is that this thick plastron would disappear in open-water conditions, where the water is mostly undersaturated and tends to take the gas away. This dissimilarity between the water tunnel and open-water conditions is most likely why apparently successful lab studies have so far never been repeated in the real marine environment. In fact, the rare studies carried out in tow tanks actually reported an *increase* in drag, rather than a reduction, with random SHPo surfaces. The increase is understandable because high peaks of the random roughness will penetrate into the water once the plastron becomes thin, impeding the flow – rather like a coating of tiny barnacles.

On the other hand, a SHPo surface that is slippery enough to produce appreciable drag reduction in macroscale applications (such as a boat) is difficult to test in open-water conditions. These types of SHPo surface have large spaces between microstructures so they lose the plastron easily, and since they must be fabricated using MEMS technologies, it is difficult to manufacture the relatively large samples (more than $1\ m^2$) used for open-water tests. Actual boats are, of course, even bigger, and their hulls are curved. But these challenges are practical rather than fundamental. By developing ways to get around them, rather than confronting them head on (which will take much longer), my lab is currently performing experiments using a motor boat that replicates field conditions as closely as possible.

Clear water ahead?

After nearly two decades of research we now understand the potential as well as the limitations of SHPo drag reduction much better than we once did. We can predict how slippery a SHPo surface is and how much drag reduction is possible at a given flow condition. Although most of our understanding deals with laminar flows, we can extrapolate this to a certain extent for turbulent flows, including open-water conditions. My group has very recently obtained a 30% reduction with a boat in ocean water. This result is preliminary, but its unprecedented success is grounded in the extensive body of scientific knowledge summarized briefly in this article. It is, perhaps, a peek into the future of marine transport.

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News & Analysis

First black hole images unveiled

The Event Horizon Telescope has taken the first-ever images of a black hole, a breakthrough that lets astronomers study the event horizon of supermassive black holes. **Michael Banks** reports

The first direct visual evidence of a black hole and its "shadow" has been revealed by astronomers working on the Event Horizon Telescope (EHT). The image is of the supermassive black hole that lies at the centre of the huge Messier 87 galaxy, in the Virgo galaxy cluster.

Located 55 million light-years from Earth, the black hole has been determined to have a mass 6.5 billion times that of the Sun, with an uncertainty of 0.7 billion solar masses. Although black holes are inherently invisible because of their extreme density and gravitational field, the researchers have managed to obtain images near the point where matter and energy can no longer escape – the event horizon.

"We are giving humanity its first view of a black hole – a one-way door out of our universe," says Sheperd Doeleman of the Haystack Observatory at the Massachusetts Institute of Technology (MIT) who is the EHT's lead astronomer. "This is a landmark in astronomy, an unprecedented scientific feat accomplished by a team of more than 200 researchers." Doeleman says that the result would have been "presumed to be impossible just a generation ago", adding that breakthroughs in technology and the completion of new radio telescopes over the past decade have allowed researchers to now "see the unseeable".

"General relativity has passed another crucial test," adds theorist Avery Broderick from the University of Waterloo, Canada, who was involved with interpreting the results from the EHT. "Seven complimentary windows have now opened on black holes. Science fiction has become science fact."

The EHT results have been published in six papers in a special issue of *Astrophysical Journal Letters,* which is published by the Institute of Physics on behalf of the American Astronomical Society. All the papers are free to read (ow.ly/Ajwa50pSHpO).

EHT Collaboration

Seeing is believing The first-ever image of a black hole is of the one that lies at the centre of the Messier 87 galaxy. Taken on 11 April 2017, the image shows a bright ring, which is the effect of photons being warped as they travel around the black hole. The central dark spot is the so-called "shadow" of the black hole, in which lies the event horizon.

Discs of glowing gas

Supermassive black holes are thought to lie at the centres of most galaxies in the universe, and astronomers are keen to decipher their key properties – such as how their extreme gravity affects the space–time around them, and how some of them fuel the massive jets of material that spew out from the galaxies that host them. A key feature of a black hole is its event horizon – the boundary at which even light cannot escape its gravitational pull, as the velocity required to do so would be greater than the speed of light, which is forbidden by Einstein's general theory of relativity. And while that theory has passed many tests, researchers want to see how well it holds up at the "ultimate proving ground" – a black hole's edge.

Despite their name, black holes are not, however, all dark. The gas and dust trapped around them in an accretion disc is so compact that it is often heated to billions of degrees even before the matter eventually succumbs to the black hole, making them glow brightly. Indeed, general relativity also predicts that a black hole will have a "shadow" around it, measuring around three times larger than the event horizon. The shadow is of great interest as its size and shape depend mainly on the mass and – to a lesser extent – on any possible spin of the black hole, thereby revealing its inherent properties.

"If immersed in a bright region, like a disc of glowing gas, we expect a black hole to create a dark region similar to a shadow – something predicted by Einstein's general relativity that we've never seen before," says Heino Falcke from Radboud University in the Netherlands, who chairs the EHT's science council. "This shadow, caused by the gravitational bending and capture of light by the event horizon, reveals a lot about the nature of these fascinating objects."

Experimental tour-de-force

To directly observe the black hole at the centre of Messier 87 – dubbed M87* – astronomers require a telescope with an angular resolution comparable to the event horizon, which is on the order of tens of microarcseconds across. But to achieve that resolution with an ordinary telescope – which is like spotting an orange on the surface of the Moon – would require a dish the size of our planet, which is clearly impractical.

EHT astronomers instead use the radio-astronomy technique of very-long-baseline interferometry (VLBI). It involves picking up radio signals from an astronomical source by a network of individual radio dishes and telescopic arrays scattered across the globe. The EHT, which first turned on in 2007, consists of eight radio telescope observatories in six different locations across the globe all operating at a wavelength of 1.3 mm. These telescopes include the Atacama Large Millimeter/submillimeter Array (ALMA) in Chile, the South Pole Telescope (SPT) in Antarctica and the IRAM 30 m telescope in Spain. The distance between individual EHT telescopes – known as the "baseline" – ranges from 160 m to 10 700 km.

The signals received at each individual telescope dish in the network are precisely tagged with a very accurate time stamp, normally using an atomic clock at each location. Each telescope produces roughly 350 terabytes per day, which is stored on high-performance helium-filled hard drives. The data are later correlated and used to build up a complete image by supercomputers that are located at the Max Planck Institute for Radio Astronomy in Bonn, Germany, and the MIT Haystack Observatory in the US. The development of one of the algorithms used to crunch all the data and produce an image of the black hole was led by MIT graduate student Katie Bouman, whose efforts were widely reported last month.

This combination of multiple radio telescopes and computer algorithms makes the EHT the highest-resolution instrument on Earth – capable of taking images up to 2000 times better resolution than the Hubble Space Telescope and able to resolve features as small as 20 micro-arcseconds.

From theory to reality

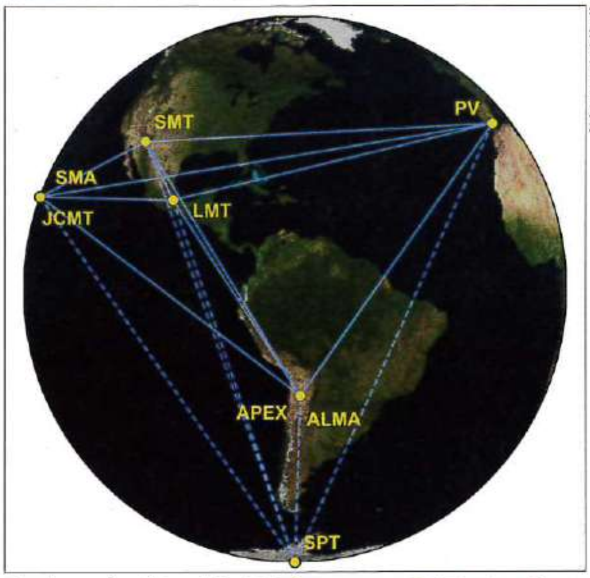
As a black hole's size is proportional to its mass, the more massive a black hole, the larger its shadow. Thanks to its enormous mass and relative proximity, M87* was predicted to be one of the largest viewable from Earth – making it a perfect target for the EHT. Astronomers observed M87* on 5, 6, 10 and 11 April 2017, with the telescope taking a series of scans of three to seven minutes in duration each day.

These multiple independent EHT observations have now resulted in the first image of a black hole including its shadow, revealing a ring-like structure with a dark central region. The diameter of the ring is 42 micro-arcseconds with a width less than 20 micro-arcseconds. EHT scientists also deduced the radius of the event horizon as 3.8 micro-arcseconds. The size of the black hole is around 40 billion kilometres across – slightly larger than our solar system.

By comparing the image with theoretical models such as general relativistic magnetohydrodynamic simulations, the observed image is consistent with expectations for the shadow of a Kerr black hole – one that is uncharged and rotates about a central axis – as predicted by general relativity. The EHT team also found that the rotation of the black hole is in a clockwise direction. The brightness in the lower part of the image is due to the relativistic movement of material in a clockwise direction as seen by us, so that it is moving towards us.

The researchers were able to deduce the mass of the M87* at 6.5 billion times that of the Sun. Previous estimates – based on models as well as spectroscopic observations of the galaxy by the Hubble Space Telescope – ranged between 3.5 and 7.7 billion solar masses. "Once we were sure we had imaged the shadow, we could compare our observations to extensive computer models that include the physics of warped space, superheated matter and strong magnetic fields. Many of the features of the observed image match our theoretical predictions surprisingly well," says Paul Ho, director of the East Asian Observatory and an EHT board member. "This makes us confident about the interpretation of our observations, including our estimation of the black hole's mass."

Sera Markoff from the Anton Pannekoek Institute for Astronomy at the University of Amsterdam says that the EHT results have now ended the "long controversy" over the mass of M87*. "Our determination of the mass lands right on top of the estimates, so this can now lead to better estimations of the mass of more distant black holes where we actually can't see the shadow."



Akiyama et al./ApJL

Global networking The Event Horizon Telescope combines the signals of eight radio telescope observatories including the Atacama Large Millimeter/submillimeter Array (ALMA) in Chile and the South Pole Telescope (SPT) in Antarctica.

> This is a landmark in astronomy, an unprecedented scientific feat

As well as revealing the properties of M87*, the EHT has lifted a veil on the event horizon, showing that it is now possible to experimentally study the region via electromagnetic waves. This, the researchers write, has now transformed the event horizon from a purely "mathematical concept" to a "physical entity". Gopal Narayanan from the University of Massachusetts at Amherst, who built the spectroscopic imaging instruments and led the team that constructed two radio astronomy receivers at the Large Millimeter Telescope in Mexico, says that the finding is "a shot in the arm to theorists" who will now be able to test their theories with experimental data. "It's very gratifying and immensely exciting to see the results coming out after years of work," he adds. "At times it looked like an impossible task. But we showed that you can collaborate on this scale and get results. The camaraderie and team spirit was a wonderful thing to see."

Astrophysicist Rob Fender from the University of Oxford, who is not part of the EHT collaboration, says that the first production of radio images with a resolution comparable to the angular size of a black hole event horizon is a "major breakthrough" in high-energy astrophysics. He adds that the EHT observations are our best look yet at the region where the jet of the black hole is formed. "The region close to the black hole, just above the event horizon, is the site of much of the most extreme astrophysics in our universe since the Big Bang," he says. "These jets carry an enormous amount of energy away from the central black hole, via processes that are not well understood."

Building on success

While this latest result is the biggest yet from the EHT, it is not the first to come from the collaboration. In 2012 scientists working on the array managed to observe, for the first



Akiyama et al/ApJL

time, the base of the jet emanating from the M87 galaxy. The work established that the black hole at the heart of M87 is spinning and that the accretion disc follows the direction of spin. Three years later, researchers on the EHT measured the first direct evidence of magnetic fields near the event horizon of Sagittarius A* – the black hole at the centre of our Milky Way galaxy lying around 26 000 light-years away but with a mass around three orders of magnitude smaller than M87*. By studying the right- and left-handed circular polarization of the incoming radio waves, they were able to infer the direction of linear polarization that traces the magnetic field, finding that it even changed on a daily basis and revealing the extreme dynamics at play at the heart of the black hole.

Astronomers now hope to carry out further observations of M87* to deduce the shape and depth of the shadow region more accurately. They are also hopeful to add more telescopes to the array that will allow for higher-resolution images. As well as M87*, the EHT team is attempting to take the first image of Sagittarius A*. But this is more difficult to resolve – despite being nearer – because it is more dynamic than M87*, changing on the scale of minutes rather than days.

Shadowlands From left to right, Event Horizon Telescope observations of M87* taken on 6 April 2017; a simulation of M87*; and a simulation convoluted to the resolution of the EHT.

Space

Israeli firm fails in private Moon landing

The first effort by a private organization to land a spacecraft on the Moon has ended in failure after contact with the Beresheet lander was lost just minutes before it was due to touch down. Had SpaceIL's craft landed successfully last month, it would have made Israel the fourth nation to reach the Moon, after the US, the Soviet Union and China.

"If at first you don't succeed, you try again," noted Israeli prime minister Benjamin Netanyahu from the control room, moments after the crash was announced. "Don't stop believing! We came close but unfortunately didn't succeed with the landing process," said the SpaceIL team via Twitter.

Beresheet was aiming to touch down within the *Mare Serenitatis* (Sea of Serenity), which lies in the northern hemisphere of the Moon. Designed to inspire an Israeli "Apollo Effect", the mission was closely followed across the world despite falling at the final hurdle.

"Beresheet offers inspiration to all those who are fascinated by space exploration but feel it is too hard or too expensive," said mission scientist Oded Aharonson from the Weizmann Institute, Israel, before the attempted landing. "In addition to learning something new about the Moon, it also offers a unique opportunity for educating young people in subjects of science and technology."

SpaceIL/IAI

Close encounters Beresheet takes a selfie during lunar landing before losing contact with mission control and crashing into the Moon.

Founded in 2011, SpaceIL is an Israeli not-for-profit organization that originally sought to win the Google Lunar XPRIZE – to build, launch and land an unmanned spacecraft on the Moon. Although Google's competition ended on 31 March 2018 with no winners, the team pushed on, making up the required $100m budget with support from philanthropists and the Israel Space Agency.

Beresheet, meaning "Genesis" in Hebrew, launched on 22 February this year on a Falcon 9 rocket. The Beresheet craft weighed 585 kg at launch and initially was put into a high-Earth orbit before being captured into the Moon's orbit on 4 April. On 11 April it began the landing process but shortly afterwards mission controllers lost contact with the mission's telemetry system before also experiencing an issue with its engine. The craft then crashed into the Moon's surface at high speed.

Not everything had been plain sailing for the SpaceIL team even before its fateful final minutes. During its journey to the Moon, Beresheet's onboard computer system crashed several times and its navigation system was blinded by the intensity of light from the Sun. "You run into the unexpected so you always have to have a plan B or some reserves because you will use that eventually," admitted Chris Russell, member of the SpaceIL Science team, ahead of the landing.

While the craft would have studied the Moon's magnetic field via a magnetometer, Beresheet's biggest legacy, however, would have been its status as the first private mission to land on our closest neighbour. In a statement after the crash, NASA nevertheless offered its congratulations for SpaceIL having sent the first privately funded mission into lunar orbit. "Every attempt to reach new milestones holds opportunities for us to learn, adjust and progress," wrote NASA head Jim Bridenstine. "I have no doubt that Israel and SpaceIL will continue to explore and I look forward to celebrating their future achievements."

James Dacey

Weighing water from space

By monitoring tiny changes to the Earth's gravitational field, the GRACE satellites have been pinpointing the distribution of fresh water on our planet for almost two decades. But as **Marric Stephens** explains, a new follow-on mission is also helping with plans for a space-based gravitational-wave detector

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Imagine the sea on a still day, calmer than you have ever seen it, with no wind to stir its surface, and no currents or tides to disturb its depths. Now imagine that the sea has risen to cover the whole face of the planet, submerging the continents and even the highest mountain peaks. What you are seeing approximates the "geoid" – a surface that joins all of the points on the Earth where the strength of gravity is the same. The geoid is the level that a hypothetical global ocean would attain in the absence of forces such as tides, winds and currents, influenced only by gravity and the rotation of the Earth.

You might expect the surface of such an ocean to be a nearly perfect sphere, albeit bulging a little at the equator due to centrifugal forces arising from its rotation. However, this hypothetical ocean is not uniform. Given that the gravitational field varies slightly across the Earth, this ocean would – in finding its natural level – flow towards areas where gravity is strongest. It would pile up over the heavy spots to produce a watery planet with a lumpy, undulating surface.

With only a few hundred vertical metres between the highest and lowest points of this ocean, it would still look remarkably featureless to the naked eye. Exaggerate the scale, however, and you would see a seascape that mirrors the geology and relief of the planet beneath. That's because the varying field strength is largely to do with the presence of topographic highs and lows, and with density differences in the crust and upper mantle. You would notice, for example, the ocean surface rising over the Andes and Himalayas – where gravity is enhanced by the huge thicknesses of continental crust – and sinking over the Indian Ocean, under which the material of the mantle appears to be unusually light.

But not all of the matter that matters is in rocks. The geoid is also shaped by more transient effects like the phase of the Moon, the motion of the oceans, and the state of the hydrosphere. Overlaid on the relatively static gravitational signature of mountain ranges, ocean trenches and deep geological features, there is therefore a constantly fluctuating signal that reveals how mass is redistributed about the Earth by dynamic processes. One of the most important contributors to this signal arises from the changing disposition of the Earth's water on a regional and global scale.

Overlaid on the relatively static gravitational signature of mountain ranges, ocean trenches and deep geological features is a fluctuating signal that reveals how mass is redistributed by dynamic processes

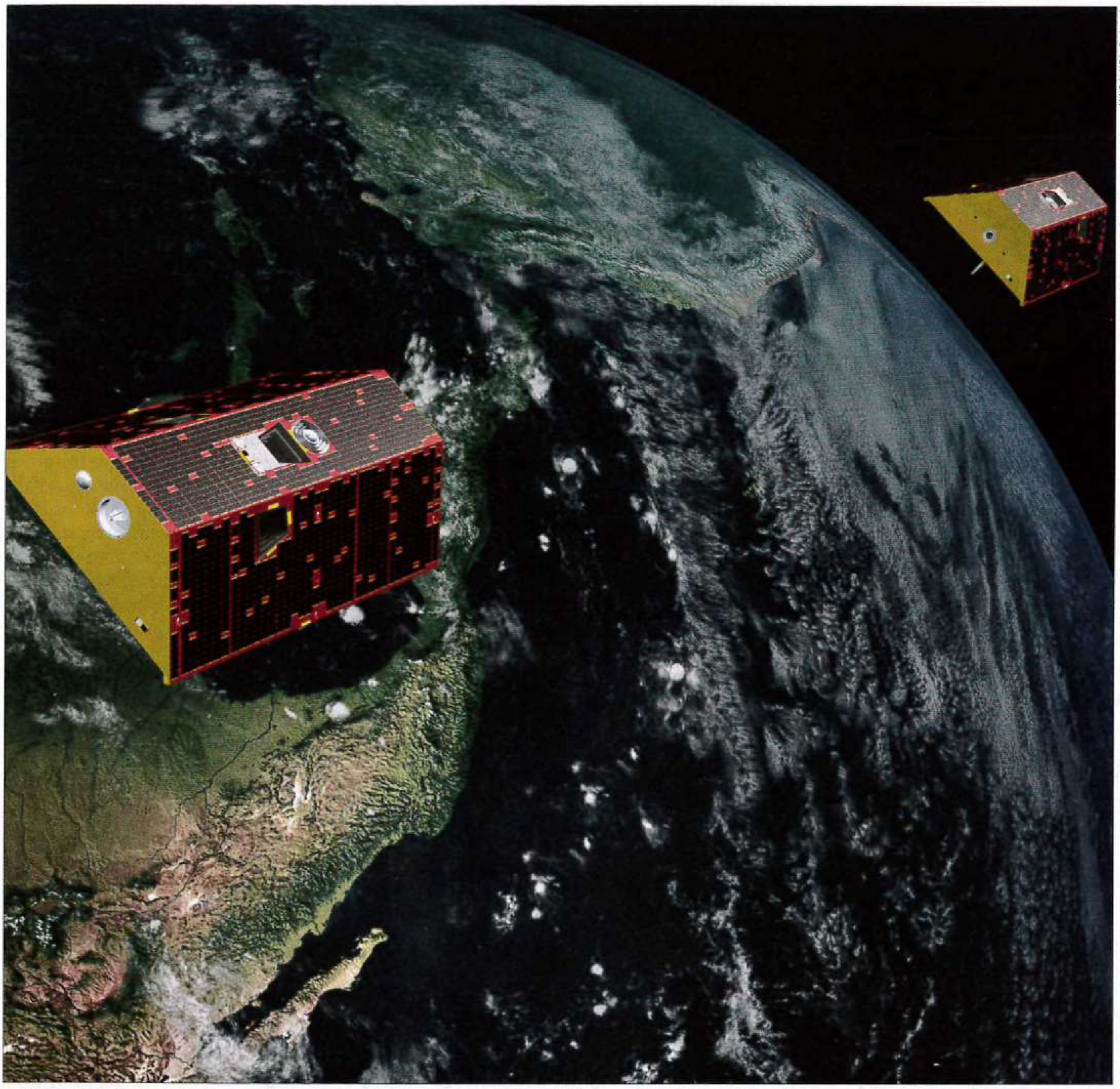
A state of GRACE

Detecting the impact of water on the geoid was one of the main aims of a space mission called the Gravity Recovery and Climate Experiment (GRACE), which was launched in 2002 by NASA and the German Aerospace Center (DLR). Its role was to map the geoid with enough sensitivity to observe tiny variations in mass distribution over months and years – the sort that allow scientists to monitor changes in sea level, ice caps and water stored on land.

Running for more than 15 years, the experiment consisted of two identical satellites, GRACE-A and GRACE-B, both orbiting the Earth's poles at an initial altitude of 490 km (although the height decayed gradually over the course of the mission). The two satellites did not fly together – instead, one orbited between 180 and 220 km ahead of its twin. The precise separation depended on various factors, including solar-radiation pressure, atmospheric drag, and any occasional forced adjustments to the crafts' trajectory to make them dodge any space debris.

However, all of these factors were just "noise" to be filtered out. The really important influence on the inter-satellite distance – the force that the experiment was designed to measure – was gravity, which increased or decreased as the spacecraft crossed contour lines on the Earth's geoid. If, say, the satellites' orbit carried them towards a mass concentration in the Earth below, the leading spacecraft would be the first to feel the greater tug of gravity, which would perturb its trajectory and extend the separation minutely but measurably. When the pair had moved on so that the mass concentration was between them, it would now be the trailing satellite's turn to feel the extra tug, while the leading satellite was pulled back in its orbit (figure 1).

Circling the Earth 16 times a day and achieving near-total global coverage every 30 days, the GRACE satellites – month by month and year by year – generated a map of where the two craft were pulled apart and where they bunched up. Mission sci-



Second step
Artist's rendering of the twin spacecraft of the Gravity Recovery and Climate Experiment Follow-On (GRACE-FO) mission, which launched in May 2018. Like its successful predecessor, GRACE-FO can monitor tiny changes in the distribution of mass on Earth.

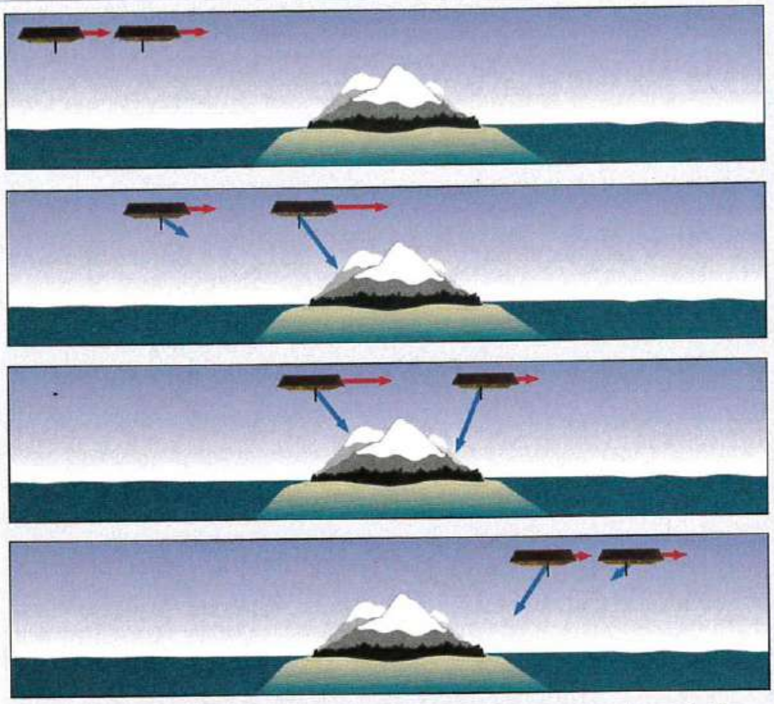
entists were then able to translate these data into an increasingly detailed picture of the planet's gravitational field (figure 2).

The initial sketch of the mean gravity field produced by the experiment depicted the Earth's large-scale geological structure: mountains, by definition, are big, and the gravitational signatures of such features were correspondingly easy to identify. Tracking the relatively small month-to-month changes in how water is distributed over oceans and continents, on the other hand, was a harder proposition.

The general locations of the satellites were measured using the Global Positioning System (GPS). But spotting the variation in satellite separation – and therefore gravity – associated with, for example, a depleted aquifer or a rising lake, required a sub-micron precision that GPS could not provide. Instead, the experiment sent a microwave beam from one satellite to the other, where it interfered with a reference beam on that craft. Changes in the inter-satellite distance would alter the relative phase of the two beams, revealing the shift as a change in the interference pattern.

Ascertaining the satellites' relative motion so precisely was only half the challenge, however. Also crucial was to carefully account for all the other, non-gravitational sources of perturbation, which would otherwise have overwhelmed the gravity signal. At the initial 490 km orbit height, the biggest of these effects was solar-radiation pressure, which fluctuated constantly as the satellites passed in and out of the Earth's shadow. This also created a cycle of warming and cooling, requiring sensors to measure each satellite's thermal expansion. Without this information, any change in the dimensions of one of the spacecraft could have been mistaken for a gravi-

1 A guide to the GRACEs



Adapted from an original by NASA/JPL-Caltech

Like the original GRACE project, the GRACE Follow-On mission measures changes in the gravitational pull (blue arrows) on the two craft resulting from changes in Earth's mass below the orbiting satellites, which can be due, for example, to variations in sea level, ice caps or the amount of water stored on land. As the satellites orbit the Earth, one following the other, these moving masses alter the gravitational pull, changing the distance between the craft very slightly. Separations have been exaggerated for effect.

tational effect on the distance between them.

Another significant non-gravitational force that had to be accounted for as the mission progressed was aerodynamic resistance on the GRACE satellites from the Earth's tenuous upper atmosphere. As with all orbiting bodies, there was a positive feedback effect: the drag made the craft lose height, which in turn made them travel through a denser atmosphere, which led to more drag and so on. Indeed, by the end of 2017 when the two craft finally stopped operating, the satellites were orbiting barely 300 km above the Earth's surface.

To compensate for all of these non-gravitational forces, each satellite had at its centre a 50 g "proof mass" suspended electrostatically within a cage. Electrodes in the walls of this container corralled the mass to within 30 μm of the satellite's centre of mass, and simultaneously measured any displacement. Non-gravitational forces applied to the satellite body deflected the proof mass relative to its cage, whereas gravitational perturbations affected the entire set-up equally. Any change in distance that could not be attributed to some non-gravitational force on one of the satellites was therefore taken to be an indicator of unevenness in the geoid.

Watery changes

With the confounding effects removed, the precision of the distance measurement meant that GRACE was sensitive to changes in strength of gravity on the order of a few microns per second squared, or less than a millionth of the value at the surface (roughly $9.81\,\text{m}\,\text{s}^{-2}$). Members of the GRACE team could therefore spot changes in gravitational strength caused by the water level in a large lake or aquifer rising or falling by as little as one centimetre. Every month, they compiled such changes in the Earth's mass distribution and made the updated map available to researchers around the world.

One of those scientists is Matthew Rodell, head of the Hydrological Sciences Laboratory at NASA's Goddard Space Flight Center in Maryland, US, who has used these data to map the changing availability of fresh water across the planet, including regions that would otherwise be hard to access. "GRACE revealed and quantified groundwater depletion associated with irrigated agriculture in northern India, the North China Plain and parts of the Middle East, among others," says Rodell.

Writing in a recent paper in *Nature* (**557** 651), he and his team quantified dozens of global trends in freshwater distribution that are expected to affect food and water security in coming decades, and that could spark conflict if not managed carefully. The changes were a mix of natural, climate-change-related and directly human-caused effects, and the work simultaneously captured processes as diverse as ice-cap loss in Greenland and Antarctica, groundwater extraction in the Middle East, and the damming of rivers in China.

This ability to spot links between regional trends, which is a vital part of GRACE's observations, is illustrated by a discovery made by Rodell's colleagues at the Jet Propulsion Laboratory (JPL) in California (*Geophysical Research Letters* **39** L19602). They found that the global mean sea level, which had previously been steadily rising by about 3 mm a year, suddenly dropped between 2010 and 2011 by 5 mm. Carmen Boening from JPL, who began pondering the puzzle with collaborators at JPL, the National Center for Atmospheric Research and the University of Colorado at Boulder, wanted to know if the drop was related to ocean cooling or whether there was simply less water in the ocean.

Using GRACE data, Boening and her colleagues found that the sea-level fall really was due to missing mass, and that it was balanced by a corresponding increase in water stored on land. It turned out that the onset of La Niña conditions in the Pacific – a cyclical variation in ocean-surface temperature – had caused so much rain over Australia, south-east Asia and northern South America that the oceans were temporarily depleted, and the continents made measurably more massive. By 2012 the effect had ended and the rising trend, associated with global warming, had resumed. "So by using GRACE to weigh the ocean, we confirmed that there was less water there, which must have moved to the continents," Boening says.

Following the trend

The two satellites making up the original GRACE mission were designed to last for just five years, with operations scheduled to end in 2007. Sensitive electronics can only take so much radiation and thermal cycling before they start to break down, and the spacecraft had no means to maintain their orbits in the face of aerodynamic drag. However, in a fashion familiar to fans of NASA's Mars rover fleet, the GRACE scientists over-delivered, and the mission kept returning data until late 2017, shortly before the two satellites fell from orbit and burned up in the Earth's atmosphere.

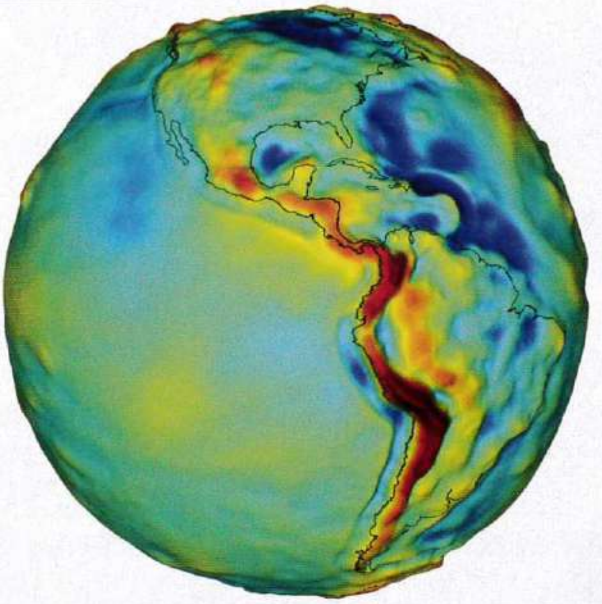
But the mission is not yet over. Recognizing the importance of obtaining further measurements, in 2011 NASA initiated a successor to GRACE, known as the GRACE Follow-On (GRACE-FO). With the US National Academy of Sciences also recognizing in its 2017–2027 Decadal Survey for Earth Science and Applications from Space that mass-change measurements are vital for tracking long-term trends in the hydrosphere, GRACE-FO was duly launched in May 2018 from the Vandenberg Air Force Base in California. To ensure continuity, GRACE-FO essentially duplicates the original experiment, but with a few minor improvements derived from lessons learned along the way.

The most novel aspect is the addition of a technology-demonstration instrument – a laser interferometer for measuring the inter-satellite separation. Developed by researchers from JPL as well as the Max Planck Institute for Gravitational Physics in Hannover and the Leibniz Universität Hannover, the device works on the same principle as the microwave-based method used by the original GRACE craft. It should, however, deliver a precision a hundred times greater because of the beam's shorter wavelength. "It's a really incredible piece of technology," says GRACE-FO project scientist Frank Webb from the JPL. "The sensitivity limit is at the hundreds-of-picometres level, which is about half the size of a water molecule – over a distance of 200 km."

In the current set-up, other sources of error are too large to do justice to the new instrument, so the increased precision cannot be fully utilized. Indeed, during routine operations, GRACE-FO will employ the same microwave-based method used by the original satellites. The laser interferometer is intended only as a validation of the technology for future missions, which will be able to make better use of the device to measure mass changes on the ground with greater accuracy and resolution. Even so, Webb will be surprised if scientists do not find some way to relate test data from the new device to mass change on the Earth. "The scientists are pretty clever, and they should be able to tease out a little more information from this new capability," he says.

GRACE-FO is slated to work for five years, but whether it can continue working for as long as its predecessor will depend in part on the strength of the next cycle of solar activity, which will start in late 2019. When the Sun is especially active, the increase in emitted ultraviolet radiation adds energy to the Earth's upper atmosphere. This makes the atmosphere "puff up", increasing the drag in low Earth orbit and accelerating orbital decay. The craft would then burn up, as its predecessors did, albeit much sooner. "A strong solar cycle will push the satellites lower, earlier," Webb says.

2 Gravitational highs and lows



NASA's Goddard Space Flight Center

A map from the GRACE mission of gravity across Earth. Red shows areas where gravity is stronger than the standard "idealized" value and blue where it is weaker. The mission was so sensitive that it could monitor month-by-month changes in the distribution of water across the planet.

Follow on following on

Not content with GRACE-FO as a successor to GRACE, there are already plans for a follow-on to the follow-on. These missions will not only continue the geoid observations, but will also incorporate design changes that minimize uncertainty from other quarters, such as the sensitivity of the accelerometer, letting scientists make full use of the increased resolution afforded by the laser interferometer. This will let researchers measure even smaller mass changes over finer spatial scales, potentially revealing additional trends not glimpsed by the satellites launched so far.

And while GRACE-FO – and its successors – will measure changes in the Earth' gravity field, a similar device could be used by another mission to observe gravitational signals from beyond. That's because the successful demonstration on GRACE-FO is a major milestone in the development of the European Space Agency's Laser Interferometer Space Antenna (LISA). Planned for launch in 2034, LISA will comprise three satellites arranged at the corners of a triangle 2.5 million kilometres on a side. Circling the Sun far from the noisy environment of low Earth orbit, the interferometer used on this mission will be able to detect a change in distance of just picometres – sufficient to spot the infinitesimal flexing of space due to the passage of gravitational waves from across the universe.

Results from the GRACE-FO demonstration will provide a practical lesson on how to operate, diagnose and, if necessary, debug the instrument after launch. Members of the LISA team – many of whom also work on GRACE-FO – will be watching keenly in preparation for when they turn their gravitational gaze outward from the Earth.

Supporting the professionals

Marc Delcroix says that amateur astronomers can play a key role when it comes to future planetary missions


Anthony Wesley

Impact zone It was once thought that objects hit Jupiter's atmosphere every 100 years, but in 2009 Australian amateur astronomer Anthony Wesley helped turn that view on its head.

Collaboration between professional and amateur astronomers has a long and successful history. Take Jupiter. In early 1994 scientists discovered that the Shoemaker–Levy 9 comet was fragmented and they observed parts of it hitting the giant planet in July that same year. It was an impressive series of collisions leaving traces visible for months. Many professional astronomers believed they had witnessed a once-in-a-lifetime event, something that would only be observed once in a century, or possibly longer.

In 2009, however, Anthony Wesley, an Australian amateur astronomer, saw an unusual small dark patch during his observations of Jupiter. He accurately identified it as the trace of an impact in the planet's atmosphere and alerted the professional and amateur communities, who later proved him right. A year later, he and other amateurs observed two flashes on Jupiter – a tell-tale sign of a smaller body hitting the planet's atmosphere – and found further impacts in 2012 and 2016.

All these discoveries were observed simultaneously by several amateurs, demonstrating the large observation coverage that they provide. The discovery has now led scientists to ponder the frequency of such impacts and to refine their model of small-body distributions, which could yield better estimates of the age of crater-hit bodies. This issue may well again be answered by amateur observations and analysis.

This remarkable progress has been due to the equipment that amateur astronomers use to observe planets in our solar system becoming ever more complex in recent years as technology has developed. In the 1990s amateurs used charged coupled devices but a decade later they were already using simple webcams to film planets. During this time, amateurs began to use techniques to image planets with a quality equal to that professionals had produced 20 years before. With imaging-sensor development driven by a huge consumer market and many industrial applications, the worldwide community of amateur astronomers now routinely uses top-quality yet affordable cameras to film planets. With their own advanced processing software, amateurs can reach the limit of their 20–40 cm telescopes and produce high-resolution images at visible wavelengths, detailing fine atmospheric phenomena.

Amateurs are estimating wind speeds, confirming cloud detections and presenting their results at professional meetings

Professional observations, on the other hand, are driven by ground-based 10 m telescopes that work in the infrared. These are mostly used to observe distant objects and are rarely used to study Earth's neighbouring planets. The best images of planets are produced by space-based instruments, such as the Hubble Space Telescope (when they are aimed at them, that is), while fly-by missions or orbiters usually focus on specific targets such as a planet's satellites, rings or magnetic field. This is where the two communities can come together: with plenty of time on their hands, amateurs can provide regular, high-quality images of planets, while professionals can call on them for support whenever they want to study a specific phenomenon.

This type of collaborative working has arisen over the last 10 years, which has seen the number of publications co-written by planetary science professionals and amateurs shoot up by a factor of 20. Every new atmospheric point of interest – be it the reddening of ovals in Jupiter's atmosphere, storm cloud sources on Saturn, or even Uranus and Neptune's cloud activity – has been studied not only from professional telescopes but amateur ones too. Amateur observations are also increasingly being used by researchers to justify bigger telescopes spending time on such sources to deliver more accurate observations.

Leading amateur astronomers are analysing their colleagues' images, estimating the wind speeds on the upper gaseous planet atmospheres, confirming cloud detections and presenting their results at professional meetings. Some are also developing advanced software for processing or analysing their own images to create equivalent tools to those professionals have used. Some lucky amateurs even have access to 1 m professional telescopes that they use for their own observations.

These and other endeavours by amateur astronomers have been recognized by planetary scientists, such as NASA's call for them to support the Juno mission to Jupiter when it arrives at the planet on 4 July. A workshop held in Nice in May – attended by 33 amateurs and senior professional astronomers – involved discussions about how to work best together on the mission and study Jupiter in general. The plans include high-resolution amateur observations of Jupiter's atmosphere during the mission to help determine what features the Juno camera will aim at. Another general target for amateurs is to continue looking for impacts on Jupiter to improve our knowledge of such events.

The contribution that amateurs play in astronomy is clear. An amateur astronomer observing planets can easily contribute by sharing observations on existing databases, user groups and on the Internet, or participating in specific observations or analyses. What a wonderful time it is now for amateurs to see their childhood passion turn into real science. For them it must feel like Christmas every day.

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Critical Point See like a solar system

Robert P Crease discusses what the science of the solar system teaches us about perception

Many years ago I read a news item in which a scientist said that a sodium cloud issuing from a volcano on Jupiter's moon Io was "the largest permanently visible feature in the solar system" (*Science News* **137** 359).

That remark stopped me cold. What does it mean to "see" a sodium cloud? More generally, what do scientists mean when saying they see dark matter or black holes? Are they speaking precisely or metaphorically? What is perception?

Questions like these were a big factor in attracting me to the philosophy of science. Perception, I decided, isn't as easy as it looks. To be a scientist is to develop an extended ability to perceive – and the science of the planets in our solar system is replete with examples.

Seeing like a rover

When scientists say they see things like sodium clouds, they speak rigorously. To perceive is not just to grasp something somewhere from a single perspective. It is also to have a sense, however rudimentary, of how that thing looks from other perspectives. Whenever I see a cup, I see only one profile of it. But thanks to our earlier experiences with cups, to see something as a real cup – rather than as a cutout or hallucination – means to anticipate other profiles; how it'll appear if I walk around it, pick it up and so on. Sometimes these profiles surprise us, or we turn out to be deceived or wrong, but to perceive is always to grasp a profile of something and have a set of expectations about other anticipated profiles. Perception, in short, has a deep structure.

The same is true for a space scientist's perception, except that it is technologically mediated. In philosophical language, scientists sometimes "embody" their instruments, seeing the world through them relatively directly, just as a blind person sees the world through a cane. When we perceive a planet or comet through an optical telescope, for instance, our previous experiences make us expect the object to be visible at other times in other locations – and that when observed through stronger telescopes it will have profiles that we might not know but that we can guess. At other times, scientists don't embody but "interpret" their instruments. Just as we say "it's cold outside" by looking at a thermometer, so a space scientist "sees" a sodium cloud with filters and spectrometers if these belong to expected profiles.

NASA/JPL

Eyeing up How do we use instruments like a NASA Mars rover to "see" other planets?

To be a scientist is to develop an extended ability to perceive

Astronomical perception involves a complex combination of these two concepts of embodiment and interpretation. An interesting case study is found in the 2013 article "Mediating Mars: perceptual experience and scientific imaging technologies" (*Foundations Sci.* **18** 75) by the philosopher Robert Rosenberger from the Georgia Institute of Technology, US. In it, he describes a debate about a rock formation imaged by NASA's Mars Global Surveyor in a Martian crater known as Eberswalde. Some scientists argued they were looking at the remains of a river delta, others an alluvial fan, still others that they were seeing the product of mudslide-like events.

Rosenberger shows that the scientists went about resolving the controversy, not by evaluating competing theories or explanations about the rock formation itself, but by appraising the different strategies that they were using to produce the images. They asked themselves, Rosenberger writes, "How does the process of transforming this object of study (i.e. a rock formation on Mars) into a specific form we are able to perceive here on Earth (i.e. images) leave these images open to interpretation in particular ways?" Without being able to move freely around the formation as they would on Earth, the scientists had to sharpen their perception of the rock formation by understanding the profiles better, and by analysing other profiles provided by shadows, laser altimeter data and so on.

Another analysis of scientific perception is found in *Seeing Like a Rover: How Robots, Teams, and Images Craft Knowledge of Mars* (2014 Princeton University Press) by the Princeton University sociologist Janet Vertesi. She based her book on two years spent as an ethnographer studying scientists in NASA's Mars Exploration Rover mission. Vertesi found the researchers' workspaces, computer screens and Powerpoints were saturated with images: filtered, false-colour, 3D, fish-eye, panoramic and more.

Taken by the two rovers Spirit and Opportunity, these images let the researchers "see" on Mars, but not with a human eye. One researcher told Vertesi that the two rovers' view of the world was like "trying to make your way through a dark cluttered room with nothing but a flashbulb". Yet the researchers became skilled at it, seeing and manipulating phenomena on the Martian surface. "When you work with the team for a while," another researcher told her, "you kind of learn to see like a rover."

Vertesi's book shows that seeing like a rover is not just a matter of grasping profiles and horizons, but also involves ways of speaking and gesturing, emotional connections, habits and even the research group's social and organizational structure. Seeing like a rover, she writes, "is…a question of seeing from *somewhere*, not adopting a view from nowhere" – with the "somewhere" referring not just to the rover's cameras but to the entire research team and its activity.

The critical point

The curse of current-day philosophy of science is the lingering but fraudulent idea among philosophers that science involves the quest to see phenomena from nowhere. Instead, it's done by people with inherited concepts using particular equipment to study topics that seem important. To perceive a scientific phenomenon involves grasping how all of the profiles you can see of it – and how others that you don't yet or never will see – hang together. Not only that, but mediated scientific perception deepens and extends our notion of what it is to perceive at all.

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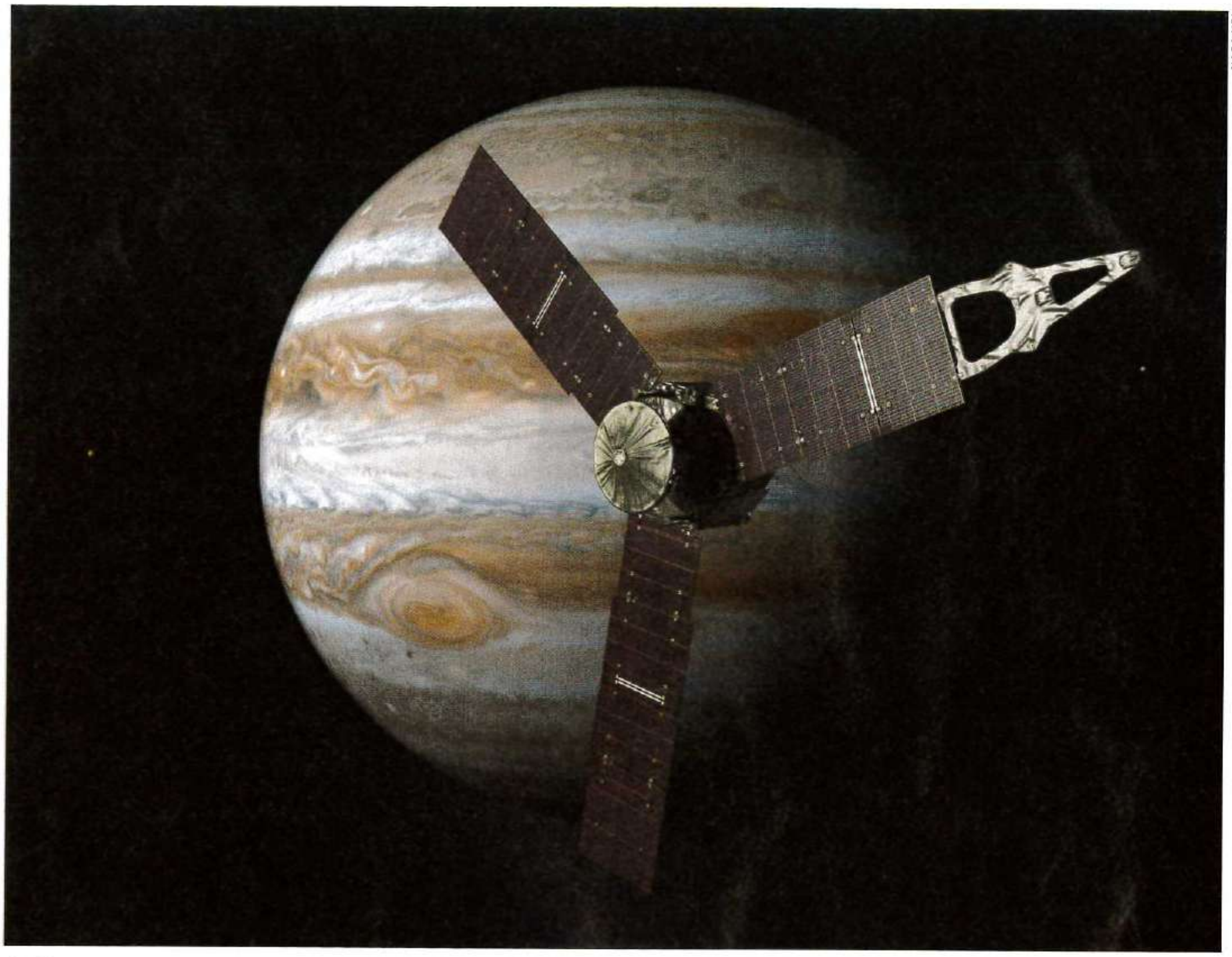


Brave new Jupiter

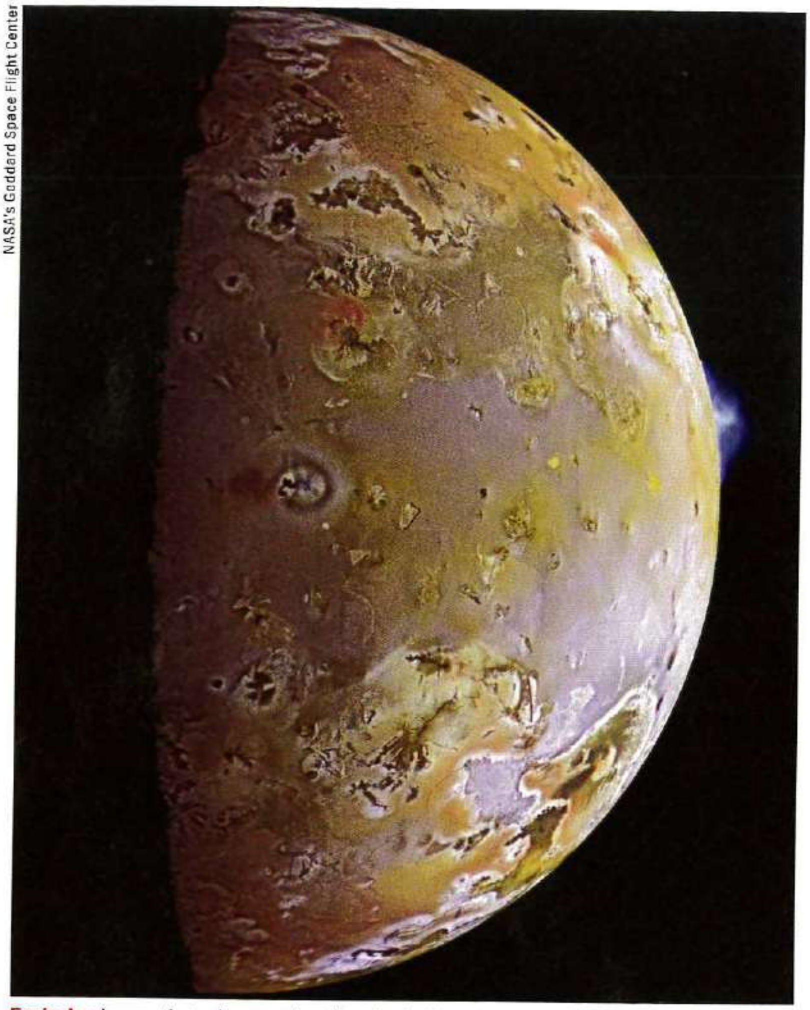
With NASA's Juno mission to Jupiter arriving this month, researchers look to our local, ancient behemoth to figure out how planets form – including our own – as **Stephen Ornes** reports

Stephen Ornes is a science writer based in Nashville, Tennessee, US, e-mail stephen@stephenornes.com

For the last five years, NASA's Juno spacecraft has been barrelling towards its final destination: Jupiter, king of the planets. On 4 July this year, the four-tonne, spinning craft – which looks like an oversized propeller that has abandoned its plane – will fire its thrusters and slow down enough to be captured by the gas giant's gravity. The burn should only last about 40 minutes, but they'll be a tense 40 minutes: during that time, as Juno shifts from orbiting the Sun to orbiting Jupiter, the rest of its devices will go quiet. (As will the physicists at NASA's Jet Propulsion Laboratory in Pasadena, California, tracking the mission from the ground and, one imagines, with fingers crossed.) Then the instruments should flicker back on, and over the course of more than 35 long, loping polar orbits throughout the following year, Juno will execute an intimate and unprecedented observation of our local colossus, beaming data back to Earth.

Scott Bolton, Juno's principal investigator, says the effort to study Jupiter is no less than a desire to understand the origin story of the solar system. Bolton is a space physicist at the Southwest Research Institute in San Antonio, Texas, and has led the \$1.1bn mission from idea to execution. "When you want to understand where we all came from and how the planets were made, you have to start with Jupiter," he says. Jupiter's composition seems more star than planet, as it is dominated by hydrogen, followed by helium. At the same time, its atmosphere is drizzled with the heavier elements – including carbon, nitrogen and oxygen – that make life on Earth possible. "Jupiter is enriched with the same stuff we're

NASA's Goddard Space Flight Center



Explosive An erupting volcano on Io, taken by the New Horizons mission.

made of," says Bolton. "We're trying to understand our own history."

Bolton's words echo a growing wisdom among astronomers: if you want to know details about how the solar system formed, or how giant, gassy worlds coalesce around far-flung stars, you have to ask Jupiter. It likely formed early and fast, sweeping up most material left behind after the Sun formed. Jupiter is more than twice as massive as all the other planets, moons, asteroids, comets and Kuiper-belt objects in our system combined. Because of its age and heft, the gas giant probably played a critical role in arranging the solar system, helping to jockey planets into their current positions. (With 67 known moons, it effectively hosts its own planetary system too.) Astronomers even credit Jupiter's gravitational oomph for diverting comets and asteroids that might otherwise have plummeted into Earth and brought a quick end to life as we know it.

Ancient civilizations watched the planet with awe and interest, and astronomers have been probing its mysteries since Galileo Galilei first studied it and its moons through a telescope more than 400 years ago – observations that showed that not everything in the heavens orbits Earth. The more scientists learn about Jupiter, the more unknowns they find. Even after centuries of inquiry, Jupiter is shrouded in mystery.

Scientists don't know the structure of its thick, hot atmosphere, or how much water that atmosphere contains. Even more mysterious is the structure of its centre, hidden far below. Brilliant light shows, called auroras, encircle the opposite poles like twin crowns – and even exist deeper in the atmosphere – though researchers disagree on how they form (see "Extraterrestrial light shows" on pp37–39). Ground-based observations of Jupiter and its moons offer tantalizing hints at answers to these mysteries, but the best way to ask how the king of planets ticks is to go there and see for ourselves.

Juno's goals are simple. How did the planet form and evolve? What hides beneath Jupiter's clouds? The answers to those puzzles may help answer even bigger questions about why planets form at all. "We're after the recipe for the solar system," Bolton says.

The story so far

In Roman mythology, Juno was the wife (and sister) of Jupiter, king of the gods, and she didn't take kindly to his extramarital interests. To conceal an affair with a mortal priestess named Io, Jupiter concealed himself with a dense cloud cover. Not to be fooled, Juno handily swept the clouds aside – an action that resonates with the modern Juno mission.

The planet Jupiter, long a source of fascination for stargazers, appears in one of the first *bona fide* science-fiction stories. In 1752 French philosopher Voltaire published *Micromégas*, a story that reads a bit like *Gulliver's Travels*, but in space. The tale follows the adventures of a 37 km-tall alien and his 2 km-tall friend as they compare experiences and explore the solar system. Their trajectory takes them to the moons of Jupiter and briefly to the planet itself: "They stopped at Jupiter and stayed for a week, during which time they learned some very wonderful secrets," which are not, unfortunately, revealed in the story. (The duo later visits Earth, but as its inhabitants are too small to be seen, they dismiss the possibility of finding intelligent life there.)

The Juno mission represents the ninth Jovian visit by a human-built ship. The most recent was New Horizons, en route to Pluto, in 2007. The first arrived in 1972, when the Pioneer 10 spacecraft snapped 300 images and took measurements as it zoomed by at 132 000 km/h, about 130 000 km above the tops of the clouds. Data from that mission helped scientists to make early hypotheses about the fluid-filled interior and to analyse plasma in the planet's giant magnetosphere – the region in which charged particles are affected by Jupiter's magnetic field. The mission wasn't entirely smooth sailing: some of the onboard instruments malfunctioned due to the intensity of the radiation surrounding Jupiter, but those problems helped guide the design of better protection for future missions. The next year, on its way to Saturn, Pioneer 11 flew by Jupiter, even lower and faster than its predecessor. The Voyager 1 and 2 missions followed in the late 1970s, sending back more data – and unanswered questions.

"The Voyager missions opened up a bunch of unknowns," says physicist Theodor Kostiuk of NASA's Goddard Space Flight Center in Maryland,

Picture this Jupiter's magnetosphere as it would appear from Earth if visible.

who uses ground-based telescopes to study Jupiter's atmosphere. Voyager, for example, identified active volcanoes on Io, a large inner moon, that affect the entire planetary system. (Prior to Voyager, astronomers didn't know that volcanic activity existed anywhere else in the universe.)

The Ulysses spacecraft measured Jupiter's magnetosphere during flybys in 1992 and 2000, when it used gravitational assists from the planet to slingshot itself towards the Sun, its primary research target. The Cassini–Huygens spacecraft, while en route to Saturn, took tens of thousands of pictures of Jupiter and made detailed measurements of its atmosphere during a six-month period in 2000 and 2001. The first orbiter to reach Jupiter was Galileo, which spent eight years circling the planet's equator and studying the Jovian moons, but it ran into problems and did not ultimately fulfil all of its scientific goals.

Planetary space physicist Fran Bagenal, of the University of Colorado at Boulder, worked on Galileo's science team and now leads the plasma research teams for both Juno and New Horizons, the mission that reached Pluto last year (see "Our new view of Pluto" on pp40–43). "Galileo's observations told us a lot about the moons, but it had this problem," she says. In 1991 Galileo's 4.8 m high-gain antenna, which was shaped like an umbrella and designed to radio data back to Earth, only partially opened. Scientists tried for five years to fix the problem from Earth, but to no avail. "It meant we couldn't do a lot at the planet itself," Bagenal recalls. The mission couldn't send back as much data as scientists had anticipated, and the spacecraft disintegrated during its intentional, final plunge into Jupiter's turbulent atmosphere.

Juno is the scientific heir to Galileo, but it differs in important ways. Galileo's price tag was about \$1.4bn, whereas Juno's estimated cost is about \$1.1bn. Where Galileo circumnavigated the equator, Juno will orbit the poles. Galileo, like most spacecraft, used nuclear fuel to travel through space. Juno relies on solar power. Its three radial arms are 9 m arrays that hold 19 000 solar cells, and in January of this year Juno set a record for the farthest distance travelled using solar power. (The record was previously held by the European Space Agency's Rosetta spacecraft, which travelled to the asteroid belt between Mars and Jupiter.)

In Earth's neighbourhood, Juno's solar cells receive enough sunlight to generate 14 kW – which could power 10 microwave ovens at once. Near Jupiter, where sunlight is weaker, the cells collect only enough light for about 400 W. That's not enough to power a hair dryer, but it's sufficient for Juno's suite of scientific instruments. "It demonstrates that solar power works in a new environment that we hadn't thought possible," says Bolton. The radiation belts around Jupiter, he says, are "one of the harshest regions in the solar system".

Preparing for the storm

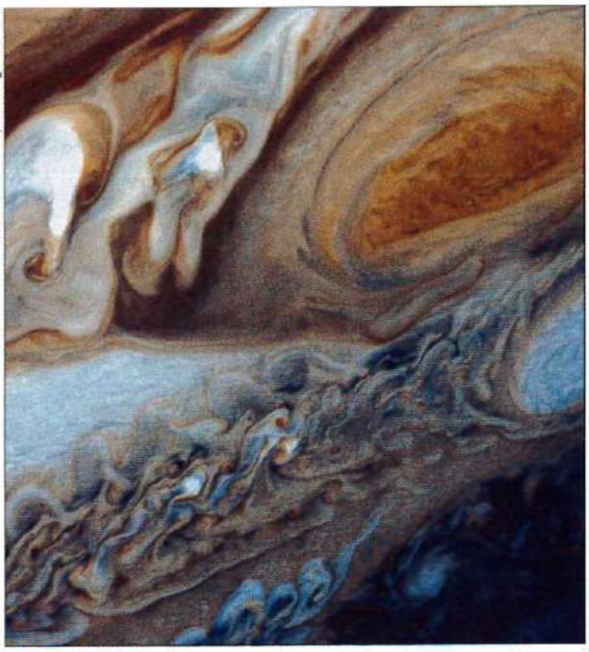
Jupiter is notoriously inhospitable. Winds blow at 650 km/h or more. Lightning strikes with 100 times the intensity of lightning on Earth. The Great Red Spot – the solar system's biggest storm, which has been raging for more than three centuries – is so big it could swallow Venus.

The planet's biggest threat to space travel, though, is radiation. Jupiter's magnetic field is 10 times stronger than Earth's. Indeed, its magnetosphere is the largest known structure in the entire solar system. If it glowed visibly, the magnetosphere would appear to observers on Earth more than twice as big as the full Moon. Such a sprawling magnetosphere traps a lot of high-energy particles, creating radiation belts that circle Jupiter, forming what must be the most hazardous doughnut in space. (The belts are similar in shape and structure to Earth's Van Allen belts.)

"You've got this stream of electrons and protons circling the planet, and they're lethal to spacecraft," says astronomer and Juno team member Tobias Owen of the University of Hawaii, whose goal is to measure oxygen in Jupiter's atmosphere. "Until now, spacecraft have been farther out. We're going to be inside it."

Instruments onboard Juno include a particle detector, magnetometer, ultraviolet and infrared spectrometers, and radio instruments for measuring fluctuations in the gravitational field. (The payload also includes three LEGO figurines, representing Juno, Jupiter and Galileo.) The electronic devices would ordinarily be crippled by the intense radiation, which is why they are safely housed in a protective vault with centimetre-thick titanium walls. Juno's flight plan, which takes it over the poles, will also reduce exposure to the powerful radiation.

The same magnetic field that makes the mission so treacherous embodies one of the planet's most pressing mysteries. Io's active volcanoes, as Voyager observed, spew sulphur-dioxide particles that



Swirling bands Jupiter's atmosphere as imaged by Voyager 1. Vibrant bands of clouds carried by winds that can exceed 650 km/h continuously circle the planet's atmosphere. Such winds sustain spinning anticyclones such as the Great Red Spot – a raging storm three and a half times the size of Earth.

become ionized and fill the magnetosphere, says Bagenal. As they accelerate to high energies, many of the particles end up bombarding the atmosphere of Jupiter – a process that's believed to contribute to the auroral light shows at the poles.

However, "we've never flown over the poles of Jupiter before, and we don't know what it's like up there" says Bagenal, whose research focuses on plasma in planetary magnetospheres. "We don't know what processes accelerate those particles into the atmosphere." Juno, she says, will be able to measure the magnetic field, charged particles and plasma waves as it looks down on the auroral emissions in the atmosphere. "We're trying to put together the bigger picture of what causes the auroras, and how they work."

Beneath the clouds

It's tempting to assume that Jupiter's auroras form like those on Earth, which arise after charged particles from the solar wind are accelerated along magnetic field lines into the upper atmosphere, where they collide with other particles and emit light. But such an explanation might be too simplistic for Jupiter. Ultraviolet images taken by the Hubble Space Telescope 20 years ago show that the auroras form round, oval-shaped structures near the poles. Though small-scale changes occasionally occur within and to each oval, "it really doesn't vary a whole lot" says Bagenal.

That might be in part because Jupiter's powerful magnetosphere protects its atmosphere from the solar wind. In that case, the auroras might be generated internally, from "an atmospheric region deep in the atmosphere" says Kostiuk. Using infrared imaging, Voyager identified a thermal aurora deeper in the atmosphere in the north hemisphere, which researchers have studied for three decades with ground-based measurements and, in 2001, data from the Cassini flyby. Kostiuk points out that recent ground-based measurements of the aurora in the infrared do show some variation with the solar cycle – suggesting a contribution from the solar wind.

From within or without? "That's the big debate," says Bagenal. She suspects the auroral emissions arise from how the plasma in the magnetosphere moves with respect to the planet, which completes a rotation in just under 10 hours. "At some point, the clutch begins to slip," she says. "We think electric currents associated with that process are partly driving the auroras." But scientists won't know for certain until Juno takes a look.

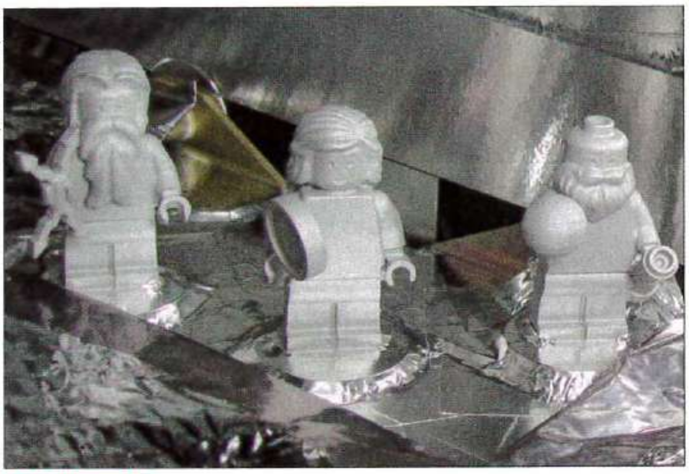
Another of Juno's scientific goals is to better understand the colourful, swirling bands of clouds. To date, it's been difficult for scientists to probe the depths because it's too hot, and the pressure is too great. Juno's multi-frequency microwave radiometer will receive thermal radiation from the depths of Jupiter's cloud cover, up to pressures about 1000 times Earth's normal atmospheric pressure at sea level. That penetration will help researchers better understand the rotation of the atmosphere relative to the core – if it exists – and what elements exist there.

"The thing that's most exciting to me is the determination of water deep in the atmosphere," says Owen. "By measuring the water we'll get an idea of the way that Jupiter came together." Many astronomers have proposed models to explain Jupiter's formation, but different models predict different levels of water. Measuring that abundance, says Owen, will help models get closer to approximating the origins of the planet. "Water abundance is key if you're trying to understand how planets are formed in our solar system," says Bolton.

Juno will also be studying what lies beneath the clouds. The planet likely contains a vast and bizarre ocean unlike anything found on Earth – and unlike anything that can even be simulated on Earth. It's made of hydrogen under so much pressure that electrons separate from protons, and the fluid conducts electricity like a metal. As this strange sea rotates with the planet, it generates Jupiter's powerful magnetic field. "We think that's where the dynamo is produced," says Bagenal.

But scientists don't know how deep the liquid metallic hydrogen extends, or what's underneath it. They hypothesize that Jupiter's core is rocky and

To date, it's been difficult for scientists to probe the depths of Jupiter because it's too hot, and the pressure is too great



NASA/JPL-Caltech/KSC

Mission mascots Three LEGO minifigures aboard Juno represent the Roman god Jupiter, his wife Juno and Galileo Galilei.

made of heavier elements. Since no device could reach the hydrogen sea – much less any core that lies beneath – Juno will map the interior structure by tracking changes in the planet's gravitational field as it orbits.

The end of Juno

Juno will spend a full year measuring and sending data to astrophysicists on Earth, but Bolton warns that definite answers about Jupiter won't show up immediately. "We're limited on how we can interpret Juno's data," he says. Using gravity measurements to map the distribution of mass will be fairly straightforward. But to connect data on variables like temperature and pressure and forge one big, coherent picture will require physicists to agree on an equation of state to describe the conditions on Jupiter. That's a challenge in and of itself: no-one has any idea how metallic hydrogen is supposed to behave. They also suspect, but can't prove, that heavier elements likely dissolve in that strange soup.

Bagenal says the equation of state is a crucial and missing piece of the puzzle. "Every time we have a meeting of the interior working group of the Juno mission, these guys come up with a new equation of state," she says. "They're always improving, and always changing their minds. Since we launched, they've changed their minds a few times."

At the same time, Juno's data will immediately be put to use by theorists who come up with models of how Jupiter formed. "All theories on how Jupiter forms will have to be consistent with what Juno sees," says Bolton. "By making these measurements, we will constrain the models." Those limitations will, in turn, lead to more refined models that more accurately represent the reality of Jupiter.

Once Juno's year-long data-gathering feast is over, it will change direction one last time. The spacecraft won't be allowed to orbit indefinitely because of the unlikely chance it might collide with and contaminate Europa, a Jovian moon with a subsurface ocean where, one day, scientists would like to look for life. So instead of drifting off, Juno will end, like Galileo before it, by disintegrating during a final plunge into the heart of its host.

Pathway to Planet Nine

Konstantin Batygin explains what led him and astronomer Mike Brown to propose the existence of a ninth planet in our solar system

Konstantin Batygin is an assistant professor of planetary science at the California Institute of Technology, US, e-mail kbatygin@gps.caltech.edu

As one of the oldest forms of natural science, astronomy has enjoyed a long and dramatic history. However, it was not until the early 1600s that the entire discipline was kicked into high gear by Galileo's adoption of the telescope as a scientific instrument. No longer bound by the resolving power of the human eye, astronomers had finally attained the freedom to search the night skies for the wandering motion of the faintest stars. The door to the discovery of additional planets that orbit the Sun had been cracked open.

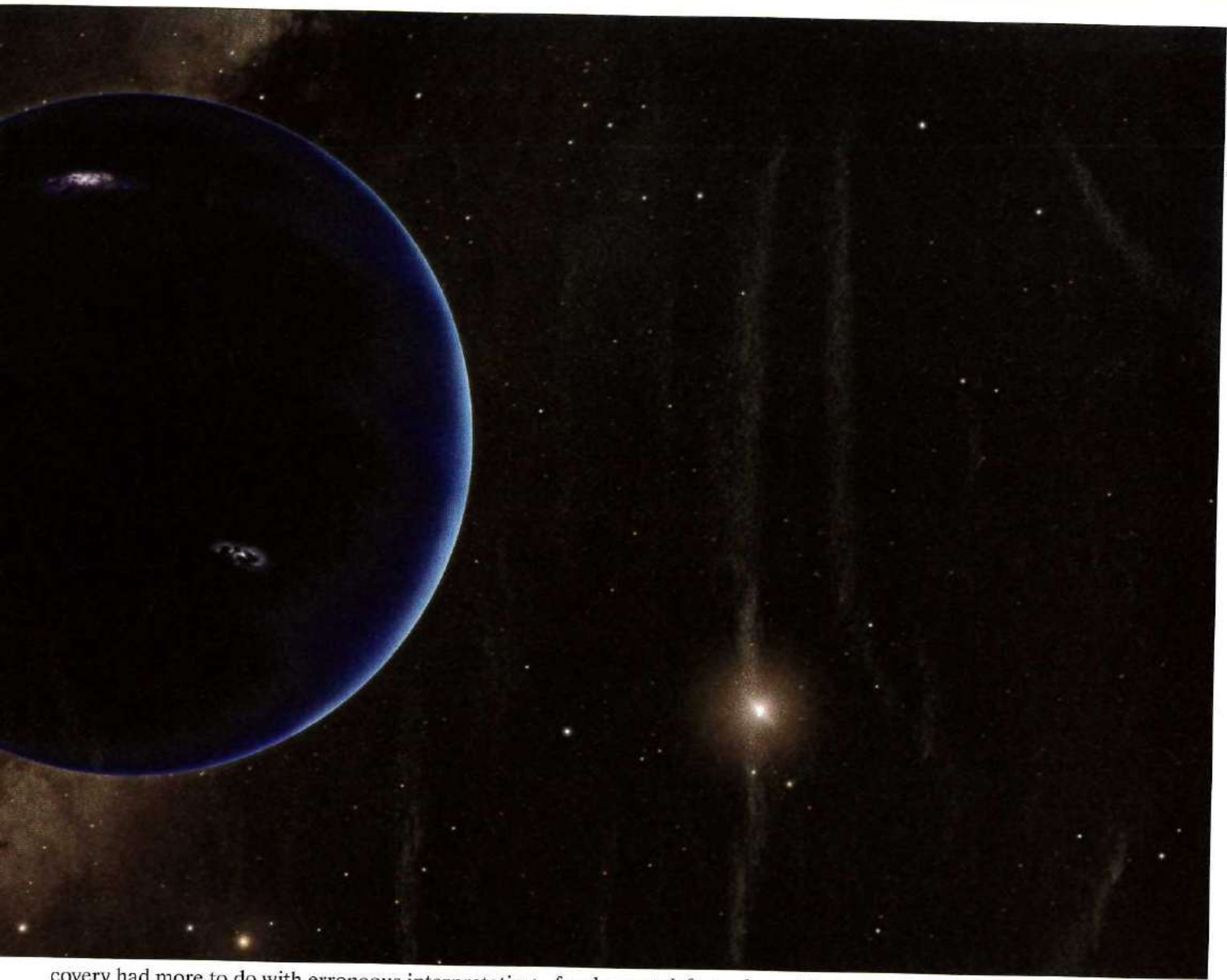
In terms of sheer numbers, efforts to expand the solar system's planetary album have yielded rather unimpressive results. Over the last four centuries, only two planets that were not known to ancient civilizations have been found. The discovery of the first of these planets, Georgium Sidus (now known as Uranus), was announced by William Herschel at the time of the American Revolutionary War, in 1781. This finding simultaneously marked the beginning and the end of purely astronomical detection of planets in the solar system. Indeed, the revelation of the next planet would rely more on celestial mechanics than on a telescope.

Soon after Herschel's announcement of Uranus, astronomers began to compute its orbital motion and flirt with the idea that an additional, more distant object could gravitationally perturb its trajectory. Among the first astronomers to lead this charge was Anders Johan Lexell. In a set of compiled astronomical tables published in 1821, which included accidental observations of Uranus that predated its formal discovery, Alexis Bouvard (then director of the Observatoire de Paris) noted that Uranus was indeed deviating from its predicted path. Without discounting the possibility of spurious data, Bouvard joined Lexell in speculating that the irregularities in Uranian motion could be caused by an additional planet.

It would take more than two decades before the promise of Bouvard's data came to fruition. In a parallel set of calculations completed in 1846, John Couch Adams and Urbain Le Verrier independently predicted the existence of Neptune. Although the computed orbital period and mass of the putative Neptune exceeded the real values by a significant margin, the calculations gave the correct location in the sky. Then, in a remarkable feat of observational confirmation of theoretical results, Neptune was spotted by Johann Galle on the first night of his observational campaign later that same year.

Once the ability to deduce the presence of an additional planet using orbital irregularities had been demonstrated, a number of contemporary mathematicians attempted to derive the existence of even more distant objects using existing data. As a result, by the early 1900s there was no shortage of hypothetical planets beyond Neptune. One particularly notable prediction was Percival Lowell's famed Planet X hypothesis, which led to the accidental discovery of Pluto in 1930 (see "Our new view of Pluto", pp40–43).

In the end, it was unmanned spaceflight that killed Planet X. Following Voyager 2's 1989 encounter with Neptune, the planet was recognized to be a fraction of a per cent less massive than previously thought. Like a Rubik's cube snapping into its orderly configuration, this small change cleansed the solar system's astronomical charts of any irregularities, and erased the theoretical need for Planet X. As history shows, the claims of additional planets following Neptune's dis-



A ninth planet? Artist's impression of Planet Nine. For the first time in 170 years, evidence for the existence of an additional massive planet in the solar system is mounting up.

Robert Hurt, IPAC/Caltech

covery had more to do with erroneous interpretation of the observational data than anything else. Every time the observations seemed to call for the introduction of another planet, further analysis revealed that the apparent anomalies could be fully reconciled within the framework of the known solar system.

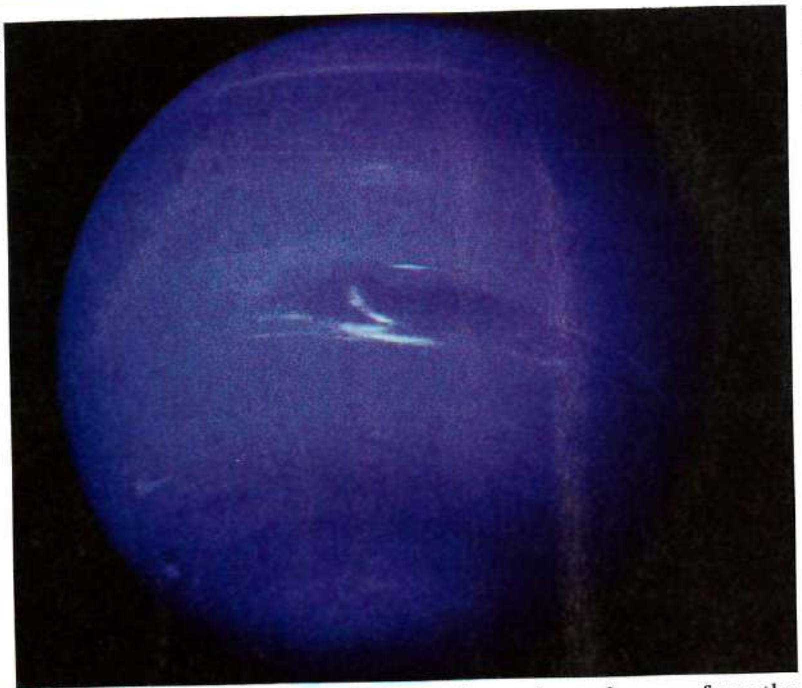
The first discoveries of icy debris beyond the orbit of Neptune, now collectively known as the Kuiper belt, conformed to this narrative well. As observational surveys began to expose the intricate dynamical structure of the Kuiper belt, it became increasingly clear that virtually every Kuiper belt object's orbital evolution could be explained through gravitational interactions with Neptune. While some objects are currently locked into orbital resonances with Neptune, others show signs of having been tethered by its gravitational pull in the past. Hence, at the turn of the 21st century, the large-scale architecture of the solar system showed no signs of abnormality whatsoever.

Kuiper belt clues

The solar system in 2016 tells a very different story. Over the course of the last 15 years, observational mapping of the Kuiper belt has revealed a simple, fundamental fact: the orbital arrangement of the most distant bodies in the Kuiper belt is incompatible with an eight-planet solar system.

The first real hint that the solar system still has some tricks up its sleeve came in 2003, when a team of astronomers led by Mike Brown discovered Sedna, a Kuiper belt object (KBO) unlike any other. Whereas most known KBOs have orbital periods not too different from the approximately 250-year period of Pluto, Sedna requires more than 11 000 years to complete its journey around the Sun. Another impressive feature of Sedna's orbit is its staggering ellipticity. At its furthest from the Sun, Sedna swings out to almost 1000 astronomical units (where one astronomical unit is the mean Earth–Sun distance, roughly 150 million kilometres).

The truly remarkable thing about Sedna, however, is that its orbit is not elliptical enough. Most KBO orbits appear to physically hug the orbit of Neptune. That is because – due to gravitational potential being conservative – any small object that has been sent on a highly elliptical trajectory by Neptune must come back to its point of origin, i.e. the orbit of Neptune. Sedna's orbit represented the first true exception to this rule: even at its closest approach to the Sun,



Data-led discovery
To date, Neptune (pictured here by Voyager 2) is the only planet in our solar system to have been predicted by theory and later confirmed by direct observation.

Sedna remains more than twice as far away from the Sun as Neptune. As a result, Sedna's origin posed somewhat of a mystery. A body that never experienced direct interactions with Neptune could not have been placed on its orbit by Neptune alone.

In a paper detailing Sedna's discovery, Brown, Chad Trujillo and David Rabinowitz speculated on the various scenarios that could potentially account for the genesis of its strange orbit, including a scenario where an undiscovered Earth-mass planet lurks beyond the orbit of Neptune (2004 *Astrophys. J.* **617** 645). Around the same time, Brett Gladman and Collin Chan independently discussed the possibility of a rogue planet shaping some features of the Kuiper belt. A similar viewpoint was adopted by yet another researcher, Rodney Gomes, in Brazil. In some sense, the discussion mirrored the Lexell–Bouvard speculation of the early 1800s, in which close examination of Uranus had given clues to the existence of Neptune. Clear echoes of a distant perturbing body were beginning to emerge.

Sedna's loneliness as an outlier finally came to an end in 2014, when Trujillo and Scott Sheppard discovered a second Sedna-like object, 2012 VP113 (*Nature* **507** 471). With a perihelion distance (a body's closest distance to the Sun) even larger than that of Sedna, 2012 VP113 confirmed that these objects are not outliers: they are members of a separate, detached population of KBOs. It was with this very paper in hand, and a facial expression showing a combination of excitement and concern, that Mike Brown walked into my office two years ago.

Could it truly be that after 170 years of false alarms and non-detections, we had stumbled upon actual evidence that the solar system's planetary catalogue is incomplete?

Gravity of the situation

"Have you seen how weird this is?" Mike asked, pointing to figure 3 in Trujillo and Sheppard's paper. Here the authors note that all KBOs with orbits with perihelion distances beyond Neptune and with periods longer than 2000 years tend to cluster in their argument of perihelion. (The argument of perihelion is a bizarre parameter: it is the angle between the point at which an orbit intersects the ecliptic plane while travelling from south to north on the sky and the point of closest approach to the Sun. Taken at face value, a collection of similarly inclined orbits that cluster in the argument of perihelion would trace out a cone-like structure.) Not swaying from tradition, Trujillo and Sheppard had speculated that this clustering could be due to an unseen, few-Earth-mass planet, with a circular orbit and a period equal to that of 2012 VP113. However, the authors simultaneously acknowledged that such a planet could not, in fact, explain the data adequately.

Intrigued, Mike and I examined the data ourselves. The clustering pointed out by Trujillo and Sheppard emerged on the computer screen. However, to our surprise, this clustering was not alone – other orbital co-ordinates were grouped as well. Immediately, it was clear that the clustering of the argument of perihelion is only part of the full picture. A closer look at the data showed that six objects that occupy the most expansive orbits in the Kuiper belt (including Sedna and 2012 VP113) trace out elliptical paths that point into approximately the same direction in physical space, and lie in approximately the same plane.

Mike and I were genuinely perplexed. Could the confinement of the orbits be due to an observational bias, or perhaps to mere coincidence? Will any theory aimed at explaining these observations suffer the same fate as Lowell's Planet X hypothesis (i.e. the need for it disappears once more accurate observations are made)? Thankfully, the probability of the observed alignment being fortuitous can be assessed in a statistically rigorous manner, owing to the large size of the comparison sample (i.e. other KBOs that are found at a similar radial distance to our objects of interest). The probability that the alignment is a fluke clocked in at only 0.007%. Not a great gamble.

Could this orbital alignment be a relic of an encounter with a passing star during the solar system's infancy? An application of simple mean-field perturbation theory showed that if allowed to evolve under the gravitational influence of Jupiter, Saturn, Uranus and Neptune, these objects' orbits would become randomly oriented on timescales much shorter than the multi-billion-year lifetime of the solar system. So the dynamical origin of the peculiar structure of the Kuiper belt cannot be outsourced to the distant past – something is holding the orbits together *right now*.

Having quadruple-checked our results, we sat on my couch and stared silently at each other. The gravity of the situation began to sink in. Could it truly be that after 170 years of false alarms and non-detections, we had stumbled upon actual evidence that the solar system's planetary catalogue is incomplete? We got to work.

Glimpse of hope

Our progress was initially anything but rapid. Coming from observational and theoretical backgrounds respectively, Mike and I don't always speak the same language, and would spend hours arguing profusely, only to later realize that we are in fact, saying the exact same thing. Then there were all the calculations that did not pan out. Ideas crowding our outtakes reel range from models where the self-gravity of the Kuiper belt itself keeps the observed structure intact, to a scenario where the orbit of a distant planet cradles the orbits of KBOs from the outside, maintaining the same average orientation. Each hypothesis failed when confronted with the data.

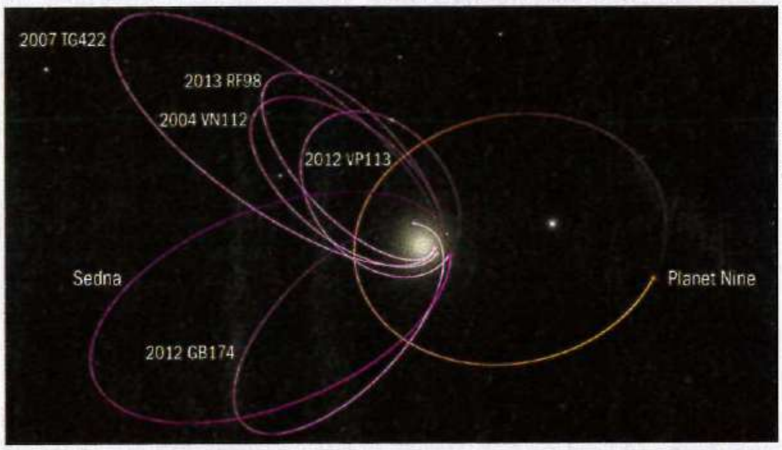
Last summer brought our first glimpse of hope. We were running a series of evolutionary numerical experiments, starting off each time with a randomized disc of planetary building blocks, or "planetesimals". We placed these objects in eccentric, Neptune-hugging orbits that were allowed to evolve under the gravitational influence of a distant perturber, which we dubbed "Planet Nine". We began to notice that groups of planetesimals emerged in orbits that were co-linear and spatially confined. Intriguingly, this would occur only if Planet Nine was chosen to be about 10 times more massive than the Earth, and to reside on a highly eccentric orbit. More unexpectedly, the confined orbits would cluster in a configuration where the long axes of their orbits are anti-aligned with respect to Planet Nine.

At first glance, this outcome was puzzling. If the trajectories of the KBOs intersect the orbit of the perturbing planet, wouldn't the objects have been scattered away at some point over the past few billion years? It turns out that the answer can be summarized in one word: resonance. Just as the overlapping orbits of Pluto and Neptune are protected from close encounters by a clockwork-like orbital period ratio of 3:2, the confined orbits of the distant Kuiper belt glean long-term stability from resonances with Planet Nine. However, the latter picture is somewhat more complex: the resonances at play are exotic and interconnected, yielding orbital evolution that is fundamentally chaotic. In other words, perturbed by Planet Nine, the distant orbits of the Kuiper belt remain approximately aligned, while changing their shape unpredictably on million-year timescales.

Surprising and unforeseen results continued to accrue. Upon a cursory examination of the simulation data, we noticed that gravitational torques exerted onto the Kuiper belt by Planet Nine would induce long-period oscillations in the perihelion distances of the confined KBOs. This naturally generated detached orbits, such as those of Sedna and 2012 VP113. Suddenly, the origins of these objects became abundantly clear: they are regular KBOs that have been pulled away from their original locations by Planet Nine. Moreover, the evolutionary calculations suggested that if we were to revisit the Kuiper belt in a hundred million years, objects like Sedna and VP would once again look like conventional, garden-variety KBOs, while some of the more typical objects would now be in detached orbits.

Finally, there was a weird, crazy twist. In every simulation that produced a synthetic Kuiper belt that resembled the real one, the model also consistently generated orbits that were nearly perpendicular to the plane of the solar system. Given that there is virtually no other way to produce such extreme inclinations in the solar system, we thought that this would be a strong prediction: if such objects were ever discovered, they would constitute tangible evidence for the existence of Planet Nine.

1 Peculiar arrangement



Kuiper belt objects that have periods in excess of 4000 years have highly elliptical orbits that cluster in physical space. This peculiar orbital arrangement can be explained by the existence of Planet Nine beyond Neptune.

Planet Nine falls into place

Caught up in our attempts to understand the dynamics of the simulations, we had forgotten to check the actual data. Then, on a sunny afternoon in October, we plotted the observed catalogue of objects on top of our model's predictions to see if, by any chance, highly inclined bodies of the type our simulations predicted had been discovered since we last checked. And there they were – five objects, accidentally detected by a near-Earth asteroid survey, exactly where our model predicted them to be. Once again, Mike and I sat in our seats and stared at each other in silence, allowing reality to slowly sink in.

For the first time in our joint scientific journey, we realized that Planet Nine is really out there. The theoretical model did not just explain the peculiar clustering of the orbital angles. It tied together three, seemingly unrelated aspects of the Kuiper belt into a single, unified picture: physical alignment of the distant orbits; generation of detached objects such as Sedna; and the existence of a population tracing out perpendicular orbital trajectories. As far as merits of a dynamical model go, it is difficult to ask for more. However, it is simultaneously important to keep in mind that until Planet Nine is caught on camera, it remains a theoretical prediction.

Fortunately, the prospects of confirming Planet Nine observationally are not as dim as the planet itself. Given our model's best estimates, Planet Nine has an apparent magnitude of 24–25 and currently lies in the vicinity of Orion's shield. Detecting its parallactic motion is well within the capabilities of the Subaru Telescope on Mauna Kea in Hawaii, and multiple groups have already set out on the observational hunt. It may take years, but I, for one, am confident that we will one day wake up to learn that solar photons that reflected off Planet Nine's frigid surface have landed onto the aperture of a terrestrial telescope.

For now, I wait anxiously for that day. ■

Between a rock and a cold place

Marc Rayman, mission director of NASA's Dawn mission, talks to **Laura Faye Tenenbaum** about what we are learning about Vesta and Ceres – the two largest objects in the main asteroid belt

Laura Faye Tenenbaum is senior science editor at NASA's Jet Propulsion Laboratory, e-mail laura.f.tenenbaum@jpl.nasa.gov

"A lot of people confuse Vesta and Ceres with asteroids and they aren't asteroids," explains Marc Rayman, mission director and chief engineer of NASA's Dawn mission. We are sitting in his office surrounded by a cacophony of spacecraft models, rocks and spinning science toys, including a custom-made box of "Ceres-O's" cereal. "I have one of the largest collections of space information and memorabilia in private hands you can find anywhere. I just love the stuff," he proclaims proudly. "When I was four years old, I saw a meteor – my parents called it a shooting star – and I've been a space enthusiast ever since."

I couldn't help, though, staring at his socks. They're cute socks, black with a planets-and-stars pattern. Rayman lifts a trouser leg just a smidge and I "Ooh" in admiration. Sure, it felt silly to start with socks. I'm busy. Rayman's busy. He's a mission director based in an office located behind a key-card-only access wing of an upper floor at NASA's Jet Propulsion Laboratory. And there's me yakking about his socks.

Still, the tone of our conversation is set: we bounce back and forth between our mutual fascination with legitimate cutting-edge science, with me being charmed by his passion for planetary science and his ability to gleefully rattle off exact numerical quantities. "10320 is the number of known asteroids in the main asteroid belt larger than 10 km in diameter," he tells me. "Ceres is around a third of the total mass of all the objects in the main asteroid belt between Mars and Jupiter," he adds.

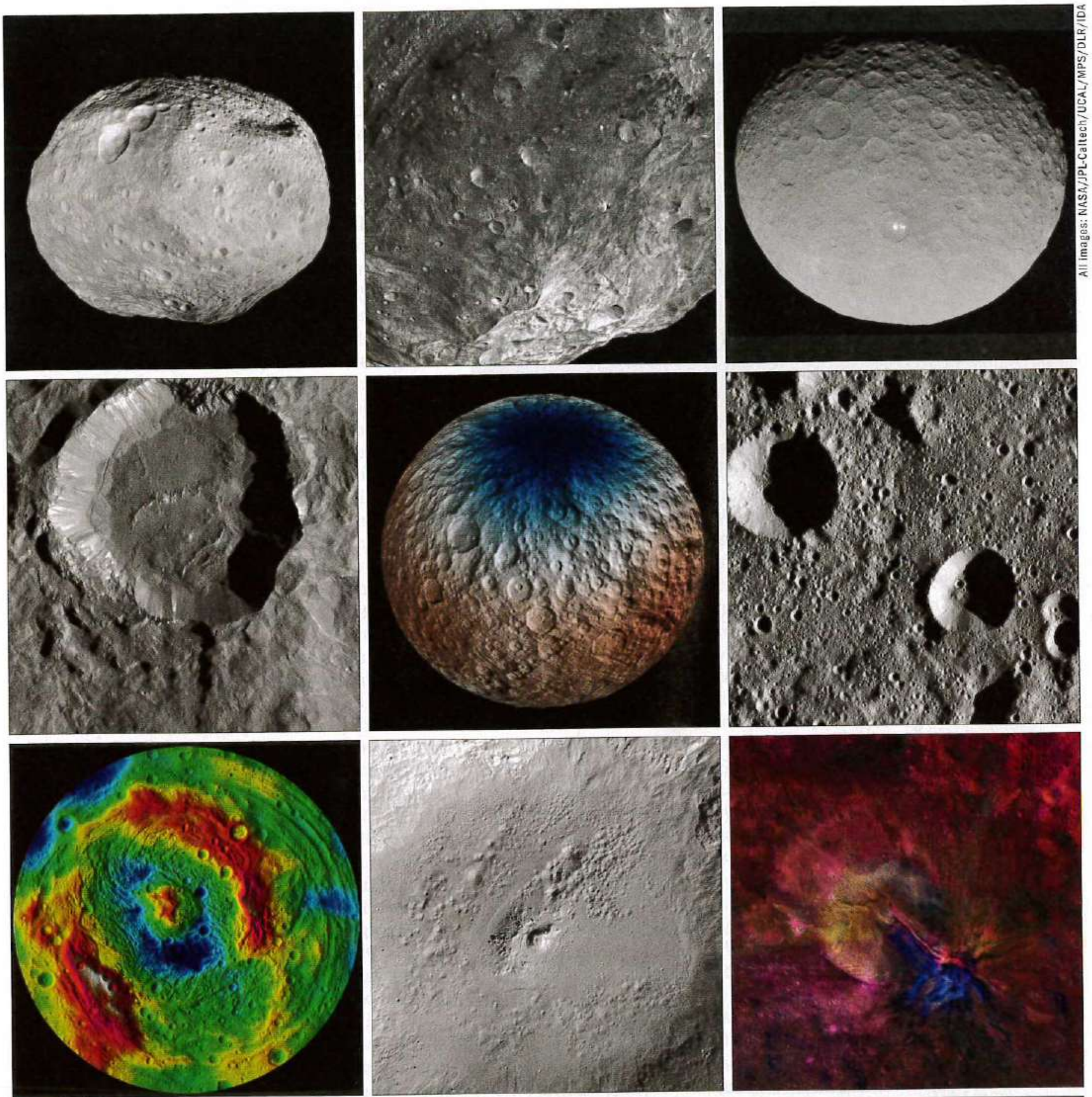
Once we get past socks, we get into history. Ceres (discovered in 1801 by Italian astronomer Giuseppe Piazzi) and Vesta (discovered in 1807 by German amateur astronomer Heinrich Olbers) are the two most massive objects in the main asteroid belt. Vesta has an equatorial diameter of 562 km, and Ceres is nearly 1000 km across. In fact, Rayman explains, "Ceres was first interpreted as the missing planet between Mars and Jupiter. You can even find schoolbooks from around the 1820s that list Ceres as well as Vesta as planets. And for almost two generations, people called them planets."

But by the mid-19th century, so many more bodies had been discovered between Mars and Jupiter that the whole lot, including Ceres and Vesta, were lumped together and called asteroids. It wasn't until 2006 that Ceres, along with Pluto, was reclassified as a dwarf planet. "Science is a process," Rayman emphasizes. "As we learn more, as scientific knowledge advances with new information, we adjust our vocabulary to reflect our new understanding."

With the exception of Ceres and Vesta, the objects in the asteroid belt never got massive enough to achieve one key requirement for definition as a planet, dwarf or otherwise: hydrostatic equilibrium. This means the shape of a planetary body is dominated by gravity and is roughly spherical. Whether or not a body's own self-gravity pulls it into a more uniform shape depends on its material composition as well as the temperatures it experiences while forming. "For example," says Rayman, "a low-viscosity material, such as a bunch of metal close to the Sun, will form into a sphere more easily than a big ball of rock." And this whole time I'd thought asteroids were just potatoes floating around in space. "There are a lot of those potatoes. And many of the little potatoes look basically the same," he assures me. "But Vesta and Ceres are not like asteroids."

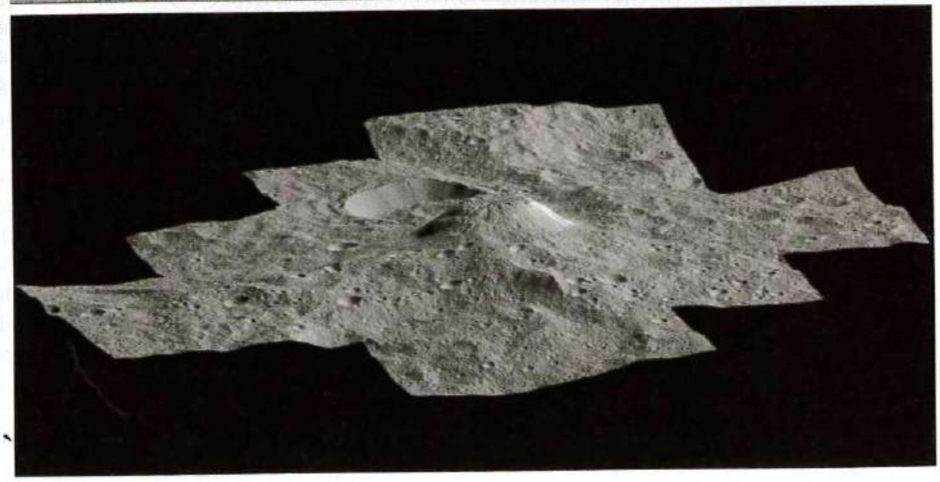
I want to know more about why some planets are planets and others are dwarfs. "Earth, we pretty much have this orbit to ourselves, whereas Ceres is embedded in the asteroid belt," Rayman explains. A planet's larger mass and gravitational pull is able to suck in or eject most other objects from its orbit, whereas a dwarf planet is not massive enough to clear its orbit of other bodies. But this definition isn't perfect. "There's a continuum," he tells me, "everything from little tiny particles, the size of a grain of sand, up to the giant planets. Some of the moons of Saturn and Jupiter are bigger than the planet Mercury and significantly bigger than Pluto, and these moons range in size down to the size of your iPhone."

It was during the formation of the solar system that Ceres and Vesta came to be protoplanets. Understanding how they formed, therefore, sheds light on the rest of the solar system too, which explains why NASA wanted to send the Dawn spacecraft to the asteroid belt to study them. "A journey to the asteroid belt is like a journey back in time," Rayman says as he stretches his arms wide, trying to convey how long ago that was. "It's like looking at a construction



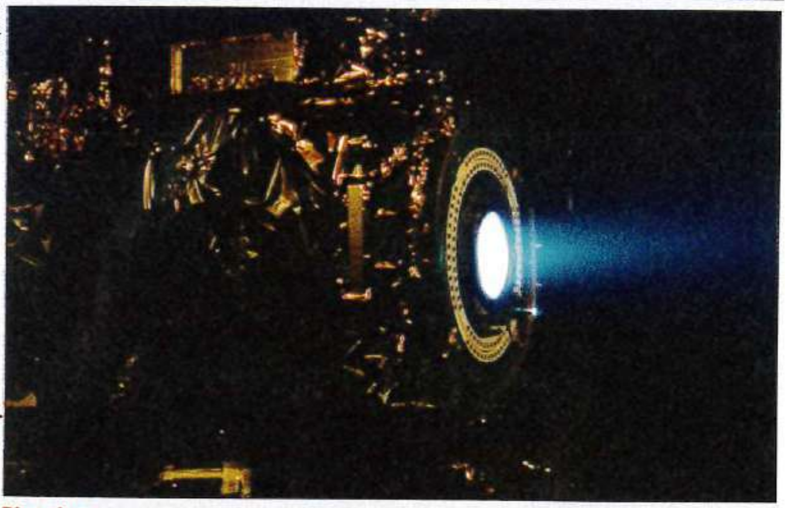
All images: NASA/JPL-Caltech/UCAL/MPS/DLR/IDA

View from a probe Top row: full view of Vesta, showing clearly its irregular shape; Vesta's south pole, pictured by Dawn's framing camera; full view of Ceres, showing its spherical shape. Second row: the Kupalo crater on Ceres, named after the Slavic god of vegetation and harvest; map of the concentration of hydrogen in the regolith, the loose surface material on Ceres, based on the number of neutrons detected per second; the Chaminuka crater on Ceres, named after the Shona spirit who provides rains during drought. Third row: topographic map of Vesta showing Rheasilvia, a large impact basin; Vesta's spectacularly preserved pitted terrain – an unexpected discovery; composite image showing the flow of material inside and outside the Aelia crater on Vesta. Right: perspective view of Ahuna Mons on Ceres.



Sci-fi becomes sci-fact

NASA/JPL



Blue glow A xenon ion engine, photographed through a port of the vacuum chamber where it was being tested at NASA's Jet Propulsion Laboratory. Dawn has three ion thrusters.

Until the late 1990s, ion propulsion as a means of interplanetary travel existed only in the realm of science fiction. It crops up in the *Star Wars* franchise in the TIE fighter's "twin ion engines". And although it has not been developed in the world of *Star Trek*, it still gets a mention from Captain Kirk. "Advanced ion propulsion is even beyond our capabilities," he once told his crew.

The Starship *Enterprise* doesn't need ion propulsion, as it can rely on its warp drive. But back in the real world, the Dawn mission would be impossible without it. To travel to one destination, get into orbit, manoeuvre there, break out of orbit, travel someplace else and break into orbit there – as the Dawn spacecraft has done – is far beyond the ability of conventional propulsion technology. Ion propulsion is 10 times as efficient as conventional chemical propulsion.

According to Marc Rayman, the Dawn mission's director and chief engineer, there's a "dirty little secret of celestial mechanics", which is that the farther you are from the Sun, the slower you go. Ion propulsion is not being used to make a spacecraft go faster, but to slow it down to match orbits. Ion propulsion is very gentle. It pushes on a spacecraft as much as a piece of paper pushes on an outstretched hand. In the microgravity, frictionless conditions of space, gradually the effect of this thrust can build up. "It would take Dawn four days to accelerate from 0 to 100 km/h, so it's not a drag racer," says Rayman. "But if you thrust for a week, or month, or year or, as Dawn now has, for five years, you can achieve fantastically high velocities. Acceleration with patience is a great way to explore the solar system."

Ion propulsion produces a cool blue glow because it uses xenon as the propellant, which happens to glow blue. Dawn's three ion engines are power hungry. A large area of solar cells is needed to capture enough light to produce sufficient electrical power to ionize and accelerate xenon. The spacecraft is dominated by two solar arrays, 20 m wingtip to wingtip. When it launched in 2007, Dawn was the largest interplanetary spacecraft NASA had ever sent into space.

site to understand what's inside a building. It's like you're seeing the pieces before they grew to be a bigger planet."

Ceres and Vesta are protoplanetary remnants. They were in the process of growing into full-sized planets when massive Jupiter cut them off. Jupiter's gravity interrupted their formation process by getting into a tug of war with the Sun. The way Rayman explains it, "Jupiter is saying, 'This stuff over here is mine, you can't mess with it.' And the stuff in the asteroid belt is saying, 'I don't know what to do. I can't be a planet. Help!'" So the asteroid belt just stayed in a jacked-up interrupted mess. And that's cool because now Ceres and Vesta have retrievable records of the conditions and the processes that were acting at the *dawn* of our solar system. *Dawn*; get it?

The Dawn mission is an opportunity to explore two of the last uncharted worlds in our solar system. It's an odd juxtaposition of the excitement of going someplace new, but that someplace happening to be super old. It's like "going into the future to study the past", Rayman muses. The Dawn spacecraft blasted off from Earth in September 2007, got a gravity boost as it flew by Mars in February 2009, and finally went onto orbit around Vesta in July 2011.

Views of Vesta

Dawn found that Vesta is mostly dry, rocky and dense. It looks planet-like, even though it's misshapen and not quite spherical. Vesta's southern hemisphere has a giant impact crater with a big mountain in the centre. Rayman explains that craters often have a mountain in there because when a big piece of interplanetary debris screams down into the surface, it hits with so much energy that the rock melts and flows away from the impact site, then sloshes back, before solidifying into place. So the peak in the centre of big craters is like a snapshot of the process of how the crater formed. About a billion years ago, after Vesta had cooled off in hydrostatic equilibrium and become rigid, an asteroid about 30–50 km across smashed into it and the impact excavated a huge amount of material. This explains why Vesta looks wonky and not so spherical, and also why one in every 16 meteorites that fall to Earth comes from that impact.

Really? Yes, Vestan meteorites make up about 6% of the total mass of all meteorite falls. And those meteorites have been studied in labs for decades. "We have more material from Vesta than from Mars or even the Moon, including the 382 kg of Moon rocks brought back by Apollo astronauts," Rayman tells me. Of course he has a small chunk of Vestan meteorite in a little box. He holds it out for me to examine, explaining that Vesta has a unique infrared spectrum, just like a fingerprint, that distinguishes it from all the other stuff out there.

Much smaller bodies called Vestoids have the same unique fingerprint too. Those were broken off when the crater was excavated and the total mass, size and number of all these bodies adds up to what you would expect from the amount of Vesta that's missing. That sounded insane to me – no wonder he wanted to go there. "We needed Dawn to clinch the case that the meteorites originated from Vesta and to provide the rest of the geological context," he says proudly, "and the Dawn mission corroborated everything."

Instruments on the Dawn spacecraft take stereophotographs for making topographic maps, and determine the atomic constituents and mineralogical compositions of the bodies studied. They also measure the gravity field, which tells us about interior structure. Vesta has a dense core of iron and nickel, surrounded by a rocky mantle, surrounded by a crust. According to Rayman, "It's not just a chunk of rock; it's really more like a mini planet."

Vesta has a network of more than 90 chasms near the equator that run for hundreds of kilometres and are kilometres deep and kilometres across. They're scars from the impact that excavated the huge crater near the South Pole. "Vesta got punched so hard," says Rayman, "that the energy reverberated inside and broke up the surface hundreds of kilometres away." This could not have happened if Vesta were just a big chunk of rock, like a typical asteroid.

Asteroid navigation

After 14 months of study, in September 2012 Dawn broke out of Vesta's orbit and, using ion-propulsion technology (see box opposite), embarked on a two-and-a-half-year-long arduous climb through the main asteroid belt, on towards its new target: Ceres.

So how did the Dawn spacecraft manage to wind its way through the crowded and perilous asteroids without being smacked by some random space pea and ending up smashed to bits? We know where all the large things are, of course, but there are uncountable millions of marbles zooming around and the probability that none will whip right into Dawn seems incomprehensible. "Yes, we're used to seeing an environment in the movies where you need Han-Solo-type piloting skills to fly in-between them," Rayman assures me, "but space is big and we're not worried about getting taken out by a space pea. We've survived." Plus, there is redundancy built into the spacecraft. "We have two cameras, two radio receivers, two transmitters and two central computers. Everywhere we have a heater, we actually have two heaters," says Rayman. "There are 11480 solar cells, and we don't need every one. So if a speeding space bean takes out one of the cells, we don't care. In fact, we wouldn't even know if we lost some." Today Dawn is four times further away from Earth than the Sun and more than a million times more distant from us than the International Space Station. If it has a problem, nobody can fix it. "I love the visceral thrill of working on a spacecraft a thousand times farther away than the Moon," he says.

Arrival at Ceres

When Dawn reached Ceres in March 2015, it became the only spacecraft in almost 60 years of space exploration to ever orbit any two extraterrestrial destinations. It began exploring Ceres at a 13600 km orbit and has since spiralled down to an altitude of merely 385 km, where it is now and where it is the closest to Ceres it's ever going to be. Today the Dawn spacecraft circles Ceres in just under 5.5 hours and is closer to the surface of Ceres than the International Space Station is to Earth.

Unlike Vesta, Ceres has a lot of water – 25% of its mass – which is low density compared to rock or metal. (For comparison, water is less than 1% of the Earth's mass.) This may be because Ceres probably formed a million or a few million years after Vesta. A million years can make a difference, especially if it was during a crucial phase of solar system development. "If you form early when there's still a lot of radioactive matter around, then you get very hot and

NASA/JPL-Caltech/UCAL/MPS/DLR/IDA

Shining bright The Occator crater, which contains the brightest area on Ceres. These reflective salt flats formed after the impact caused salty water to rise to the surface, which then froze.

if you have water, it boils away," explains Rayman. "If you form a few million years later, much of the raw material is decayed so you hang on to your water."

There's even some evidence that Ceres may not have formed where it is now. Dawn detected ammonia on Ceres, which is very volatile and, during solar system formation, wouldn't have existed as a solid in the warm inner part of the solar system where Ceres is now. This means Ceres may have formed farther from the Sun, incorporated the ammonia as ice, and then as the planets underwent gravitational jostling and moved around, Ceres may have been perturbed or pushed to where it is now. So Ceres and Vesta may have formed at slightly different times and at very different locations, which would explain why they would have incorporated different materials.

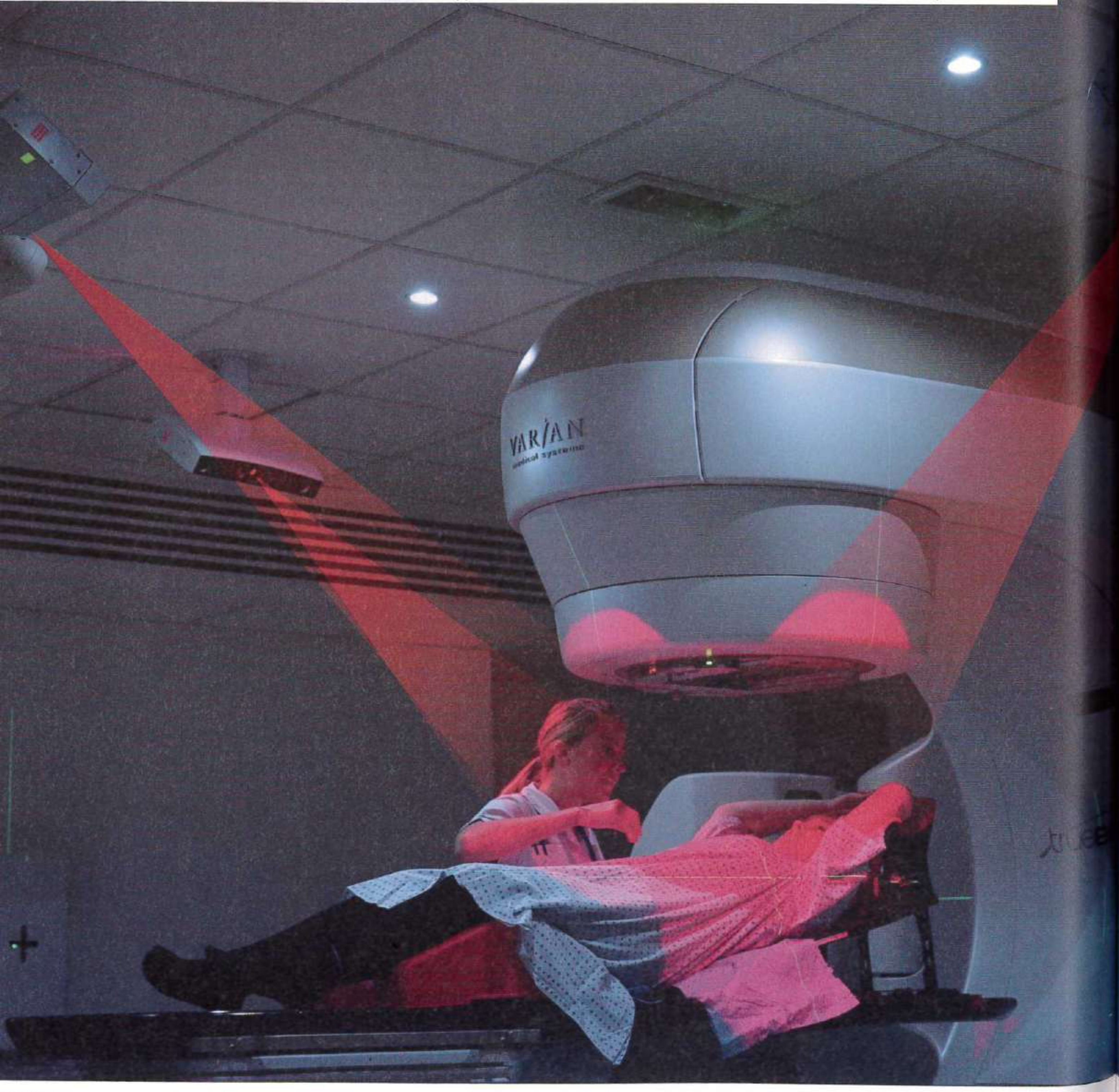
One feature on Ceres mesmerizes Rayman, as he puts it, "like a light casting forth on the cosmic seas". The shiny 92 km diameter Occator crater glows brightly due to reflective salt flats that formed in an impact crater. After the impact, briny salt water made its way to the surface of Ceres and froze in the cold vacuum of space. The water molecules then turned to gas and departed, and all that was left behind was the salt. They're the only extraterrestrial salt flats we know of.

Dawn's stereoscopic view has shown us that Ceres has a variety of terrains and about 130 locations of the bright material. The science team decided to name all of the dwarf planet's features in line with Ceres' own name – Ceres is the Roman goddess of agriculture – drawing from deities and festivals related to agriculture around the world. As well as Occator, named after a Roman helper god of Ceres, other crater names include Toharu, the Pawnee god of food and vegetation, and Rao, the Mangarevan god involved in the planting of turmeric.

"It's so profound that humankind has the capability to send spacecraft to destinations so distant you really cannot imagine it. These are robotic emissaries to the stars," Rayman declares. "I've been looking up at the sky since I was four years old and to think we have spacecraft out there is amazing." ■

ACCURATE TARGETING OF TUMOURS

lignRT system monitors the patient's surface area during radiotherapy treatment, using ceiling-mounted stereoscopic cameras. This allows the patient to be oned accurately and tracks for any movement during radiation therapy



Radiotherapy is a widely used treatment for cancer, allowing doctors to shrink tumours that cannot be surgically removed because of their size. However, there is a risk of damage to healthy tissue from radiation beams if the patient moves. Science writer Tereza Pultarova talked to Norman Smith from Vision RT, a finalist for the 2017 MacRobert Award, about its technology that accurately tracks a patient's position before and during treatment.

In the UK, one in every two people are likely to receive a cancer diagnosis at some point in their lives, and about 50% of them will require radiation therapy, which often successfully cures patients.

Radiotherapy works by targeting high-energy photon beams into tumours to destroy cancer cells. However, the radiation can also be harmful to healthy tissue, so healthcare professionals do their utmost to ensure that only the target area receives the dose. However, slight movements that patients naturally make during treatment can be almost impossible to detect, and these can lead to radiation damage to healthy tissue and potential long-term health problems.

Generally, in radiotherapy treatment, laser beams are used to help position the patient's body before the treatment begins. Since several radiotherapy fractions (a series of treatment sessions that make up the entire course) are usually spread over a number of weeks, repeated accurate positioning is needed. To aid this, the patient is often given tattoo marks in several places so that the radiotherapist can determine the exact location of the tumour inside the body, based on a previously taken CT (computerised tomography) scan.

In order to ensure that patients can be set up precisely and radiotherapy is delivered as accurately as possible, technology company Vision RT developed AlignRT, a completely non-contact system that continuously tracks the patient's position in 3D before and during treatment with better than one millimetre accuracy.

A pseudorandom pattern is projected onto the patient's body, which is picked up by the cameras and used to develop a 3D image of the patient's surface area to precisely calculate their location during treatment

CREATING A SYSTEM

After graduating in electrical engineering from the University of Cambridge and completing a PhD in medical image processing at Imperial College London, Norman Smith, CEO of Vision RT, joined a startup that was developing stereoscopic imaging techniques for various applications. Stereoscopic imaging systems mimic human visual perception to see surroundings in three dimensions; they consist of two cameras positioned at a known distance from each other, in the same way that humans have two eyes to perceive depth.

During this period, Smith visited a few radiotherapy clinics and was surprised at how primitive some of the techniques for setup and monitoring were. He was confident that stereoscopic technology would be able to monitor not just a few tattoo marks, but the entire patient before and during radiotherapy treatment, and would also remove the need for the tattoo reference points on the body, which can remain a permanent reminder for patients of their cancer. While it was here that the idea for the AlignRT technology was born, the company that Smith worked for was not interested in developing the idea further.

In 2001, Smith and co-founder and CTO Ivan Meir began operating as Vision RT from the attic of Smith's parents-in-law's house in North London. Gideon Hale, Vice-President Operations, joined the organisation 18 months later. The journey from vision to reality was not straightforward; at the time, there were no suitable 'off-the-shelf' stereoscopic camera systems available so they designed and engineered their own proprietary cameras, electronic hardware, processing software and user interface. Apart from the camera chips and lenses, all the system's components are manufactured in the UK.

The system continuously monitors the patient surface using three separate 3D camera modules that are ceiling-mounted in the radiotherapy treatment room and view the treatment table from different angles. The camera modules also contain a projector that illuminates the patient's body with a pseudorandom pattern on the surface of their body. This pattern is detected by the cameras and custom-written stereo-matching software to find corresponding points between pairs of calibrated stereo camera images. Through the process of triangulation, 3D coordinates are calculated for each set of 2D image points, which results in a 3D surface model comprising tens of thousands of points. The data from all 3D cameras is combined and the surface position is determined to sub-millimetric accuracy at a rate of 2 to 10 hertz (Hz) to precisely define the location of the patient as they undergo radiotherapy.

This accurate surface map is then dynamically matched to a reference surface model derived from a CT image, and treatment is planned using this. This allows the location of the tumour, based on the patient's body surface, to be tracked in all six degrees of freedom (the freedom of movement of a rigid body in a three-dimensional space) to ensure that the treatment is being delivered correctly.

ACCURACY, EASE AND COMFORT

In addition to real-time 3D mapping of the surface, the company has developed an easy-to-use interface software that, via a simple colour bar display, gives directions that allow the radiotherapy operator to position the patient faster and more accurately. Moreover, instead of relying on a human operator to stop the beam manually if

eiling-mounted modules contain a projector and stereoscopic camera. These need to be precisely aligned

unexpectedly system senses the movements from and automatically ation delivery, damage to healthy ther benefit of ment is that it s the need for to be held in ariety of invasive ion frames or ch were previously o restrict patient Historically, aff would also patient's position the tattooed osed-circuit om a neighbouring patient moved, manually stop n beam, but this nable to pick up ments and requires gilance.

surface mapping camera positions to aligned and rigid. d by the fact that radiotherapy treatment rooms are usually solid concrete-walled and roofed structures, but it also means that the camera/ projector modules must be rigid and thermally stable, which can be very difficult to achieve with an optical measurement system. The company solved this through careful mechanical design and an innovative thermal management solution, which ensures that the modules are operated at a controlled temperature.

Any detected motion must be synchronised to the treatment delivery machine (the linear accelerator), so the company has designed its own electronics modules for this and interfaced these, in collaboration with the different manufacturers, to their radiotherapy treatment delivery machines. The processing software, which at its core uses a mathematically complex and computationally challenging matching technique, must be both fast and accurate over the whole image to track any patient movement at high frame rates. The whole system calibration is checked daily and to do this the company has designed easy-to-use calibration phantoms that are mounted on the treatment couch, with built-in hardware and software consistency checks.

CONFIRMED RESULTS

The company's first prototype was tested at the Royal Marsden Hospital, a cancer specialist centre, in 2002. This confirmed that the system could track an object to within a millimetre, and the following year, Vision RT submitted its data to the American Society of Radiation Therapy and Oncology (ASTRO), the world's largest professional radiotherapy organisation. On acceptance of its paper for oral presentation, the company decided to attend ASTRO 2003 to exhibit its prototype technology at the associated industry trade show.

A chance encounter at the event between Smith and Meir and Dr George Chen, a professor at the Department of Radiation Oncology at Harvard Medical School and a leading authority in the field, initiated a productive relationship. Dr Chen had been attempting to develop something similar with MIT for three years, but had been unsuccessful. Within a year, Vision RT had installed a prototype system at Massachusetts General Hospital, Harvard Medical School's largest teaching hospital and one of the world's leading biomedical research facilities.

By 2005, two scientific papers were published in peer-reviewed journals; soon after, the company received clearance to market the technology in the US and the first units were sold, initially to leading academic

The software aids healthcare professionals in rapid positioning of patients prior to radiotherapy, and highlights any movement they make from their intended position during treatment

institutions that were focused more on technical efficacy than usability. As Vision RT's market expanded, feedback from more routine users complaining about the ergonomics of the system required the engineers to completely redesign the system's user interface to make it very easy to operate. During subsequent years, a new and improved camera/projector system was developed, as well as an enhanced calibration technique to enable the exceptional accuracy that is required for radiosurgery treatment where the radiation beam is both narrower and of significantly higher intensity.

EXPLORING NEW AREAS

Vision RT long ago left behind the cramped attic of Smith's parents-in-law's home and its team of five has expanded to more than 150 people. Now, the technology is used in 70% of the top 50 cancer centres in the USA and in more than 30 countries around the world. The system is becoming a standard in radiotherapy treatment. The company has pioneered a new field of surface-guided radiotherapy treatment (SGRT) and has built up and supported a 'SGRT community', consisting of almost 1,000 healthcare professionals, which trains, shares and helps develop the technique and the AlignRT product.

The company has also signed a distribution agreement with Varian Medical Systems, the world's largest manufacturer of radiotherapy treatment delivery systems, through which a Varian-branded version of the AlignRT 3D surface-imaging technology has become part of the company's offering; it is incorporated into many of Varian's radiotherapy systems.

The accuracy and rapid mapping of the system has enabled an advanced form of treatment for left-breast tumours. Because of the closeness of the heart to these tumours, damage to cardiac blood supply is a common complication. However, if the patient takes a deep breath, this moves the heart away from the chest wall. By monitoring when the patient is holding their breath, the AlignRT technique can ensure that the radiation is only delivered to the tumour during this period and not to the heart. Using the simple alignment software that had already been designed to enable the radiographer to position the patient, Vision RT has produced a simple tablet-based bar graph display that the patient can use to ensure that they have taken a deep enough breath to move the chest wall, and hence the tumour and heart, into the right position for treatment.

The treatment of left-breast cancer has been a success story; a recent clinical study in North Carolina showed that no patients who were treated with the guidance of AlignRT experienced damage to heart blood supply, in comparison to 27% in a previous study using traditional techniques.

With Vision RT having over 50 patents to date and its clinical evidence being evidenced in more than 60 peer-reviewed papers, it seems that the practice of SGRT can only go from strength to strength.

BIOGRAPHY

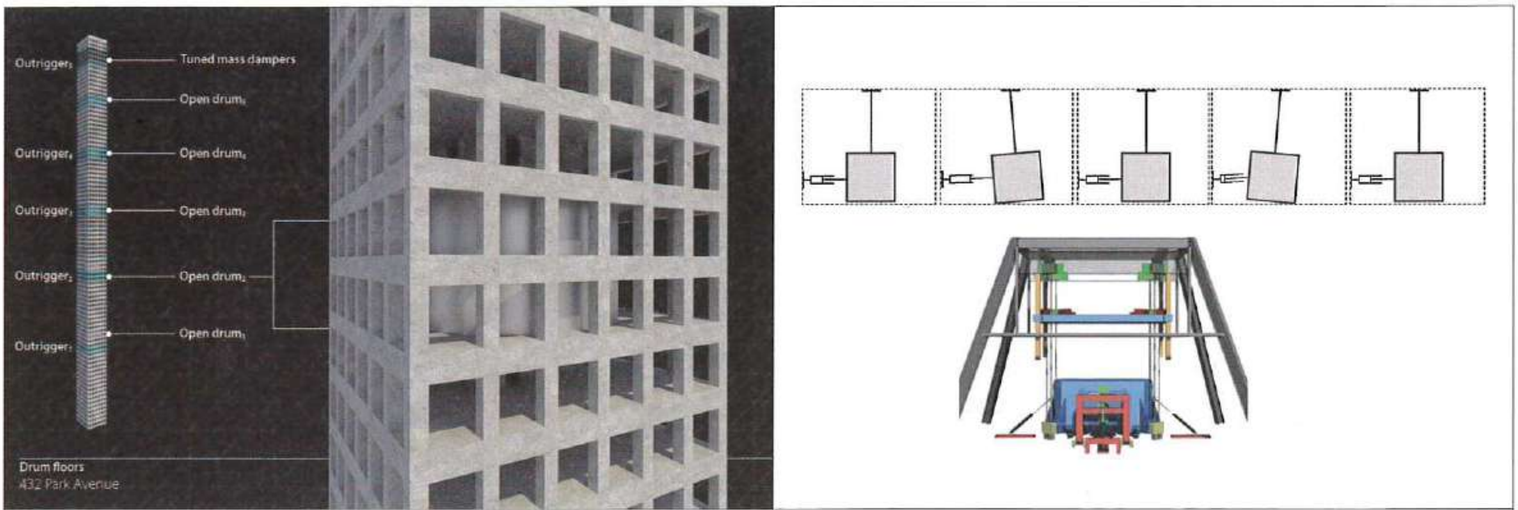
Dr Norman Smith is CEO and co-founder of Vision RT. He studied engineering at the University of Cambridge and holds an MSc and PhD in medical imaging from Imperial College London. Dr Smith is a Fellow of the Institution of Engineering and Technology, and is named inventor on several patents.

The author would also like to thank Professor David Delpy CBE FREng FRS for his help in putting together this article.

THE SKY'S THE LIMIT

Avenue is one of the most recent additions to the New York City skyline. The building stands at 426.5 storeys) high. At about every 12th storey, the building has been left windowless and 'open' floors have ced, to reduce vibration in windy conditions © Nicola Evans

Tall buildings are rising in cities all over the world, at a rate and with a variety never seen before. Engineer and writer Hugh Ferguson talked to skyscraper designer and London-based structural engineer Kamran Moazami, about one of New York's most recent skyscrapers – 432 Park Avenue – and the engineering challenges of designing tall buildings.



(Left): The square-shaped concrete core of the building is connected by 'outriggers' (double-storey concrete frames) to the exterior girder-column grid at roughly every 12th floor so that they act monolithically and increase the overall lateral stiffness. (Right): Two mass-tuned dampers were also added to the top of the building to control its movement. A tuned-mass damper is a mass that is supported by a pendulum arrangement and connected to the structure by means of springs and/or dampers

In the heart of Manhattan, where land values are among the highest in the world, stands 432 Park Avenue, one of New York City's newest skyscrapers and a towering example of what new engineering techniques, combined with high-performance materials, can achieve.

New York was the birthplace of the skyscraper, notably with the Chrysler Building and Empire State Building in the early 1930s, both of which are steel-frame buildings of rectilinear shapes. In the 1960s, the skyscraper had something of a resurgence with the introduction of tubes (usually in steel) for perimeter and interior columns and perimeter diagonal bracing, which greatly reduced the weight of steel required and hence the cost. Many buildings over 40 storeys that have been constructed since the 1960s use a tube system adapted from the structural engineering principles of Bangladeshi-American structural engineer Fazlur Khan. This included the John Hancock Center in Chicago, and the World Trade Center twin towers in New York (1973).

Tall buildings continue to spring up throughout the world in response to many factors, such as increasing urbanisation, a desire to create iconic buildings, and the discovery that people across the world love grand views. Now, more than half of the world's 20 tallest buildings are in China (with four elsewhere in the Far East, three in the USA and two in the Middle East). Before 2000, two thirds of skyscrapers were commercial: since then, more than two-thirds have been residential or mixed-use.

SLENDER AND STABLE

At 426.5 metres, 432 Park Avenue is one of the world's 20 tallest buildings. It is the second tallest building in New York City and the third tallest in the USA. The building is entirely residential and currently stands as the tallest residential tower in the Western Hemisphere. However, much more significantly, with its compact 28.5 metre-square footprint, it has an aspect ratio (height/width) of 15:1, a slenderness that would have been unthinkable a decade ago. This has helped the developer to gain more usable space out of a comparatively small plot, and to give the residents unrivalled views of the Manhattan skyline. It also presented interesting challenges for the building's engineers.

432 Park Avenue had to be designed to transfer its own weight down to the foundations, to resist seismic and wind loads, and – most challenging of all with such a slender building – to manage the movement of the building under wind loading so that no movement would be detectable. All tall buildings move in the wind, but occupants, particularly residents, do not want to feel the sway. That meant keeping the lateral acceleration of the building (its movement in wind) below around 0.1 m/s2 (metre per second squared) in a 'once-in-a-year' gale.

On most tall buildings, extensive wind tunnel testing is an integral part of the structural analysis and design. This may include: high-frequency force balance tests; multi-degree of freedom aeroelastic modelling, which explores the independently acting factors that influence the interaction between inertial, elastic, and aerodynamic forces that occur when the building is exposed to wind; wind studies done at large scale on just one section of the building at a time; cladding pressure and wind studies to ensure that cladding panels, windows and fixings will not be blown off in a gale; pedestrian-level comfort testing, to ensure downdraft wind on the building and any street-level wind tunneling does not create discomfort for nearby pedestrians; and wind-induced aeroacoustic studies, to check and ensure that wind will not cause any whistling or noise-generated issues.

At the time, 432 Park Avenue was one of the most slender buildings in the world, designed by engineering consultant WSP. Therefore, wind-related testing was particularly rigorous, with

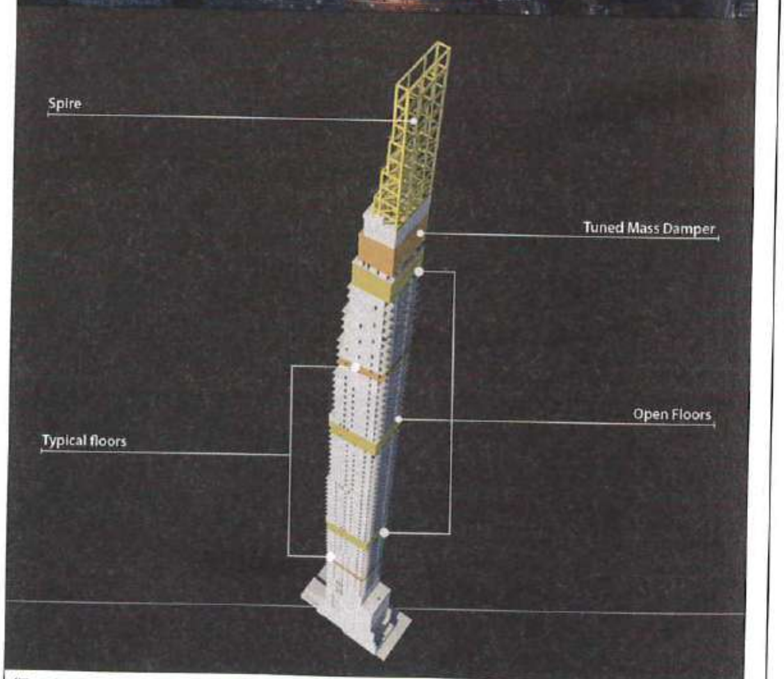
On most tall buildings, extensive wind tunnel testing is an integral part of the structural analysis and design

ound five force-balance tests
cked up by several all-day
orkshops to establish the
timum configuration, followed
a few aeroelastic tests on the
al design.

First, the building had to made as stiff as possible to sist overall bending, while still ving large, open areas on the or plan for attractive living aces. This was achieved by eating a 9 metre by 9 metre ncrete shear-wall core around e lift shafts, which formed e building's central spine. s conventionally, the strong e was supplemented by a mework of concrete columns d spandrel beams forming building envelope and iding the need for externally unted cladding. Cast-in-situ rs span between the central e and the perimeter frames. ensure that the concrete imeter framing and interior e respond monolithically to ctural demands, they were nected by massive double-ey concrete frames, known utriggers', which were cealed in the plant rooms at ut every 12th floor.

Despite its lateral stiffness, tower still tended to move cceptably in varying wind ditions, and the lateral--carrying system had e 'tuned' to minimise lacement, acceleration and vibration. This was solved by adding mass to the upper levels, achieved by increasing the typical 250-millimetre thickness of the concrete floors to 450 millimetres. Two 650 tonne mass-tuned dampers were also added to the top of the building: these are large steel weights supported by cables as pendulums and connected laterally to the structure by viscous dampers, which absorb the energy of the moving weights and transfer the associated forces into the building structure, slowing the building's acceleration to acceptable limits.

Early wind tunnel testing had shown that the building exhibited significant 'vortex shedding', which is an oscillating airflow phenomenon that can occur when wind flows past a building at certain velocities. Vortices are formed at the back of the building, detaching periodically from either side. Each low-pressure vortex tends to pull the building towards the centre of the vortex, so the effect can be to induce vibration, particularly if the frequency of vortex shedding approaches the resonant frequency of the structure. The solution at 432 Park Avenue was ingenious and innovative: by leaving the building windowless at precise levels and creating 'open' floors – a technique comparable to making



(Top): A rendering of 111 West 57th Street. Its aspect ratio of 24:1 will make it the world's most slender building. (Bottom): Shear walls run the full length of the east and west exteriors and have been thickened to as much as one metre. Like 432 Park Avenue, it also has open floors that the wind will pass through to reduce vibration

holes in the sail of a boat – the wind loading was reduced and so was the vortex shedding. Conveniently, these levels could be designed to coincide with the plant rooms, and to some extent the outriggers, at about every 12th floor.

Although these 'open' floors take up valuable floor space, this is more than compensated for by the additional stability that allowed the height of the building to be increased to 86 storeys.

The concrete required for the building was no ordinary concrete. For the lower sections carrying larger vertical and lateral loads, it had to have a specified compressive strength of more than 100 MPa ('high strength' concrete is considered to have a compressive strength of 50 MPa or more). All structural concrete was designed for enhanced durability by minimising air content and the water/cementitious materials ratio, and with a higher-than-normal modulus of elasticity to minimise deformation. To aid placing and to improve the finished appearance, it had to be able to be pumped to great heights. Since there was to be no cladding to cover potential blemishes on the exterior of the building, architectural exposed concrete was specified in an attractive white colour and was placed with near-perfect workmanship. Although the exterior concrete should be maintenance-free, the exterior of the building required access for glass cleaning and replacement, which will be done using a telescopic building maintenance unit (BMU) mounted on the roof.

EVEN TALLER TOWERS

Now, just three blocks away, 432 Park Avenue's successor is nearing completion. Not content with an aspect ratio of 15:1, 111 West 57th Street has a ratio of 24:1, which will make it the world's most slender building. At 438 metres, it is slightly taller than its neighbour and shares many features: both are high-end residential buildings, both use high-strength concrete, and both have a tuned mass damper on the top and gaps allowing the wind to pass through to minimise vortex shedding.

The main difference is that the architect and developer for the latter wanted clear, unobstructed views northwards over Central Park and south towards Lower Manhattan. The solution was a concrete core, linked to two concrete shear walls (a structural system composed of braced panels) running the height of the building on the east and west sides and allowing clear views north and south, so that the overall structure resembles a giant cantilevered concrete I-beam standing on its end. The tower is slightly tapered, and the external walls will be clad in terracotta and bronze rather than architecturally exposed concrete. Even more slender buildings are now being planned, although an aspect ratio of 24:1 is approaching the limit of what is currently feasible.

EVOLUTION OF SKYSCRAPERS

The buildings at 432 Park Avenue and 111 West 57th Street are responding to the specific parameters of the New York City market, including strong demand for high-end residential properties with outstanding views, and locally available high-strength coarse aggregates needed for making ultra-high-strength concrete (a similar compressive strength in London, for example, would require imported aggregate, and is therefore a much more expensive concrete). In different parts of the globe, the parameters or constraints are likely to be different and so too are the solutions.

The original rectilinear steel buildings most usually associated with skyscrapers have largely been succeeded by concrete or composite concrete-and-steel, and the equally ubiquitous office buildings have given way to varieties of mixed-use towers: modern skyscrapers are

SAFER SKYSCRAPERS

Skyscrapers are now among some of the safest buildings in the world, if only because they must be thoroughly engineered to the finest detail, and not just for structural safety. For example, the fixings of every external cladding panel should allow for sufficient movement to accommodate the building's sway under anticipated conditions.

Most modern tall buildings have a fire strategy, which covers such issues as means of escape, detection and alarm, fire suppression, smoke control, firefighter access and facilities, compartmentation, and internal and external spread of fire. Most high-rises now include dedicated fire stairs and some have lifts that can be used for fire escape, which require special details such as pressurised lobbies where people can wait for the lifts and an independent cooling system for the lift motor room. The main fire element of tall building design is ensuring that compartmentation is maintained until the fire is exhausted or controlled. Spread of fire on the exterior of a building is usually mitigated by choosing insulation materials of limited combustibility, provision of adequate fire-stopping, cavity barriers around openings, and main compartmentation lines. For example, in August 2016, a fire burned dramatically for two hours on the outside of Dubai's 84-storey Torch Tower, where some of the cladding systems were not of limited combustibility and fire-stopping was inadequate. Despite this, other elements of the fire strategy worked well, particularly suppression that ensured safe escape for occupants, and consequently there were no casualties.

...NGINEERING CHALLENGES OF TALL BUILDINGS

...d

...d in 2012, the Shard is London and the UK's tallest ...at 310 metres and 95 storeys ('Building the Shard' *Ingenia* ...haped as an irregular pyramid with highly complex .... The Shard is a modern mixed-use tall building with retail ...e, offices in the lower levels, a hotel in the middle and ...l near the top. A strong central concrete core and a hat ... engages all perimeter columns form the spine of the ...and the remainder varies with use: structural steel was the ... solution for the offices, where deep steel beams could ...nty of space for the extensive services required. For the ... flats, where acoustic separation was more important and ...ans were shorter owing to the tapering of the building, ...floors were preferred, which were thinner so that two ...eys could be added within the same overall building ...e tower then reverts to steel for the viewing gallery and

...dland

...netre high, 60-floor Newfoundland residential tower in ...narf, London, straddles the Jubilee Line tube tunnels, ... close to the surface at the tower's location. This required ...ing between and beside the tunnels, as well as a major ...fer structure to connect the building's loads with the ...vever, to avoid disturbance to the tunnels, it was necessary ...e weight of the building to a minimum. The solution was ... of diagonally placed steel members on the outside of the ...reating a diamond pattern, and the use of post-tensioned ...loor slabs connecting the diagrid to the concrete ...diagrid was so light and strong that the thickness (and ...the weight) of the shear walls in the concrete core could ...d to just 300 millimetres.

Torre Mayor

The 225-metre high, 55-storey Torre Mayor in Mexico City was Latin America's tallest building when it opened in 2003 and is now Mexico's fourth tallest. The design priority for this building was seismic design, as it stands in the lakebed area where the heaviest damage from the catastrophic Michoacán 1985 earthquake was recorded. Designed to resist an 8.5 Richter earthquake, Torre Mayor has an arrangement of 96 viscous fluid dampers (with technology borrowed from the shock-absorbers of a car) attached to the diagonal cross-bracing on the perimeter of the building, which safely absorb the earthquake's energy. A 7.6 Richter earthquake in 2003 did no damage: indeed, the occupants did not even notice that a tremor had taken place. The structure also responded as expected when the 7.1-magnitude earthquake of Axochiapan occured in September 2017.

22 Bishopsgate

When it is completed in 2019, 22 Bishopsgate will be London's second tallest building at 278 metres with 62 floors. This will be even more of a 'vertical village' including retail, restaurants, an auditorium, various leisure facilities, including a climbing wall on the 25th floor, and offices. It will be the first building in the UK to be accredited for standards to improve building users' health and wellbeing. The main challenge here was to address what had gone before: a previous scheme was abandoned in 2012 because of the recession, with just the foundations and seven storeys of the core constructed. The core was deconstructed, but for the new tower to be economical, a way had to be found to place an entirely different building on the existing foundations (including London's deepest piles). This required elaborate analysis and design at the base of the building to transfer the loads to the foundations such that the full strength of the existing piles was mobilised, but no individual pile was overloaded.

...self-contained cities in ...n a mix of residences, ...els, restaurants and ...s.

...stronger materials ...ed, there is no ...mit to how high ... could be built. ...reseeable limits are ...the larger footprints ...ground level and ...rt links to service the ...pants, for example the constraints of providing fast and efficient vertical transportation, and economic viability: larger buildings cost more, take longer to build, and take longer to let, so developers need even deeper pockets. However, with increased urbanisation, a public appetite for tall buildings and the ingenuity of engineers, the surge in skyscrapers worldwide is set to increase still further – and some of them will be even taller.

BIOGRAPHY

Kamran Moazami is Managing Director, Property and Buildings, at WSP. He studied structural engineering at Columbia University in New York. After obtaining his master's degree, he joined New York skyscraper firm, Cantor Seinuk, where he worked on several high-rise buildings. In 1996, he relocated to the UK to head up the company's London office. Later on, Cantor Seinuk was acquired by London-based firm WSP.

Hugh Ferguson also talked to Nick Offer, Head of London Building Services for WSP.

All projects mentioned in this article have been structurally designed by WSP

Glossary

Acceleration Rate of change of velocity. Units are m/s^2.

Activity/count rate The number of radioactive emissions in a certain time.

Air resistance (drag) The force from the air that opposes movement.

Alpha (radiation/particles) A type of radioactive emission with low penetrating power blocked by paper.

An alpha particle is made up of two neutrons and two protons (a helium nucleus).

Alternating current (a.c.) A current that changes direction as the supply voltage changes from + to −, or − to +.

Ammeter An instrument used to measure the size of an electric current.

Ampere (amp) The unit of electric current.

Amplitude The maximum displacement of a wave from the equilibrium position.

Analogue signals Signals carried as continuous waves that vary in frequency and amplitude.

Anode The positively-charged electrode.

Artificial satellite A satellite put into orbit from the Earth.

Atmosphere The layer of gases around the Earth.

Atom The smallest part of an element that can exist. Atoms have a nucleus consisting of protons and neutrons around which are shells of electrons.

Atomic number The number of protons present in an atomic nucleus (and the number of electrons present in the neutral atom).

Background radiation The radioactivity that is always present in the environment.

Battery A number of electrical cells joined together.

Beta (radiation/particles) A type of radioactive emission with moderate penetrating power blocked by thin sheets of aluminium.

Beta particles are high-energy electrons.

Big Bang theory A theory that considers that the Universe started from a gigantic explosion.

Black hole An object in space that is so dense and its gravitational field so strong that light and other forms of electromagnetic radiation cannot escape from it.

Braking distance The distance a vehicle travels before stopping after the brakes have been applied.

Capacitor A device designed to store electrical charge.

Cathode The negatively-charged electrode.

Cell (electrical) The single unit from which batteries are made.

Centre of mass The point where all the mass of an object can be thought to be concentrated.

Chain reaction A reaction in which a nucleus is split and neutrons released that can split other nuclei to produce a continuous chain of events.

Charge A feature of atomic particles. Protons and electrons have a charge. Electrons have a negative charge and protons have a positive charge. Opposite charges attract; like charges repel.

Circuit breaker A device that uses the action of an electromagnet to switch off an electrical supply very rapidly.

Comet An object made of ice and rock which orbits the Sun in a different plane to the planets.

Compression (forces) The process of being squashed.

Compression (waves) The region in a longitudinal wave where the vibrating particles of the medium are closer together than normal.

Conduction (electrical) The transfer of electrical energy along a material by free electrons.

Conduction (heat) The transfer of heat energy along a material.

Conductor (electrical) A substance allowing electrical charge to pass through.

Conductor (heat) A substance allowing heat energy to pass through.

Convection The transfer of heat energy in a liquid or a gas (fluid) caused by differences in density. Warmer, less dense fluids rise. Cooler, more dense fluids sink.

Converging lens Any lens that is thicker in the middle than it is at the edges.

Core (Earth) The innermost part of the Earth.

Core (electromagnets) The central part of electromagnet around which current-carrying coils of wire are wound.

Cosmic ray Rays and particles from space reaching Earth.

Coulomb Unit of electric charge.

Crest The point of maximum displacement in a transverse wave.

Critical angle When a ray of light, travelling in a more dense medium, hits the boundary between the more dense medium and the less dense medium and only just emerges by refraction, the angle of incidence of the ray in the more dense medium is called the critical angle.

Crust The surface layer of the Earth.

Cycle (vibration) For a transverse wave one cycle is one trough and one crest. In a longitudinal wave one cycle is one compression and one rarefaction.

Decay (atomic) The break up of unstable nuclei resulting in the production of radioactive emissions.

Decelerate To slow down.

Diffraction The spreading out of a wave as it passes through a narrow gap or moves past an object.

Digital signals Signals carried as a series of 'on' and 'off' pulses.

Diode An electrical device that only conducts electricity in one direction.

Direct current (d.c.) Electric current that does not change direction.

Diverging lens Any lens that is thicker at the edges than it is the middle.

Drag The force from the air that opposes movement.

Dynamo (generator) Device supplying a voltage from the relative motion of a conductor with a magnetic field.

Earthing The linking of a low resistance wire to a metal object to provide a low resistance path to the Earth's surface for an electric current.

Efficiency The ratio of useful energy transferred by device to total energy transferred to device.

Elastic collision A collision that involves no overall change in kinetic energy.

Elastic potential energy The energy stored in an object when work has been done to change its shape.

Electric charge A quantity of electricity.

Electric current The flow of electrons or ions. The rate of transferring electric charge. Units are amperes (amps)

Electrical energy Energy transferred by a charge.

Electrode A negatively or positively charged conductor.

Electrolysis The process of splitting up a chemical compound using an electric current.

Electromagnetic induction The production of a voltage or current across a conductor in relative motion within a magnetic field.

Electromagnetic spectrum The range of frequencies and wavelengths of electromagnetic waves.

Electromagnetic waves Transverse waves that have a common speed in air or a vacuum.

Electrons Negatively-charged sub-atomic particles orbiting in shells around the atomic nucleus.

Electrostatic forces Forces due to stationary electric charge. Like charges repel, unlike charges attract.

Element A substance made up of atoms which contain the same number of protons so contain only one type of atom, and which cannot be broken down into anything simpler by chemical means.

Fetus The name given to an unborn child more than 8 weeks after conception.

Focus The point through which parallel rays of light incident on a converging lens will be refracted.

Fossil fuels The non-renewable energy resources: crude oil, natural gas and coal.

Free electron The electrons in metals that move around inside the metal and do not remain in orbit around a nucleus. The presence of these free electrons allows the metal to conduct electricity and heat.

Frequency The number of cycles (vibrations) per second. Units are Hertz (Hz).

Friction A force which opposes the movement of an object.

Fuse A wire fitted in plugs that is designed to melt if too large a current flows through it.

Fusion (atomic) The joining of small nuclei to form a large nucleus. The process transfers heat energy to the surroundings.

Galaxy A vast number of star systems held together by gravitational forces.

Gamma (radiation) A type of radioactive emission with high penetrating power blocked by concrete/lead. Gamma radiation is part of the electromagnetic spectrum and has a very high frequency.

Geiger–Müller tube (GM tube) A detector of radioactive emissions.

Generator (dynamo) Device supplying a voltage from the relative motion of a conductor within a magnetic field.

Geostationary satellite A satellite which takes 24 hours to orbit the Earth.

Geothermal energy The energy produced in the Earth by natural heating process.

Global warming An international problem caused partly by the increase in the amounts of carbon dioxide and methane in the atmosphere which results in an increase in the average temperature of the Earth.

Gravitational potential energy The energy stored in an object due to the vertical height through which it has been lifted.

Gravity (gravitational force) A force of attraction that acts between all objects.

Greenhouse effect The effect in the atmosphere of heat energy being absorbed by gases such as carbon dioxide and methane.

Half-life The time taken for half a given number of radioactive atoms to decay to different atoms.

Hertz (Hz) The unit of frequency.

Hydroelectric Electrical power generated by the flow of moving water.

Input sensors Devices that detect changes in the environment.

Insulator (electrical) A substance not allowing an electric current to flow and charges to move.

Insulator (heat) A substance not allowing a transfer of heat energy from a hot region to a cold region.

Ion An atom or group of atoms which have lost or gained electrons to become positively or negatively charged.

Ionise To remove or add electrons to atoms or groups of atoms so giving them positive or negative charges.

Isotope Atoms of the same element which contain different numbers of neutrons.

Joule The unit of energy.

Kilowatt 1000 watts.

Kilowatt hour A unit of electrical energy.

Kinetic energy The energy possessed by an object due to its motion.

LDR (light dependent resistor) An electrical component the resistance of which decreases when light shines on it.

Light year The distance a light ray travels in one year.

Lithosphere The outer shell of the Earth made from the crust and the upper part of the mantle.

Logic gate A type of electronic switch used to process information.

Longitudinal wave A wave in which the vibrations of the particles are in the same direction as the energy transferred along the wave.

Magma Molten rock below the Earth's crust.

Magnet An object that attracts magnetic materials such as iron, steel, nickel and cobalt.

Magnetic field The region around a magnet where a magnetic material experiences a magnetic force.

Mantle The layer of the Earth between the crust and the core.

Mass number The total number of protons and neutrons in an atomic nucleus.

Mass The amount of matter an object contains Units are kg.

Milky Way The galaxy containing our solar system.

Moment The size of the turning effect of a force, measured in Nm.

Momentum Defined as mass × velocity. Units are kg m/s.

Moon A natural satellite in orbit around a planet.

Motor effect The motion of a current-carrying conductor in a magnetic field

Neutron A particle with no electrical charge found in the nucleus of most atoms. Its mass is similar to that of a proton.

Newton The unit of force (N).

Non-renewable (finite) energy resources Energy resources that, once used, cannot be replaced.

Normal The line drawn at right angles to a surface.

Nuclear fission The breaking up of a large atomic nucleus to release energy.

Nucleon The protons and neutrons in the nucleus of an atom.

Nucleus (atom) The central part of an atom that contains positively-charged protons and uncharged neutrons.

Ohm The unit of electrical resistance.

Orbit The regular path taken by an object which passes around another object.

Output devices The part of an electronic system controlled by the processor. It transfers electrical enery to other forms of energy.

Parallel circuits Closed electrical circuits that provide several pathways for an electric current.

Pivot A point that objects turn around.

Planet A very large object which orbits the Sun.

Potential difference The voltage between two points in a circuit.

Potential divider A combination of resistors in series, used to split the voltage of a battery into two parts.

Power The rate of transfer of energy. Units are watts.

Primary coil The input coil in a transformer.

Processor The part of an electronic system that decides what action is needed.

Proton A positively-charged particle found in the nucleus of an atom. It has a mass similar to that of a neutron and the number of protons present decides which element is present.

P waves Longitudinal seismic waves which travel through solids and liquids.

Radiation (heat transfer) A process by which heat is transferred.

Radiation (nuclear) The random emission of energy from an atomic nucleus as the result of the breakdown of unstable nuclei.

Radioactive (radiocarbon) dating The use of half-life to date ancient organic objects.

Radioactive decay The emission of particles or rays from an unstable atomic nucleus.

Radioactive emissions The particles and rays produced as the result of the breakdown of unstable nuclei.

Radioactive tracer A radioactive substance, usually with a relatively short half-life, that is passed into the body and used to detect, for example, the presence of cancers, tumours or the direction of blood flow. Tracers can be used in the treatment of cancers and tumours. Tracers can also be used to monitor the flow of liquids and gases in underground pipes.

Radioactivity The random emission of energy from an atomic nucleus as the result of the breakdown of unstable nuclei.

Radioisotope A radioactive isotope.

Radionuclides Materials which produce ionising radiation, such as X rays, gamma radiation, alpha particles and beta particles.

Random Spontaneous not regular.

Rarefaction The region in a longitudinal wave where the vibrating particles of the medium are further apart than normal.

Real image An image that can be shown on a screen.

Red giant A relatively cool giant star.

Red shift The effect on the spectrum of a galaxy moving away from us.

Refraction The change in direction of a wave when it passes from one medium to another due to a change in speed when passing from one medium to another.

Relay An electromagnetic switch.

Renewable energy resources Energy resources that will always be available.

Resistance A measurement describing the difficulty of electric current flow in a conductor. Units are ohms.

Resistor A device for controlling the current in a circuit.

Satellite An object which orbits a planet.

Secondary coil The output coil in a transformer.

Seismic waves These are waves created in the Earth by vibrations due to earthquakes.

Series circuits Closed electrical networks giving only one pathway for an electric current.

Solar system A system made up of the Sun, planets, moons, asteroids and comets.

Speed The distance an object travels in a unit of time. Units are m/s.

Star A source of light due to heat caused by nuclear fusion.

Sun A star at the centre of a solar system.

S waves Transverse seismic waves that can only travel through solids.

Tectonic plates The separate slow-moving adjacent sections of the Earth's lithosphere that move because of convection currents within the Earth's mantle caused by the natural radioactive processes within the Earth.

Terminal velocity The constant speed reached by a falling object when the forces acting on it (in the direction of its motion) are balanced.

Thermistor An electrical component in which the resistance decreases when it gets warm.

Thinking distance The distance travelled by a car during the driver's reaction time.

Total internal reflection This takes place at the boundary of two materials when light travelling in the more dense material strikes the boundary at an angle of incidence greater than the critical angle.

Transformer A device that changes the size of an alternating voltage.

Transistor A device that can be used as a high speed electronic switch.

Transverse wave A wave in which the vibrations of the particles are at right angles to the direction of the energy transferred along the wave.

Trough The point of maximum displacement in a transverse wave in the opposite direction to a peak.

Turbine A device that turns a generator.

Ultrasound Sound of too high a frequency to be heard by humans.

Universe Made up of innumerable galaxies.

Velocity The speed of an object in a particular direction. Units are m/s.

Vibration The movement needed to produce a wave.

Virtual image An image that cannot be shown on a screen.

Volt The unit of potential difference 1 volt = 1 joule/coulomb.

Voltage The electrical energy difference of a unit charge moved across two points in a circuit; energy transferred per unit charge.

Voltmeter An instrument used to measure potential difference.

Watt The unit of power.

Wavelength The distance between adjacent crests in a wave equivalent to the distance taken by one complete cycle.

Waves Vibrations that transfer energy but not matter.

Wave speed The distance travelled by a wave in a second. Units are m/s.

Weight The force due to gravity on an object. Units are newtons.

White dwarf A small very dense star.

Work That which is done when a force moves an object a certain distance. Units are joules.

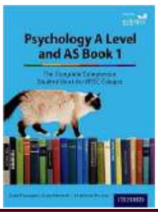
Transition Task: Psychology

Subject	Qualification	Examination Board
Psychology	A level	Eduqas
Additional Information:		

Task Overview:
The attached pack takes you through a variety of activities to introduce you to psychology Including – Books. Films and TED talks, Activities involving note taking skills, Topics to research and Online learning courses
Success Criteria:
Please see the pack for details
Resources:
Please see the pack for details
How the work produced will fit into subsequent work and the specification as a whole?
This will give you an introduction to the wide and varied field on Psychology. Some of it will tie directly into your course, other aspects will give you an opportunity to gain a wider understanding of this fascinating subject
How the work should be presented?
You can present the work either typed or handwritten. For the Mooc course, either a printed end of course notification or a screen shot is fine.

Who to contact if you should require further assistance with the work before the end of term?	If you have any queries please contact Mrs Kitching, Head of Psychology at h.kitching@gildredgehouse.org.uk
Length of time expected to complete tasks:	These tasks should last you around 12 weeks. If you find you want more to do, you could always choose another Mooc.
Submission Requirements:	

What equipment will be needed for the subject?



The Complete Companions for Eduqas Year 1 and AS Psychology Student Book (Psychology Complete Companion) Paperback – 16 Jul 2015
by Cara Flanagan (Author), Rhiannon Murray (Author), Lucy Hartnoll (Author)

Optional Extension Task/Further Reading

Extension work:

a) What is the difference between Systole and Diastole? Link this to the cardiac cycle.
b) What is a risk factor for heart disease?
c) What are the features of a good study?
d) Explain the what causes the symptoms of Cystic fibrosis.
e) What are the different types of stem cell and suggest ethical issues with using of medical research
f) What is the difference between Systole and Diastole? Link this to the cardiac cycle.
g) What is a risk factor for heart disease?
h) What are the features of a good study?
i) Explain the what causes the symptoms of Cystic fibrosis.
j) What are the different types of stem cell and suggest ethical issues with using of medical research

TRANSITION PACK FOR A-LEVEL PSYCHOLOGY

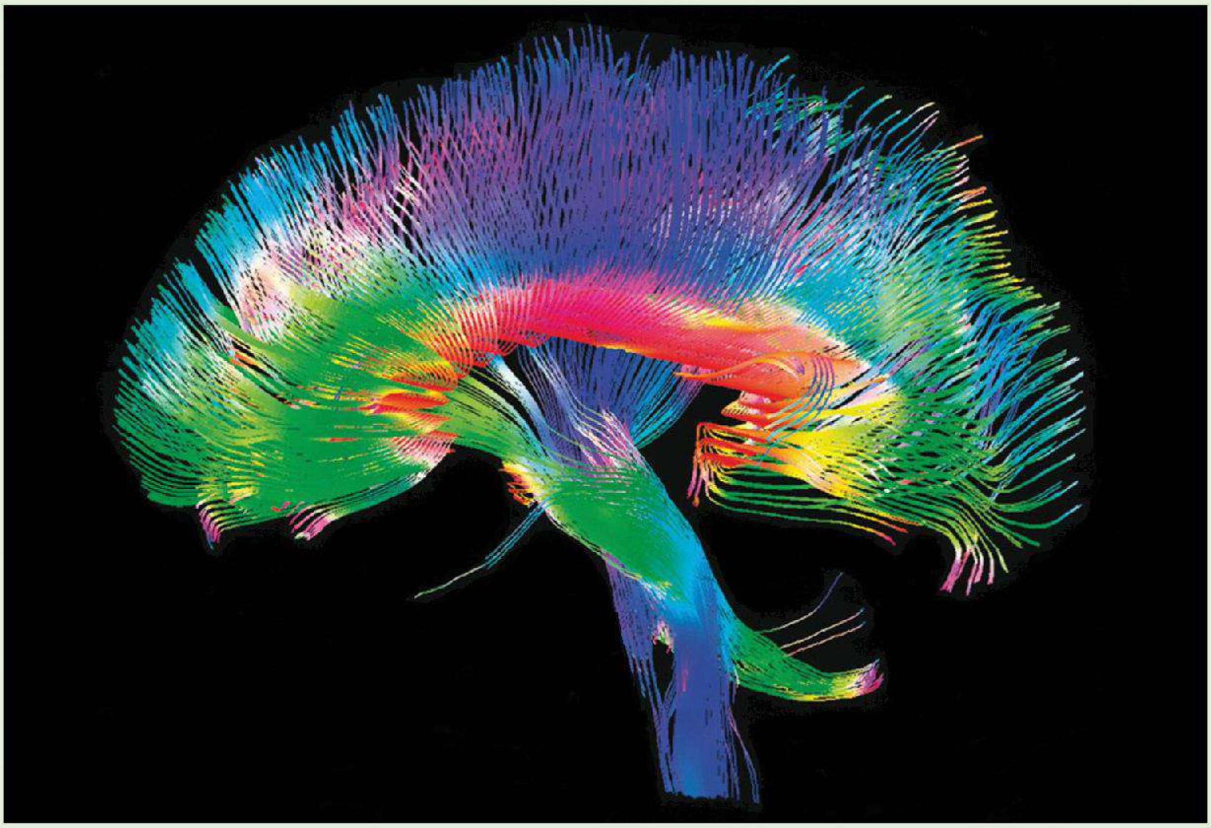
A guide to help you get ready for A-level Eduqas Psychology at Gildredge House featuring

- ✓ Books
- ✓ Films and TED talks
- ✓ Activities involving note taking skills
- ✓ Topics to research
- ✓ Online learning courses

ATP

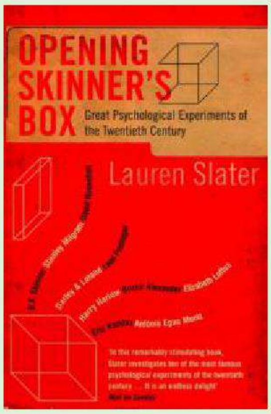
For the Teacher by the Teacher

This pack contains a programme of activities and resources to prepare you to start an A-level in Psychology in September. It is aimed to be whilst you are unable to go to school to ensure you are ready to start your course in September.

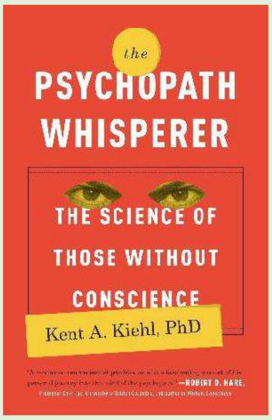


You could read this

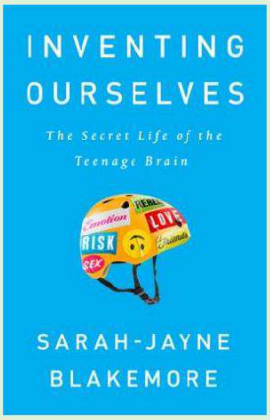
These books are all popular books about psychology, and great for extending your knowledge and understanding.



In Opening Skinner's Box, Lauren Slater sets out to investigate the twentieth century through a series of ten fascinating, witty and sometimes shocking accounts of its key psychological experiments. Starting with the founder of modern scientific experimentation, B.F. Skinner, Slater traces the evolution of the last hundred years' most pressing concerns - free will, authoritarianism, violence, conformity and morality.



We know of psychopaths from chilling headlines and stories in the news and movies from Ted Bundy and John Wayne Gacy, to Hannibal Lecter and Dexter Morgan. As Dr Kent Kiehl shows, psychopaths can be identified by a checklist of symptoms that includes pathological lying; lack of empathy, guilt, and remorse; grandiose sense of self-worth; manipulation; and failure to accept ones actions. But why do psychopaths behave the way they do? Is it the result of their environment how they were raised or is there a genetic component to their lack of conscience?



Drawing upon her cutting-edge research Professor Blakemore explores: What makes the adolescent brain different? Why does an easy child become a challenging teenager? What drives the excessive risk-taking and the need for intense friendships common to teenagers? Why it is that many mental illnesses – depression, addiction, schizophrenia – begin during these formative years. And she shows that while adolescence is a period of vulnerability, it is also a time of enormous creativity and opportunity.

You could watch this

What happens when you put good people in an evil place? Does humanity win over evil, or does evil triumph? These are some of the questions we posed in this dramatic simulation of prison life. In 1971, Stanford's Professor Philip Zimbardo conducts a controversial psychology experiment. Twenty four male students are selected to take on randomly assigned roles of prisoners and guards in a mock prison situated in the basement of the Stanford psychology building.

McMurphy has a criminal past and has once again got himself into trouble and is sentenced by the court. To escape labour duties in prison, McMurphy pleads insanity and is sent to a ward for the mentally unstable. Once here, McMurphy both endures and stands witness to the abuse and degradation of the oppressive Nurse Ratched, who gains superiority and power through the flaws of the other inmates. McMurphy and the other inmates band together to make a rebellious stance against the atrocious nurse.

Forensics: The Real CSI follows a crack team of forensic specialists from Northumbria Police, this BBC 2 series charts the fascinating journey of individual pieces of evidence from the moment they are discovered at the crime scene, through to microscopic analysis at specialist laboratories - and shows the pivotal role the findings have on each investigation.

The Real Rain Man documents Kim Peek, the genius who inspired Dustin Hoffman's character in Rain Man. Classed as a mega-savant, having memorised 12,000 books, including the entire Bible, he also has autism, finding it hard to do day-to-day tasks.

Best 7 TED talks

1. How we read each other's minds, Rebecca Saxe

According to Saxe, a professor of neuroscience at MIT, you don't need tarot cards or ESP to read people's minds. A functioning right temporo-parietal junction will do just fine. In her talk, Saxe explains how this brain region allows humans to be uncannily good at sensing other people's feelings, thoughts, and motivations.

2. The riddle of experience vs. memory, Daniel Kahneman

If you're looking for highly credentialed TED speakers, Kahneman's résumé won't fail to impress. A Nobel Prize-winning psychologist and bestselling author, Kahneman uses his 20 minutes on the TED stage to explain that there are actually two flavours of happiness: the kind we experience in the moment and the kind we experience in our memories. Maximizing your own well-being in life means keeping both in mind.

3. The paradox of choice, Barry Schwartz

More choice is always better, right? Not according to Schwartz, a psychologist who argues that having to decide which of approximately 6,000 brands of similar toothpaste to buy "has made us not freer but more paralyzed, not happier but more dissatisfied.

4. Are we in control of our own decisions?, Dan Ariely

This talk "uses classic visual illusions and Ariely's own counterintuitive (and sometimes shocking) research findings to show how we're not as rational as we think when we make decisions.

5. Flow, the secret to happiness, Mihaly Csikszentmihalyi

In this talk, legendary psychologist Csikszentmihalyi dares to ask one of life's biggest questions: What makes us happy? The answer isn't fame or money, he insists, but flow -- that lost-in-time feeling you get when you focus intensely on work you're good at.

6. The power of vulnerability, Brené Brown

One of the top-five-most-popular TED Talks of all time, this moving account of Brown's own struggles with shame and control weaves together sometimes hilarious personal anecdotes with hard research to convince viewers that forging real connections requires the bravery to be vulnerable.

7. The psychology of evil, Philip Zimbardo

Psychology isn't all happiness and flourishing, of course. The discipline also delves into the darker sides of human nature and what drives us toward unethical or even downright evil behaviour. That's the topic of this talk by 'superstar' Zimbardo in which "he shares insights and graphic unseen photos from the Abu Ghraib trials."

But don't worry, it's not all gloom and doom. He also 'talks about the flip side: how easy it is to be a hero, and how we can rise to the challenge.'

How to take notes

Making effective notes in lessons is an essential skill for A-level Psychology. Practice producing notes using the Cornell System by summarising four of the TED talks you have listened to. Complete your notes in the following format and show them to your teacher.

<table>
<tr><td>TOPIC</td><td>SUBJECT

DATE</td></tr>
<tr><td colspan="2">LESSON FOCUS</td></tr>
<tr><td>QUESTIONS AND CUE-WORDS</td><td>NOTE TAKING

1. Record: During the lecture, use the note-taking column to record the lesson using concise sentences and abbreviations.

2. Questions: As soon after class as possible, write questions in the left hand column based on the notes in the note taking column. Writing questions helps to clarify meanings, reveal relationships, establish continuity, and strengthen memory. Also, the writing of questions sets up a perfect stage for exam studying later.

3. Recite: Cover the note-taking column with a sheet of paper. Then, looking at the questions or cue-words in the question and cue word column only, say aloud, in your own words, the answers to the questions, facts, or ideas indicated by the cue-words.

4. Reflect: Reflect on the material by asking yourself questions, for example: “What’s the significance of these facts? What principle are they based on? How can I apply them? How do they fit in with what I already know? What’s beyond them?

5. Review: Spend at least ten minutes every week reviewing all your previous notes. If you do, you’ll retain a great deal for current use, as well as, for the exam.</td></tr>
<tr><td colspan="2">SUMMARY

After class, use this space at the bottom of each page to summarize the notes on that page.</td></tr>
</table>

Adapted from: How to Study in College 7/e by Walter Pauk, (2001) Houghton Mifflin Company

Topics to research

Memory

Memory helps make individuals who they are. Without the help of memories, someone would struggle to learn new information, form lasting relationships, or function in daily life. Memory allows the brain to encode, store, and retrieve information.

Different areas of the brain affect different aspects of memory. The hippocampus, for instance, is related to spatial memory, which helps the brain map the surrounding world and find its way around a known place. The amygdala, on the other hand, is linked to emotional memory.

https://www.psychologytoday.com/gb/basics/memory

https://www.simplypsychology.org/memory.html

Obedience

Obedience is a form of social influence where an individual acts in response to a direct order from another individual, who is usually an authority figure. It is assumed that without such an order the person would not have acted in this way.

Stanley Milgram (1963) wanted to investigate whether Germans were particularly obedient to authority figures as this was a common explanation for the Nazi killings in World War II.

https://www.simplypsychology.org/obedience.html

https://www.simplypsychology.org/milgram.html

Criminal psychology

Criminal psychology looks at the interaction between psychology and criminology and criminal justice. It is concerned with using psychological research and theories to analyse and improve the criminal justice system. Try the 'You be the Judge' activity by following this link. YOU hear the case, YOU decide the sentence!

http://ybtj.justice.gov.uk/

Issues in mental health

What do we mean by mental health? Which behaviours should be classified as 'sane' or 'insane', and which should be medicated or treated non-biologically? Perhaps we shouldn't treat any mental illness at all and simply accept the diversity of human behaviour? Follow these links to find out more about the explanations and treatments of mental illness.

Drug use and psychotic disorders

Virtual reality as a cure for fear of heights

https://www.bps.org.uk/what-psychology/understanding-psychosis-and-schizophrenia

MOOCs

A Massive Open Online Course (MOOC) is an interactive step-by-step course aimed at reaching an unlimited number of participants worldwide to create a community of lifelong learners. There are many different MOOC providers that cover a huge variety of different subject and topic interests.

Typically a MOOC will involve 2-3 hours study per week for 6 weeks or so. MOOCs are free of charge. All required course materials will be provided for you online, which is also 100% free! Each course is open to anyone with internet access across the world and all you need is your wonderful brain!

Here are a few that you may wish to try.

Forensic psychology

Understanding depression and anxiety

Exploring sport coaching and psychology

Your Summer Tasks!

To make sure you are fully prepared for A-level Psychology, you have some tasks to complete over the summer holiday.

- ✓ Complete your Cornell notes on four of the TED talks you have watched. (Do watch more, but you don't need to make notes on them unless you want to!)
- ✓ Read at least one of the suggested books and summarise what you learned from it.
- ✓ Watch at least two of the suggested films (as long as you can access them).
- ✓ We will be discussing the books and films when you come back in September, so make sure you don't forget what you have read and watched!
- ✓ Complete a psychology-related MOOC – there are plenty to choose from online – it doesn't have to be one of the ones suggested here but it must relate to psychology
- ✓ Start to engage with and enjoy the world of psychology, starting with the ideas in this pack! It's fantastic, and we hope you enjoy your studies!

I look forward to welcoming you all in September.
Mrs Kitching, CPsychol, AFBPsS
Head of Psychology Gildredge House School

Transition Task: Sociology

Subject	Qualification	Examination Board
Sociology	A-Level	OCR
Additional Information:	Sociology is the scientific study of society. Learning about Sociology helps you to think about the ways that people in our world act, think and interact. There are so many different topics that we study, from media to crime to culture and Identity, that there's always something new and interesting to learn.	

Task Overview:	
Research one of the famous sociologists to discover more about them, their views and theories in preparation for a class discussion.	
Success Criteria:	
○ Complete each stage of the research as set out in the task sheet.	
Resources:	
http://www.podology.org.uk/#/education/4556339398 http://www.sociology.uk.net http://www.sociology.org.uk/ http://www.sociologystuff.com http://www.s-cool.co.uk/ http://www.chrisgardner.clara.net/sls1/home.htm http://atschool.eduweb.co.uk/barrycomp/bhs/index.htm http://www.le.ac.uk/education/centres/ATSS/sites.html	
How will the work produced will fit into subsequent work and the specification as a whole?	
It is a starting point to understanding sociological theory which is integral to the whole course. Various perspectives will be studied during the course and this research will from the basis of understanding one of those.	
How should the work should be presented?	
At students' discretion i.e. could be an essay, a mind map, a leaflet, poster or powerpoint presentation.	
Who should you contact if you should require further assistance with the work before the end of term?	Mr Rogers: c.rogers@gildredgehouse.org.uk
Length of time expected to complete tasks:	Up to 4 hours
Submission Requirements:	In person when the course begins.

What equipment will be needed for the subject?
The recommended book for Year 1 of the Sociology course is: OCR A-Level Sociology Book 1 Katherine Roberts et al 2015 Hodder Education ISBN 9781471839481

SOCIOLOGY

Welcome to Sociology! It's a fantastic course that will help to open your mind to new ways of thinking about the ways that we live. Sociology is the scientific study of society. Learning about Sociology helps you to think about the ways that people in our world act, think and interact. There are so many different topics that we study, from media to crime to culture and Identity, that there's always something new and interesting to learn.

This transition task will introduce you to one of the major Sociologists that you will study throughout the course. I hope you enjoy finding out about them!

You are expected to purchase the course text book in advance of starting the course or as soon as possible upon commencing the course (it doesn't need to be a new book!).

The recommended book for Year 1 of the Sociology course is:

OCR A-Level Sociology Book 1
Katherine Roberts et al 2015
Hodder Education
ISBN 9781471839481

<u>TRANSITION TASK</u>

Choose one of the following famous Sociologists to research:

- Emile Durkheim (a 'founding father' of Sociology, functionalist)
- Karl Marx (a 'founding father' of Sociology, the original Marxist!)
- Talcott Parsons (Functionalist)
- Germaine Greer (Feminist)

You should find out about:

- Two or three of their major theories/ideas.
- Famous books they wrote and a summary of their content.
- Information about their life - country of origin, birth/death, interesting events.
- Which Sociological perspective they held (given in brackets beside their name) and what that means.
- Some famous quotes -discuss what each quotation suggests about the nature of society.

Record what you find in a method of your choice – you may prefer to make notes, produce a leaflet or even design a short PowerPoint. You will find these notes very useful when you learn about your chosen Sociologist throughout the course.

Transition Task: Sport BTEC

Subject	Qualification	Examination Board
BTEC Sport	Level 3 National Extended Diploma	Edexcel
Additional Information:		

Task Overview:

You will select one team and one individual sport.

You need to provide a report that can explain how players comply with the rules/laws and regulations in each sport.

The report you provide should include;

- Rules/laws as regulated by the national or international governing body for the two selected sports
- Competition rules/laws and regulations
- Unwritten rules and/or etiquette specific to sport
- Regulations for sports under competition rules

You must also include situations where the officials, who referee the sport, have applied rules/laws both legally (correctly), and illegally (incorrectly).

You should assess the effectiveness of the officials in their application of the rules and regulations, and discuss their roles and responsibilities when applying the rules/laws and regulations.

It is important to consider the impact of the decisions made by the officials, giving your own view of how the decisions have affected the game.

Success Criteria:

Summarise how participants comply with the rules/laws and regulations in individual and team sports.

Assess how participants comply with the rules/laws and regulations and the impact on individual and team sport.

Resources:

Textbooks

Edwards J, Badminton: Technique, Tactics, Training (Crowood Sports Guides), The Crowood Press Ltd, 1997 ISBN 9781861260277

Griffin LL, Mitchell SA and Oslin JL, Teaching Sport Concepts and Skills: A Tactical Games Approach, Leeds: Human Kinetics, 1997 ISBN 0880114789

Jones S, Rugby: Passing, Catching, Kicking (Know the Game Skills), London: A&C Black, 2009 ISBN 9781408114100

Parkhurst A, Tennis: A Complete Guide to Tactics and Training (Sporting Skills), First Stone Publishing, 2005 ISBN 9781904439479

Redknapp H, Soccer Skills and Tactics, Paragon, 2002 ISBN 9780752590448
Volleyball England, Volleyball (Know the Game) (Third Edition), London: A&C Black, 2006 ISBN 071367900X

Woodlands J, The Netball Handbook, Human Kinetics Europe Ltd, 2006 ISBN 9780736062657

Videos

A range of videos involving elite sports competition are suggested for this assignment;

Advanced Badminton (DVD) (NTSC) – United States National Champion, Kevin Han, demonstrates how to become a top-level badminton player.

Basketball Power Forward Skills and Drills – Optimize Your Skills – Become a Better Player – Techniques and skills to achieve high-level basketball performance.

Cilene Drewnick: Building an All-Around Volleyball Athlete with Brazilian Training Methods (DVD) – Training techniques to be successful in volleyball.

FA Cup Final: 2006 – The Gerrard Final (DVD) – Features the classic 2006 FA Cup final between Liverpool and West Ham United.

London 2012: Gymnastics – Going for the Gold (DVD) – Features performances from the 2012 London Olympics.

Rugby World Cup 2015: The Final (DVD) – Coverage of the 2015 Rugby Union World Cup final between Australia and New Zealand.

The Australian Open Tennis Championships 2012: Men's Final (Novak Djokovic V Rafael Nadal) (DVD) – Features the classic men's final from 2012.

The Greatest moments in track and field – Sprint and Field Events (DVD) – Features a range of world-class performances in track and field athletics.

Websites

www.badmintonengland.co.uk – Badminton Association of England
www.britishcycling.org.uk – British Cycling
www.british-gymnastics.org.uk – British Gymnastics
www.britishswimming.org – Amateur Swimming Association
www.britishvolleyball.org – British Volleyball Association
www.englandbasketball.co.uk – English Basketball Association
www.lta.org.uk – The Lawn Tennis Association
www.olympics.org.uk – The British Olympic Association
www.rfu.com – The Rugby Football Union
www.sportsofficialsuk.com – Sports Officials UK
www.thefa.com – The Football Association
www.ukathletics.net – UK Athletics
www.uksport.gov.uk – UK Sport

How will the work produced will fit into subsequent work and the specification as a whole?

This task fits directly into Unit 7 Practical Performance in Sport assessment criteria.

The work you produce will contribute to your overall performance within this unit.

How should the work should be presented?

Word Document
Calibri (Body)
Size 12 Font

Who should you contact if you should require further assistance with the work before the end of term?	t.addems@gildredgehouse.org.uk f.ocallaghan@gildredgehouse.org.uk

Length of time expected to complete tasks:	10hours
Submission Requirements:	Submit on first day of year 12
What equipment will be needed for the subject?	
Large Lever arch file, plastic wallets, USB memory stick, A4 Notepad, Pens, Sports Clothing	